Overall, the vast majority of students attempted every question. The general difficulties were problems with index notation; not understanding how to compute the deformed position from the undeformed position and displacement; and not being able to spot sensible functional forms for solutions.

A1 On the whole this question was well answered by the vast majority of students.

(i) That said, this part caused a surprising amount of trouble. A common mistake was to assume that $\epsilon = 1$, it should be small; many people also forgot that the deformed position is $\boldsymbol{R} = \boldsymbol{r} + \boldsymbol{u}$,the undeformed position plus the displacement (not just the displacement); finally, there were a non-trivial number that got the x_1 and x_2 axes mixed up.

(ii) This was generally fine apart from a few silly algebraic slips and missing factors of a half. Remember that the strain tensor is symmetric (by construction) and the rotation tensor is anti-symmetric.

(iii) The main problems were remembering the quadratic formula and the formula for the determinant of a 2×2 matrix. As always, a few people forgot the all important factor of ϵ. The principal strains are $\frac{9}{2}\epsilon$ and $-\frac{1}{2}\epsilon$ with principal axes $(3, 1)$ and $(1, -3)$.

A2 In general this question revealed weaknesses in understanding index notation and summation when an index is repeated.

(i) It is straightforward to see that for the given stress field, when $i \neq j$, the strain is also zero. The problem is then to work out the diagonal terms, which requires use of the relation $\tau_{kk} = (3\lambda+2\mu)e_{kk}$. Most people remembered or derived the relation, but did not necessarily use the fact that in **this** case you know that $\tau_{kk} = 5$. It then follows that

$$e_{(i)(i)} = \frac{(3\lambda + 2\mu)\tau_{(i)(i)} - 5\lambda}{(3\lambda + 2\mu)\, 2\mu},$$

where the brackets around the index indicates that no summation is taken.

(ii) Those that had problems with part (i) made this harder than it needed to be. When $\lambda = \mu = 1$, then $e_{11} = e_{33} = \frac{1}{2}$ and $e_{22} = 0$. The displacement field can be recovered by direct integration of these diagonal terms, but a large number of people forgot to check that the result also satisfies the equations from the off-diagonal entries of the strain tensor. Many also assumed that the arbitrary functions of two coordinates were constant without justification and a few forgot that the problem is three-dimensional (not two-dimensional).

A3 A straightforward question that was well answered. A few forgot to use the fact that $\nabla^4\phi = 0$ and there were some sign errors in the normal. The body is in the domain $x \leq 0$, so the outer unit normal at $x = 0$ is in the positive x-direction $(1, 0)$.

A4 (i) The easiest explanation here is to state the assumption that $\boldsymbol{u} = u_r(r)\, \boldsymbol{e}_r$ and then solve the simplified Navier–Lamé equations to find

$$u_r = Ar + \frac{K}{r^2}.$$

The displacement must remain finite as $r \to \infty$, which means that $A = 0$ and hence the result. Note that a large number of people did not correctly cancel both the $\sin\theta$ terms in the divergence and then tried to argue that $\sin\theta$ is constant to get the correct answer ... this is not true!

(ii) This was answered well by just about everybody.

(iii) A number of silly algebraic slips here, most commonly forgetting the factor of 2μ, but there were also some sign errors.

$$\tau_{rr} = \lambda \mathrm{div}\, \boldsymbol{u} + 2\mu\frac{\partial u_r}{\partial r} = -4\mu\frac{K}{r^3},$$

which when combined with the boundary condition $\tau_{rr}\big|_{r=a} = -p_0$, gives $K = p_0a^3/(4\mu)$.

*** A large number of people didn't read the end of the question, so did not give the deformed radius of the inclusion. Many that did just stated the displacement, which is the same conceptual error as in A1(i). The deformed radius is the undeformed radius plus the displacement at the edge of the inclusion:

$$R = a + u(a) = a + \frac{p_0a^3}{4\mu}\,\frac{1}{a^2} = a\{1 + p_0/(4\mu)\}.$$

B5 (i) This part of the question was very similar to known examples and previous exam questions, so, not surprisingly, it was answered well. Almost everybody got to the form $u_r = Ar + B/r$, but a worryingly large number did not calculate the coefficients A and B. The question asks you to find the displacement field which means that you need to know A and B in terms of the physical variables.

(ii) Nearly everybody correctly computed the stress tensor and most had a reasonable attempt at explaining why $\tau_{zz} \neq 0$. The "best" answer would be to say that the body is in a state of plane strain and therefore requires an out-of-plane stress (in general). This is "the Poisson effect", which was another acceptable answer. Those that simply said that it was a three-dimensional problem were also judged correct.

(iii) This turned out to be the hardest question on the paper, which revealed a lack of understanding of index notation and summation convention. The stress tensor is diagonal, say $\tau_{rr} = R$, $\tau_{\theta\theta} = \Theta$ and $\tau_{zz} = Z$. Consider the expression

$$\frac{1}{2}\left(\tau_{ij}\tau_{ij} - \frac{1}{3}\tau_{ii}\tau_{jj}\right) > 4;$$

in order for it to make sense all terms must be scalars, which means that we are summing over the repeated indices. The only non-zero terms are the diagonal terms so

$$\tau_{ij}\tau_{ij} = \tau_{rr}^2 + \tau_{\theta\theta}^2 + \tau_{zz}^2 = R^2 + \Theta^2 + Z^2.$$

and

$$\tau_{ii} = \tau_{jj} = \tau_{rr} + \tau_{\theta\theta} + \tau_{zz} = R + \Theta + Z,$$

so

$$\tau_{ii}\tau_{jj} = (R + \Theta + Z)^2 \neq \tau_{ij}\tau_{ij}.$$

The two expressions are not zero and you do have to take them both into account. If you follow through the algebra, you will find that

$$p_a > \frac{2(b^2 - a^2)r^2}{a^2 b^2},$$

which has the maximum value when $r = a$ and $p_{a\max} = 2(1 - a^2/b^2)$. (This last bit is very hard, so don't feel too bad.)

B6 This question caused a great deal of confusion, but it is closely related to the periodic load question in the lecture notes. The only difference is that we have a sum of periodic loads. The sensible functional form to try is

$$\Phi_n(x, y) = \cos(n\pi x) g_n(y),$$

which includes the correct x-dependence. Substituting this into the biharmonic equation gives a fourth-order constant coefficient ODE for $g_n(y)$, which has to be exponential. (It's actually almost the same ODE as in the example in lectures.) In order for there to be decay as $y \to \infty$, we must have the form

$$g_n(y) = (A_n + B_n y)\mathrm{e}^{-n\pi y}.$$

Once you have that, it's straightforward to apply the boundary conditions and find that $A_n = -\frac{1}{n^3\pi^2}$ and $B_n = -\frac{1}{n^2\pi}$. The tricky part here was coming up with the correct functional form. Most people did get the $\cos(n\pi x)$, but then couldn't find the form for $g_n(y)$.