In general, the coursework problem was answered well. Although the algebra in question 3 was lengthy, most people were applying the correct method and therefore obtained the majority of the marks. Here are the most common mistakes:

- **Not following the summation convention when answering 1(b).**
  Setting $\tau_{ij} = 0$ in the thermoelastic constitutive law and rearranging gives

$$2\mu\, e_{ij} = \left[(3\lambda + 2\mu)\alpha(T - T_0) - \lambda e_{kk}\right] \delta_{ij}. \quad (1)$$

  The summation convention means that the term $e_{kk} = e_{11} + e_{22} + e_{33}$. Hence, setting $i = j$ in the above equation gives

$$2\mu\, e_{ii} = \left[(3\lambda + 2\mu)\alpha(T - T_0) - \lambda e_{kk}\right] \delta_{ii},$$

  where $\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33} = 3$ and rearranging gives

$$(2\mu + 3\lambda)e_{kk} = 3(3\lambda + 2\mu)\alpha(T - T_0),$$

$$\Rightarrow \quad e_{kk} = e_{11} + e_{22} + e_{33} = 3\alpha(T - T_0);$$

  this does not mean that each individual diagonal term is equal to $3\alpha(T - T_0)$ as some people wrote.

  If you wish to indicate that you are **not** summing the indices you should use brackets

$$2\mu\, e_{(i)(i)} = \left[(3\lambda + 2\mu)\alpha(T - T_0) - \lambda e_{kk}\right], \delta_{(i)(i)},$$

  and $\delta_{(i)(i)} = 1$.

- **Fudging the derivation of the ∇T term in the modified Navier–Lamé equations**
  Following the derivation in the lecture notes, the method was to take the derivative of the thermoelastic constitutive law with respect to x_j so

$$\frac{\partial \tau_{ij}}{\partial x_j} = \tau_{ij,j} = \frac{\partial}{\partial x_j}\left(\lambda\delta_{ij}e_{kk} + 2\mu e_{ij} - (3\lambda + 2\mu)\alpha(T - T_0)\,\delta_{ij}\right).$$

  The only new term is the last term which is

$$\frac{\partial}{\partial x_j}\left[(3\lambda + 2\mu)\alpha(T - T_0)\delta_{ij}\right].$$

  Everything is a constant apart from the temperature T, which is a function of position, so we have

$$\frac{\partial}{\partial x_j}\left[(3\lambda + 2\mu)\alpha(T - T_0)\delta_{ij}\right] = (3\lambda + 2\mu)\alpha\delta_{ij}\,\frac{\partial T}{\partial x_j} = (3\lambda + 2\mu)\alpha\delta_{ij}T_{,j}.$$

  Using the index-switching property of the Kronecker delta gives

$$(3\lambda + 2\mu)\alpha T_{,i},$$

  which is indeed $\boldsymbol{\nabla} T$ provided that i is the free index in the vector equation. A few people had equations that did not make sense because the free indices did not match on all terms in the equation.

  There were also people who mixed index and dyadic notation in the same equation which really makes no sense at all. I suspect that this was inspired by "pattern matching" to reach the desired solution.

- **Using the wrong formula for $\nabla^2 T$**
  In cylindrical polars for a function of r only, $T(r)$, then

$$\nabla^2 T = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T}{\mathrm{d}r}\right) \quad \textbf{NOT} \quad \frac{\mathrm{d}^2 T}{\mathrm{d}r^2}.$$

  If the working was correctly carried through after this mistake no further marks were deducted.

- **Not stating all assumptions**
These were easy marks that many people lost. The assumptions in part 3(b) are that

$$\boldsymbol{u} = u_r(r)\hat{\boldsymbol{r}},$$

  - The displacement is **only** in the radial direction.
  - The displacement is a function **only** of r.
  - There are no body forces.

  Under these assumptions, it **follows** that curl$\boldsymbol{u} = \boldsymbol{0}$. The fact that the curl of the displacement field is zero is not an (independent) assumption.

- **Poor graphs**

  - **Plotting the wrong range**
  The solution is only valid within the pipe walls $1 \leq r \leq 2$. It is meaningless to plot the solutions outside that range. I did not deduct marks for this, unless the range of interest was too small to see the significant features within it.
  - **No labels on graph axes**
  You should always label your graphs. If you are sketching then you should label key values and you should certainly label the axes. I had hoped that this would have been drummed into you by now. Mind you, I still get academic papers to review that don't do this, such is the way of the world.
  - **Graphs too small, or large but fuzzy**
  The purpose of a graph is to convey information. If it's too small, then it's almost impossible to read.

- **No physical explanation**
Many people did not attempt a physical explanation which, of course, lost marks. A physical explanation is more than simply describing the shape of the graph.

  The explanation for question 1(b) was straightforward. It was a uniform dilation with volume change given by $3\alpha(T - T_0)$.

  The complete explanation for 3(b) is quite subtle, but some people got very close. Essentially, the pipe expands, but it does so non-uniformly because the temperature is not constant throughout the pipe. It expands the most at the inner wall, where the temperature is greatest and actually contracts slightly at the outer wall where there is no thermal forcing. In order for the pipe to expand at the inner wall material originally at $r > 1$ must be displaced further than that originally at $r = 1$, hence u_r increases with distance from the inner wall. One way to see this from the maths is to note that the volumetric expansion is $e_{rr} + e_{\theta\theta} + e_{zz} = \operatorname{div} \boldsymbol{u} = \partial u_r/\partial r + u_r/r$, which gives a logarithmic decrease in expansion throughout the pipe (consistent with the logarithmic decrease in temperature).

  The reason for the maximum in the displacement just before the outer wall is that no stress at the outer wall implies that

$$(\lambda + 2\mu)\partial u_r/\partial r + \lambda u_r/r = 0 \quad \Rightarrow \quad \partial u_r/\partial r = -u_r/6.$$

  Thus, the gradient must become negative **before** the outer wall giving a turning point inside the pipe.

  Marks were awarded for any reasonable statement that mentioned **expansion** of the elastic wall.