MATH35021: EXAMPLE SHEET[1] IV

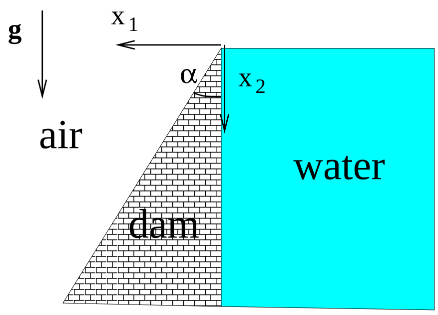
1.) A cylinder which occupies the region $x_1^2 + x_2^2 \leq R^2$ and $-L \leq x_3 \leq 0$ is in static equilibrium. The stress tensor in the body is given by $\tau_{11} = \tau_{12} = \tau_{22} = 0$ and $\tau_{13} = -ax_2,\ \tau_{23} = ax_1,\ \tau_{33} = bx_3$, where a and b are known (small) constants.

a) Use the equation of equilibrium to determine the body force (force per unit volume acting inside the body) F_i.

b) Find the stresses/tractions on the three surfaces of the cylinder (top: $x_3 = 0$, curved sides: $x_1^2 + x_2^2 = R^2$, bottom: $x_3 = -L$).

c) Determine the resultant forces on all three surfaces and check the overall equilibrium of forces (the body is in equilibrium iff the sum of the resultant body force and the applied forces on the surfaces vanishes). Think carefully about the sign of the relevant stress components at the bottom of the cylinder and the directions of the resultant surface force.

2.) An angular dam separates water (on the right) from air (on the left). The vertical right edge of the dam is located at $x_1 = 0$. The densities of water and the dam are ρ_w and ρ_0, respectively.



The stress field in the dam has the form:

$$\tau_{11} = ax_1 + bx_2; \quad \tau_{22} = cx_1 + dx_2 - \rho_0 g x_2; \quad \text{and} \quad \tau_{12} = \tau_{21} = -dx_1 - ax_2,$$

where a, b, c and d are constants. (The problem is two-dimensional; we can assume that $\tau_{i3} = \tau_{3i} = 0$). Determine a, b, c and d from the boundary conditions. [Hints: The hydrostatic pressure on the right edge is $p = \rho_w g x_2$; the left edge is exposed to air (at pressure zero)].

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