

- A1 Apart from a few arithmetic errors, nearly everybody got (i) and (ii), but part (iii) proved to be a problem. The majority got to the correct eigenproblem:

$$\begin{pmatrix} 2\epsilon(x_2 - 1) & 0 \\ 0 & \epsilon x_2 \end{pmatrix} \mathbf{v} = \lambda \mathbf{v},$$

but did not know (or realise) that the eigenvalues of a diagonal matrix are simply the diagonal entries. After writing down the characteristic equation

$$(\lambda - 2\epsilon(x_2 - 1))(\lambda - \epsilon x_2) = 0,$$

you **have** the roots

$$\lambda = 2\epsilon(x_2 - 1) \quad \text{and} \quad \lambda = \epsilon x_2.$$

Those that multiplied out and used the quadratic formula wasted a lot of time. The eigenvectors are then simply the unit vectors in the x_1 and x_2 directions.

- A2 The majority started to find the displacement field and showed a contradiction, but the simplest way into part (i) was to show that the strain compatibility equations were not satisfied by picking, say, $i = j = 1$ and $k = l = 2$.

Part (ii) was generally answered well by those that attempted it. The off-diagonal term $e_{12} = 0$ could be used to show that $f(x_1, x_2) = -3x_2x_1^2$.

- A3 This deceptively simple question showed that many people had memorised, rather than understood, how to find the strain as a function of stress. The expression for e_{kk} can simply be found by setting $i = j$ in the equation (1)

$$\begin{aligned} \tau_{jj} &= \tau_{jj}^0 + 3\lambda e_{kk} + 2\mu e_{jj} = \tau_{jj}^0 + (3\lambda + 2\mu)e_{jj} \\ \Rightarrow e_{jj} &= \frac{\tau_{jj} - \tau_{jj}^0}{3\lambda + 2\mu}, \end{aligned}$$

after relabelling the dummy indices. This result should then be used to replace e_{kk} in the expression for e_{ij} . Many left e_{kk} in the expression for e_{ij} , which is not incorrect, but not helpful. Once the expression for e_{ij} as a function of τ_{ij} is established the rest of the question is straightforward.

- A4 This was generally well answered, the main error was not giving the correct **displacement** boundary condition $u(a) = b - a$, but trying to derive a **stress** condition.

- B5 The vast majority of people got this question largely correct.

- B6 Too many people did not read the bottom of the question and assumed that the biharmonic equation in polar coordinates is simply the Cartesian version with x and y exchanged for r and θ . Apart from that parts (i), (ii) and (iii) were answered well. Parts (iv) and (v) were not so well answered, but were the hardest questions on the paper. The main problem in part (iv) was expressing the traction vector on the semi-circle in Cartesian coordinates

$$\mathbf{t} = \tau_{rr} \mathbf{n}_r = - \begin{pmatrix} \tau_{rr} \sin \theta \\ \tau_{rr} \cos \theta \end{pmatrix}.$$

In part (v), the methods are standard, but the off-diagonal term $e_{r\theta}$ leads to two non-trivial ODEs to solve after separating variables, which seemed to fox people.