Overall, the vast majority of students attempted every question. The general difficulties were a lack of confidence in differentiation and solving standard ODEs, index notation and the derivation/imposition of boundary conditions.

A1 (i) This was attempted well by the vast majority. There were a some errors in the differentiation and a surprising number of students had to multiply out the bracket $(x_2-1/3)^3$ in order to differentiate with respect to x_2, which cost valuable time. Relatively few people correctly remembered the definition of a homogeneous deformation. This deformation is NOT homogeneous because it varies with position (is a function of x_i).

(ii) As in previous years, most got to the correct eigenproblem:

$$\begin{pmatrix} \epsilon\, x_1(1-x_1) & 0 \\ 0 & \epsilon\, \frac{1}{4}-(x_2-\frac{1}{3})^3 \end{pmatrix} \boldsymbol{v} = \lambda \boldsymbol{v},$$

but did not know (or realise) that the eigenvalues of a diagonal matrix are simply the diagonal entries and that the eigenvectors are then simply the unit vectors in the x_1 and x_2 directions. There were also a few people who forgot the factor of ϵ.

(iii) This part well the least well attempted. The first principal strain depends only on x_1 and the second only on x_2 so the maximisations can be done independently $\max_{x_1\in[0,1]}[x_1(1-x_1)] = 1/4$ at $x_1 = 1/2$ (use A-level calculus or your knowledge of quadratic functions);
$\max_{x_2\in[0,1]}\left[1/4-(x_2-1/3)^2\right] = 1/4$ at $x_2 = 1/3$ (observation or calculus). Thus the maximum sum of strains is $\epsilon/2$ at $(1/2, 1/3)$.

A2 (i) This caused a surprising number of problems. By far the easiest approach is to calculate $\nabla^2\phi = 2(T-N)e^{\alpha y}\sin(\alpha x) = \Psi$, say. It is then easy to show that

$$\nabla^4\phi = \nabla^2(\nabla^2\phi) = \nabla^2\Psi = -\alpha^2\Psi + \alpha^2\Psi = 0,$$

so that ϕ is a valid Airy stress function.

(ii) The most common problem here was a failure to work out surface traction in terms of the stresses using the condition $t_i = \tau_{ij}n_j$. The outer unit normal is $\boldsymbol{n} = (0,1)$, so the applied traction must be $t_x = t_1 = \tau_{12}$, $t_y = t_2 = \tau_{22}$. This means that you only need to work out τ_{12} and τ_{22} and find their values at $y = 0$. Remembering to substitute $y = 0$ into the answer was another common problem. The correct answer should have been $t_1 = -T\cos\alpha x$ and $t_2 = -N\sin\alpha x$. Thus, the surface is loaded harmonically with maximum tangential amplitude T and maximum normal amplitude N.

A3 This question was a little tricky for some. A few people just wrote down and worked with plane strain (as in the lecture notes) instead of plane stress, but the most common error in part (i) was not realising that in plane stress τ_{ij} only has non-zero components in the x-y plane. Thus the stress tensor has the form

$$\begin{pmatrix} \tau_{11} & \tau_{12} & 0 \\ \tau_{12} & \tau_{22} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Part (ii) was answered well, although a common mistake was to forget the summation convention and put $\delta_{ii} = 1$ (in three-dimensional space $\delta_{ii} = \delta_{11} + \delta_{22} + \delta_{33} = 3$).

For part (iii), you can use the expression derived in (ii) to show that $e_{33} \neq 0$, which shows that the body is not necessarily in a state of plane strain.

A4 In general, this was not well answered in any part, perhaps due to a lack of understanding of spherical polars and circular motion? For part (i), the rotation gives displacement in the angular $\hat{\boldsymbol{\phi}}$ direction only. The distance travelled by a point on the surface is simply $R\Omega$, where R is the distance from the axis of rotation, $R = a\sin\theta$; hence the result.

In part (ii) the vast majority of people just wrote down that curl $\boldsymbol{u} = \boldsymbol{0}$ without thinking or checking using the formulæ given. If one simply substitutes the form of the displacement into the expressions given you will find that div $\boldsymbol{u} = \boldsymbol{0}$ and that the curl curl term leads to the required ODE.

Actually solving the ODE in (iii) was another problem. The ODE is in Euler form, which means that you can pose a solution of the form $f(r) = r^n$, which leads to a quadratic characteristic equation and the solution $f(r) = Ar + Br^{-2}$.

Despite all this, in (iv) most people correctly deduced that there is no stress because the displacement is a rigid-body rotation.

B5 This was probably the most consistently answered question in the paper. Most people got to the correct displacement field at the end of part (ii). Most that attempted part (iii) had the right idea, but some got lost in the algebra. You should find that the constant $C \to 1$ as $a \to b$.

B6 A few people had failed to read the bottom of the question and assumed that the biharmonic equation in polar coordinates is simply the Cartesian version with x and y exchanged for r and θ. Apart from that parts (i) and (ii) were answered well. Once again, the quickest way through the algebra is to calculate $\nabla^2\Phi$ and then take the Laplacian again.

Correctly stating the outer unit normal was the common problem in part (iii). It was in the negative θ direction on $\theta = 0$ and the positive θ direction on $\theta = \alpha$, which means that the stress boundary conditions were

$$\tau_{r\theta} = 0, \quad \tau_{\theta\theta} = -12r^2 \quad \text{on } \theta = 0.$$

$$\tau_{r\theta} = 0, \quad \tau_{\theta\theta} = 0 \quad \text{on } \theta = \alpha.$$

Those that got that far managed a good attempt at solving the simultaneous equations; the most common problem being starting from the wrong equations because the boundary conditions weren't quite correct.