MATH35021: EXAMPLE SHEET[1] III

1.) The strains $e_{ij}(\mathbf{r})$ are given throughout a body. Show that the corresponding displacement field $u_i(\mathbf{r})$ is only determined to within a rigid-body displacement. Use the following two steps: (i) Show that if two displacement fields u_i^1 and u_i^2 correspond to the same strain field, then the strain field corresponding to $u_i^\Delta = u_i^1 - u_i^2$ is $e_{ij}^\Delta = 0$. (ii) Given (i), we need to show that $e_{ij} = 0$ corresponds to a rigid body displacement. [Hint: Use the fact that $e_{ij} = 0$ implies $\omega_{ij} = const.$ (see question 3 on the previous example sheet)].

2.) Find a displacement field corresponding to the strains

$$e_{ij} = \epsilon(a + bx_3)\delta_{ij}$$

where $\epsilon \ll 1, a$ and b are constants. Note that we're not interested in the most general solution, so try dropping constants of integration whereever possible.

3.) Consider the infinitesimal tetrahedron used in the derivation of the stress tensor and show that

$$\mathbf{n}_i ds_i + \mathbf{n} ds = 0$$

where the $\mathbf{n}_i$ are the outside unit normal vectors on the three faces on which $x_i = const.$, the ds_i are their areas, $\mathbf{n}$ is the outside unit normal vector on the fourth (general) face and ds is its area.

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