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Formation-containment tracking and scaling for multiple quadcopters with an application to choke-point navigation

Yu-Hsiang Su and Alexander Lanzon

Abstract— **This paper investigates the cooperative control problem of choke-point navigation for multiple quadcopters when only their subgroup is equipped with obstacle detecting sensors. We define a quadcopter as a leader if it is equipped with an obstacle detecting sensor; otherwise, it is a follower. In addition, we introduce a virtual leader agent to create the group motion. First, we apply the leader-follower approach and propose a formation-containment tracking controller for multiple quadcopters to track the time-varying velocity of the virtual leader agent. At the same time, the leader quadcopters form the prescribed formation while the follower quadcopters converge inside a safe region, which is the convex hull spanned by those leaders. Then, we introduce a scaling vector into the displacement-based formation constraints. When the leader quadcopters identify the choke-point via their obstacle detecting sensors, they update the scaling variable to adjust the size of the formation (i.e. the safe region) and guide all quadcopters to safely pass through the choke-point. The proposed cooperative controllers are distributed because each quadcopter's control command only relies on the information states from its neighbours. Finally, two autonomous flight experiments, including formation-containment tracking and choke-point navigation, are provided to validate the effectiveness of the proposed cooperative control laws.**

I. Introduction

Over the past two decades, cooperative control of multi-robot systems (MRS) has gained significant research attention in both the control and robotics communities due to its various potential applications in civilian and military areas [1]. For example, mapping and localization [2], target enclosing and surveillance [3], search and rescue [4], smart farming and agriculture [5], cooperative transportation [6], [7], and vehicle platoons [8]. The study of cooperative control of MRS aims to design the associated cooperative control law depending on the robot dynamics and the interaction topology among robots to achieve the group objective. Compared with a single complex robot, having multiple simple robots to work cooperatively to solve the group task provides robustness to individual faults, improves operational effectiveness, reduces the cost, etc [9]. In this work, we focus on the cooperative control of multiple quadcopters.

Formation control and containment control are two of the most fundamental cooperative control techniques for controlling MRS. For the formation control problem, all robots are required to achieve and maintain a prescribed geometric shape [10]. In [11], the formation control problem for quadcopters was studied and a trajectory planning algorithm for aggressive formation flight was proposed. In [12], a consensus-based controller was proposed to solve the time-varying formation problem for multiple quadcopters. A similar consensus-based approach was applied in [13] to solve the formation problem for quadcopters flying in a constrained environment. [14] proposed a decentralized hybrid controller which achieved formation flying, rotation, tracking and inter-agent collision avoidance properties for multiple quadcopters. For the containment control problem, the states of the follower robots are driven into a convex hull spanned by multiple leader robots [15]. The containment control problem for agents with double-integrator dynamics subjected with both stationary and dynamic leaders were solved and the results were validated on mobile robots [15]. In [16], the containment control problem for networked quadcopters was studied and the simulation results were provided.

Based on formation control and containment control, the formation-containment control problem has arisen, which has advantages in some particular applications. For example, consider a group of autonomous quadcopters flying in an unknown environment, where only the leader quadcopters have sensors to detect obstacles. Specifically, the leader quadcopters will be deployed to form a safe region for those follower quadcopters which do not have obstacle detecting sensors and guide all quadcopters flying in an unknown environment. At the same time, the follower quadcopters will converge inside the safe region (i.e. the convex hull spanned by those leaders) during the flight. The formation-containment control problem for multiple quadcopters was first investigated in [17]. Recently, in [18], a novel control framework was proposed to solve the clustering, formation tracking and containment problems for MRS and validated on a group of ground mobile robots. However, other coordinated behaviours for MRS under formation-containment control, such as formation scaling, choke-point navigation, and obstacle avoidance, are still open topics that require further research.

Motivated by the aforementioned limitations and challenges, we aim to develop formation-containment tracking and scaling control laws for multiple quadcopters. In addition, we demonstrate that our proposed cooperative control algorithm solves the problem of choke-point navigation for multiple quadcopters when only their subgroup is equipped with obstacle detecting sensors. Choke-points are a type of environmental obstacle such as windows, doors, or narrow

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canyons where a group of quadcopters must adjust its formation to navigate through it [19]. In this paper, if a quadcopter is equipped with an obstacle detecting sensor, we define it as a leader; otherwise, we define it as a follower. First, we apply the formation-containment control technique on multiple quadcopters since only their subgroup is equipped with obstacle detecting sensors. Leaders are required to achieve the prescribed formation while followers are expected to converge inside the safe region, which is the convex hull spanned by those leaders. To manoeuvre the whole group of quadcopters, we introduce a virtual leader agent with exogenous time-varying velocity. A formation-containment tracking controller is then proposed, so all quadcopters track the time-varying velocity of the virtual leader agent while maintaining the formation-containment behaviour. Next, we introduce a set of scaling variables in the displacement-based formation constraints to create the formation scaling property. When the leader quadcopters detect the choke-point, they update the scaling variables and adjust the size of the formation (i.e. the safe region). Finally, we integrate the obstacle detecting and formation scaling functionalities with the formation-containment tracking controller and propose a novel cooperative control algorithm that solves the choke-point navigation problem. Furthermore, we show that under the proposed control algorithm, all quadcopters safely navigate through the choke-point when only their subgroup is equipped with obstacle detecting sensors.

The contribution of this work is twofold. First, in contrast to the existing studies of cooperative control of multiple quadcopters (e.g. [11], [12], [13], [14], [16], [17], [19]), the proposed cooperative control laws in this paper solve the formation-containment tracking and scaling problems simultaneously for multiple quadcopters. Second, we demonstrate two autonomous flight experiments. One is the formation-containment tracking experiment where the quadcopters achieve the formation-containment while tracking the time-varying velocity of the virtual leader agent. The other is the choke-point navigation experiment where a team of quadcopters safely navigate through the choke-point when only their subgroup is equipped with obstacle detecting sensors. To the best of our knowledge, this work is the first one to integrate obstacle detecting and formation scaling properties into the control algorithm for cooperative control of multiple quadcopters.

II. Preliminaries and Problem Formulation

A. Graph theory

A weighted directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$ is used to describe the interaction topology among each quadcopter, where $\mathcal{V} = \{1, ..., N\}$ is the node set, $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the edge set and $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ is the associated adjacency matrix respectively. The edge $(j,\ i) \in \mathcal{E}$ denotes that the information passes from node i to node j, which means that node i is a neighbour of node j. The set of all neighbours of node i is denoted by $\mathcal{N}_i$. a_{ij} represents the weight of edge $(j,\ i)$. The adjacency matrix $\mathcal{A}$ is defined as $a_{ii} = 0$, $a_{ij} > 0$ if edge $(j,\ i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N \times N}$ associated with $\mathcal{G}$ is defined by $l_{ii} = \sum_{j \neq i} a_{ij}$ and $l_{ij} = -a_{ij}$ when $i \neq j$. A directed graph is said to have or contain a directed spanning tree if the graph has at least one node with directed paths to all other nodes [9].

Lemma 1 ([9]): The Laplacian matrix $\mathcal{L}$ of a directed graph $\mathcal{G}$ has a simple zero eigenvalue with an associated eigenvector $1_n = [1, 1, ..., 1]^T$ and all of the other nonzero eigenvalues are in the open right half plane if and only if the directed graph has a directed spanning tree.

B. Problem formulation

Consider a group of N quadcopters, which consists of M followers and $N-M$ leaders. Denote that $F = \{1, 2, ..., M\}$ is the follower set and $E = \{M+1, M+2, ..., N\}$ is the leader set. Let a single-integrator model describes the dynamics of each quadcopter after local velocity tracking controllers were applied. That is

$$\dot{p}_i = u_i, \tag{1}$$

where $p_i \in \mathbb{R}^3$ and $u_i \in \mathbb{R}^3$ are the position and the associated control input of the i^{th} quadcopter respectively. In addition, we introduce a virtual leader agent labelled as $N+1$. The virtual leader agent is injected with an exogenous velocity which is independent of all other quadcopters. Its dynamics are described by

$$\dot{p}_{N+1} = v_{ref}, \tag{2}$$

where $p_{N+1} \in \mathbb{R}^3$ and $v_{ref} \in \mathbb{R}^3$ are the position and the velocity reference of the virtual leader agent respectively. Let $\delta_{ij} \in \mathbb{R}^3$ denote the displacement constraint between the i^{th} and j^{th} quadcopters where $i \in E,\ j \in E \cup \{N+1\}$ and edge $(i, j) \in \mathcal{E}$. The interaction topology is described by a directed graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\}$.

Remark 1: Inspired by [12], due to the fact that the attitude dynamics are much faster than the translation dynamics, the highly nonlinear and coupled dynamics of the quadcopter can be linearized to a single-integrator model in (1).

Assumption 1: The interaction topology contains a directed spanning tree with the virtual leader agent being the root node. Assume the neighbours of a leader quadcopter are leaders or the virtual leader agent, and the neighbours of a follower are either leaders or followers. In addition, for each follower quadcopter, at least one leader has a directed path to that follower.

Assumption 2: We assume that $\delta_{ij} = -\delta_{ji}$ for all $i \in E,\ j \in E \cup \{N+1\}$ and edge $(i, j) \in \mathcal{E}$, which implies that the prescribed formation is feasible among the leader quadcopters under the displacement-based constraints.

The main objective of this work is to design a high-level cooperative control algorithm such that: (i) the leader quadcopters achieve the displacement-based formation constraints; (ii) the follower quadcopters converge inside the convex hull spanned by those leaders; (iii) all quadcopters track the time-varying velocity of the virtual leader agent; and (iv) the leader quadcopters detect the choke-point and adjust the size of the formation so that all quadcopters can safely navigate through it.

Definition 1 (Formation control): The leaders are said to achieve the prescribed formation if each position of the leader $p_i(t)$ satisfies

$$\lim_{t\to\infty} p_i(t) - p_j(t) = \delta_{ij} \tag{3}$$

for all $i \in E,\ j \in E \cup \{N+1\}$ and edge $(i,j) \in \mathcal{E}$. This is equivalent to

$$\lim_{t\to\infty} \left(\sum_{j\in\mathcal{N}_i} a_{ij} \Big(p_i(t) - p_j(t) - \delta_{ij} \Big) \right) = 0. \tag{4}$$

Definition 2 (Containment control [17], [20]): The followers are said to achieve containment if for any $k \in F$, there exist nonnegative constants α_{kj} $(j \in E)$ satisfying $\sum_{j=M+1}^{N} \alpha_{kj} = 1$, such that

$$\lim_{t\to\infty} \left(p_k(t) - \sum_{j\in E} \alpha_{kj} p_j(t) \right) = 0. \tag{5}$$

III. Main Results

In this section, we propose the cooperative control protocols to achieve the aforementioned group objective for multiple quadcopters.

A. Formation-containment tracking protocol design

For the leader quadcopters, we define the formation tracking error ξ_i for all $i \in E$ with respect to its neighbours as

$$\xi_i = \sum_{j\in\mathcal{N}_i} a_{ij}(p_i - p_j - \delta_{ij}). \tag{6}$$

In addition, for the follower quadcopters, we define the containment error ξ_k for all $k \in F$ with respect to its neighbours as

$$\xi_k = \sum_{j\in\mathcal{N}_k} a_{kj}(p_k - p_j). \tag{7}$$

To solve the formation tracking problem for leaders and containment tracking problem for followers, we propose the distributed control laws

$$u_i = -\frac{1}{\gamma_i}\Big(k_p \xi_i - \sum_{j\in\mathcal{N}_i} a_{ij}\dot{p}_j \Big) \quad \forall i \in E \tag{8}$$

and

$$u_k = -\frac{1}{\gamma_k}\Big(k_p \xi_k - \sum_{j\in\mathcal{N}_k} a_{kj}\dot{p}_j \Big) \quad \forall k \in F \tag{9}$$

where $\gamma_i = \sum_{j\in\mathcal{N}_i} a_{ij}$, $\gamma_k = \sum_{j\in\mathcal{N}_k} a_{kj}$ and $k_p > 0$ is the control gain.

Theorem 1: Given a directed graph $\mathcal{G}$ which satisfies Assumptions 1, and displacement-based formation constraints which satisfy Assumption 2, the distributed control laws in (8) and (9) solve the formation-containment problem for leader and follower quadcopters respectively. Moreover, all quadcopters track the time-varying velocity of the virtual leader agent.

Proof: The dynamics of the quadcopter in (1) are decoupled along each axis, which means that the analysis on one dimensional case is valid for three dimensional motions [21]. Therefore, without loss of generality and for brevity, we can assume $p_i \in \mathbb{R}^1$ and $u_i \in \mathbb{R}^1$ in the rest of this proof.

First, we obtain the closed-loop dynamics for leaders by substituting the control law in (8) into (1), yielding

$$\dot{p}_i = -\frac{1}{\gamma_i}\Big(k_p \xi_i - \sum_{j\in\mathcal{N}_i} a_{ij}\dot{p}_j \Big) \quad \forall i \in E. \tag{10}$$

Multiplying γ_i on both sides of (10) gives

$$\sum_{j\in\mathcal{N}_i} a_{ij}(\dot{p}_i - \dot{p}_j) = -k_p \xi_i. \tag{11}$$

By differentiating ξ_i, equation (11) can be rewritten as

$$\dot{\xi}_i = -k_p \xi_i. \tag{12}$$

Since $k_p > 0$, ξ_i asymptotically converges to zero. If $\lim_{t\to\infty} \xi_i(t) = 0$ is established for all $i \in E$, then it implies that the condition in (4) holds when having a virtual leader agent with time-varying velocity. In addition, as $\lim_{t\to\infty} \dot{\xi}_i(t) = 0$ for all $i \in E$, the velocity of each leader quadcopter asymptotically converges to the velocity reference of the virtual leader agent. Thus, we conclude that the leader quadcopters achieve and maintain the prescribed formation while tracking the velocity reference of the virtual leader agent.

Subsequently, we derive the closed-loop dynamics for followers, which gives

$$\dot{p}_k = -\frac{1}{\gamma_k}\Big(k_p \xi_k - \sum_{j\in\mathcal{N}_k} a_{kj}\dot{p}_j \Big) \quad \forall k \in F. \tag{13}$$

Following the same method as mentioned above, we derive that ξ_k asymptotically converges to zero. If $\lim_{t\to\infty} \xi_k(t) = 0$ for all $k \in F$, then it implies that the condition in (5) holds when having a virtual leader agent with time-varying velocity. Furthermore, as $\lim_{t\to\infty} \dot{\xi}_k(t) = 0$ for all $k \in F$, the velocity of each follower quadcopter asymptotically converges to the velocity reference of the virtual leader agent. In other words, all follower quadcopters converge inside the convex hull spanned by those leaders while tracking the velocity reference of the virtual leader agent.

As a result, we conclude that the formation-containment tracking problem is solved by the proposed distributed control laws in (8) and (9). ■

B. Formation-containment scaling protocol design

First, we introduce a scaling vector $s = [s_x, s_y, s_z]^T$ into the displacement-based constraints, which gives

$$\hat{\delta}_{ij} = [s_x\delta_{ij}^x, s_y\delta_{ij}^y, s_z\delta_{ij}^z]^T, \tag{14}$$

where s_x, s_y and s_z are the scaling variables for x, y and z directions respectively. Then, the formation scaling error $\hat{\xi}_i$ for all $i \in E$ with respect to its neighbours is defined as

$$\hat{\xi}_i = \sum_{j\in\mathcal{N}_i} a_{ij}(p_i - p_j - \hat{\delta}_{ij}). \tag{15}$$

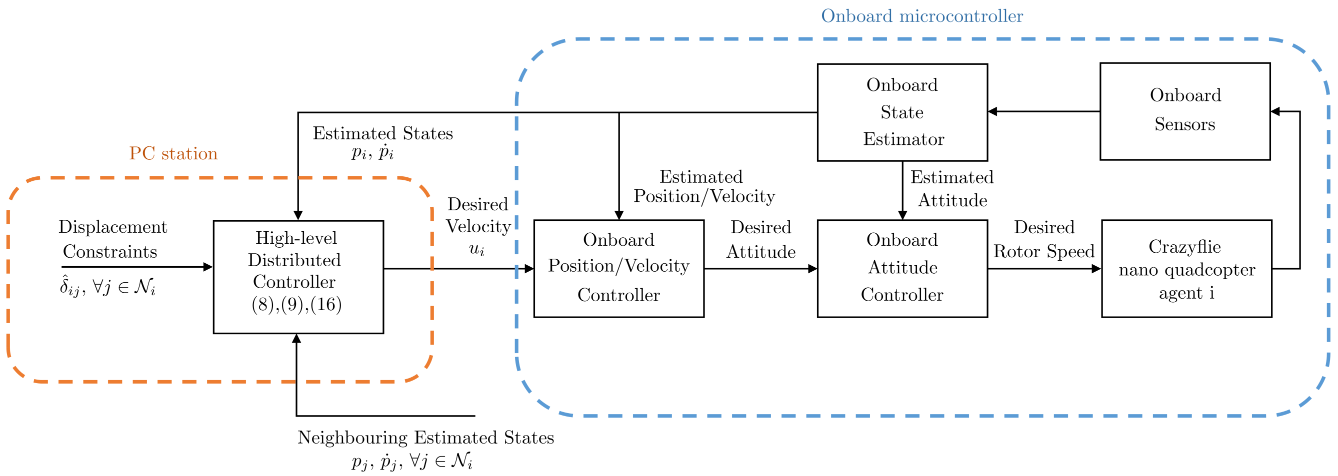


Fig. 1. Control structure associated with the flight experiment. For the i^{th} leader, the high-level distributed controller in (8) or (16) computes the desired velocity u_i given its estimated states p_i and $\dot{p}_i$, its neighbouring estimated states p_j and $\dot{p}_j$ $\forall j \in \mathcal{N}_i$ and the displacement constraints δ_{ij} or $\hat{\delta}_{ij}$ $\forall j \in \mathcal{N}_i$. For the k^{th} follower, the high-level distributed controller in (9) computes the desired velocity u_k given its estimated states p_k and $\dot{p}_k$ and its neighbouring estimated states p_j and $\dot{p}_j$ $\forall j \in \mathcal{N}_k$.

To solve the formation scaling problem for leaders, we propose the following distributed control law

$$u_i = -\frac{1}{\gamma_i}\Big(k_p\hat{\xi}_i - \sum_{j\in\mathcal{N}_i} a_{ij}\dot{p}_j\Big) \quad \forall i \in E. \tag{16}$$

Since the containment scaling error remains the same as (7), the distributed control law in (9) solves the containment scaling problem for followers.

Theorem 2: Given a directed graph $\mathcal{G}$ which satisfies Assumptions 1, and displacement-based formation constraints which satisfy Assumption 2, the distributed control laws in (16) and (9) solve the formation-containment scaling problem for leader and follower quadcopters respectively. Moreover, all quadcopters track the time-varying velocity of the virtual leader agent.

Proof: The proof of Theorem 2 follows the same procedure as Theorem 1, with the only change of replacing (8) with (16). ■

C. Choke-point navigation

In this section, we apply the aforementioned formation-containment tracking and scaling control laws and propose a control algorithm to achieve the choke-point navigation mission for a team of quadcopters when only the subgroup is equipped with obstacle detecting sensors.

First, a quadcopter is defined as a leader if it is equipped with an obstacle detecting sensor; otherwise, it is a follower. Next, we apply the formation-containment technique so that the leaders act as guardians and form a safe region for those followers which are not equipped with obstacle detecting sensors. Moreover, a virtual leader agent with exogenous velocity reference v_{ref} is introduced to create the group motion. Specifically, when the leaders detect the obstacle in front of the group's pathway and identify it as a choke-point, they adjust the scaling variable to rescale the size of the formation (i.e. the safe region). Then, leader and follower quadcopters compute their control inputs based on the proposed formation-containment scaling law in (9) and (16) respectively. Consequently, all quadcopters safely navigate through the choke-point while tracking the velocity of the virtual leader agent. The above procedure is summarised in Algorithm 1.

Algorithm 1: A control algorithm that applies formation-containment tracking and scaling control laws to achieve a choke-point navigation mission for a team of quadcopters

```
initialization;
while true do
    if choke-point = true then
        set v_ref = 0 for the virtual leader agent;
        adjust the scaling vector s;
        compute the control law u_i given in (16) ∀i ∈ E;
        compute the control law u_k given in (9) ∀k ∈ F;
    reset v_ref for the virtual leader agent;
    compute the control law u_i given in (16) ∀i ∈ E;
    compute the control law u_k given in (9) ∀k ∈ F;
end
```

IV. Experimental Validation

To demonstrate the effectiveness of the proposed cooperative control laws, we implement them on Crazyflie 2.1 nano quadcopters and conduct two flight experiments. The flight experiment consists of two main components: (i) the Crazyflie 2.1 nano quadcopter; and (ii) the Loco positioning system, both developed by Bitcraze [22]. Figure 1 presents the control structure of the experiment. The velocity and attitude controllers are implemented onboard the Crazyflie, while the proposed high-level distributed controllers are implemented in the PC station. Specifically, the PC station collects the information states from and sends the velocity control command to all Crazyflies via the Crazyradio dongle [22]. However, we can remove the PC station if each quadcopter can directly share its information states with its neighbours via Bluetooth or Wi-fi. Since the proposed cooperative controllers in (8), (9) and (16) are completely distributed, only requiring the neighbouring information states. Figure 2 shows the setup of the experiment. The Loco positioning anchors are placed at each corner of the flight arena to provide the absolute position of each Crazyflie. A video of two autonomous flight experiments can be found in the Supplementary Material.

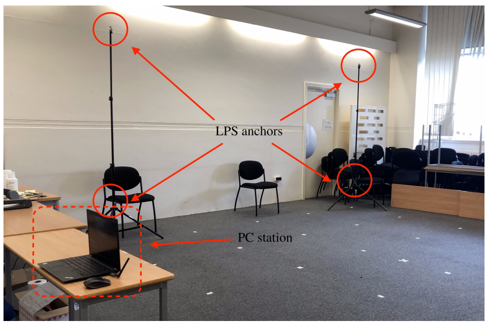


Fig. 2. The setup of the experiment includes the Loco positioning system and a PC station. Four LPS anchors are shown in the picture, while the other four are placed behind the camera.

A. Experiment 1: Formation-containment tracking

In experiment 1, six quadcopters are used to perform the formation-containment tracking task. They are divided into four leaders and two followers. In addition, we introduce a virtual leader agent with time-varying velocity. Figure 3 describes the directed interaction topology among all quadcopters and the virtual leader agent that satisfied Assumption 1. The goals are: (i) the four leaders should achieve the prescribed rectangular formation; (ii) the two followers should converge inside the convex hull spanned by those leaders; and (iii) all quadcopters should track the time-varying velocity of the virtual leader agent.

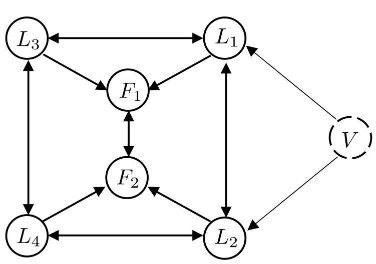


Fig. 3. The interaction topology. Agents F_1 and F_2 are the follower quadcopters, agents L_1 to L_4 are the leader quadcopters and agent V is the virtual leader agent.

Figure 4 presents the trajectories of all quadcopters, whereas Figure 5 and Figure 6 shows the formation error and the velocities in the x-direction of the virtual leader agent and all quadcopters respectively. From Figure 5, we can see that the formation error nearly converges to zero. It is noted that the accuracy of the Loco positioning system is around 0.1 m [22]. The results validate that the group of quadcopters achieve the formation-containment tracking task under the proposed control laws in (8) and (9).

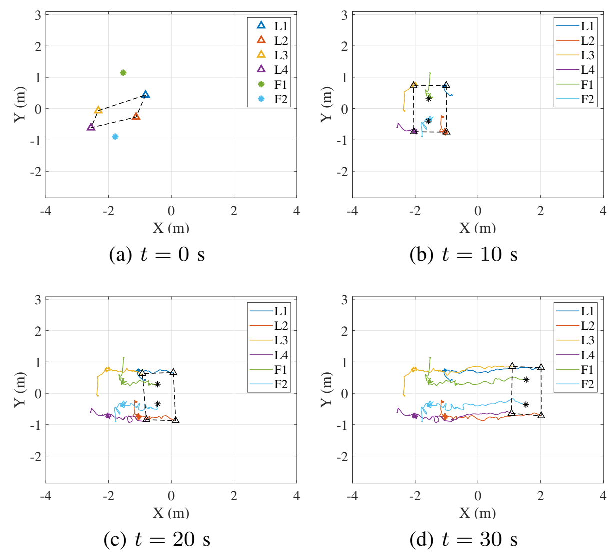


Fig. 4. Trajectories of all quadcopters in the x-y plane at time instants $t = 0$ s, $t = 10$ s, $t = 20$ s and $t = 30$ s. The Δ and $*$ represent the positions of the leaders and followers respectively. The black dashed line represents the formation of the leaders. (a) At $t = 0$ s, the leaders start from random positions while the followers are outside the convex hull. (b) At $t = 10$ s, the leaders achieve the prescribed rectangular formation and the followers converge inside the convex hull spanned by those leaders. (c) At $t = 20$ s, the leaders and followers maintain the formation-containment while tracking the velocity of the virtual leader agent. (d) At $t = 30$ s, mission complete.

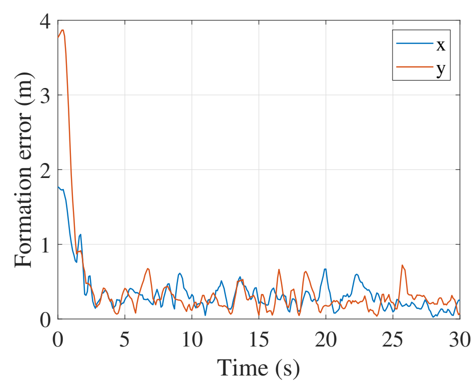


Fig. 5. Formation error in x and y directions. The formation error in each direction is defined as the summation of $|\xi_i|$ for all $i \in E$.

B. Experiment 2: Choke-point navigation

In experiment 2, we implement the proposed formation-containment tracking and scaling laws on a team of quadcopters to perform the choke-point navigation task. Like experiment 1, we consider six quadcopters and one virtual leader agent. However, only the leaders are equipped with multi-ranger decks [22] (i.e. the obstacle detecting sensors).

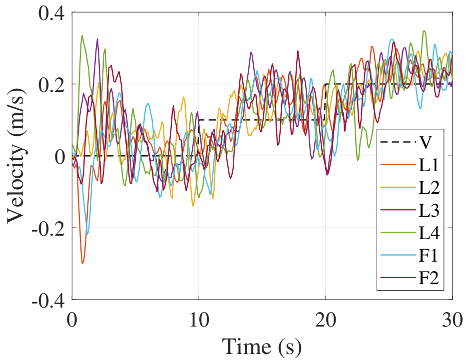


Fig. 6. Velocities in the x-direction of the virtual leader agent and all quadcopters. In the first 10 s, the velocity of the virtual leader agent is 0 m/s. Then, from 10 to 20 s, the velocity is 0.1 m/s. Finally, between 20 to 30 s, the velocity is 0.2 m/s. The result shows that all quadcopters track the time-varying velocity of the virtual leader agent.

The goals are: (i) the four leaders should achieve the prescribed rectangular formation; (ii) the two followers should converge inside the convex hull spanned by those leaders; (iii) when the leaders detect the choke-point, they should rescale the formation so that the size of the formation is smaller than the choke point; and (iv) all quadcopters should track the time-varying velocity of the virtual leader agent and safely navigate through the choke-point.

Figure 7 shows the formation error, whereas Figure 8 presents the snapshots from the flight experiment and the trajectories of all quadcopters. The results validate that all quadcopters safely navigate through the choke-point under the proposed Algorithm 1.

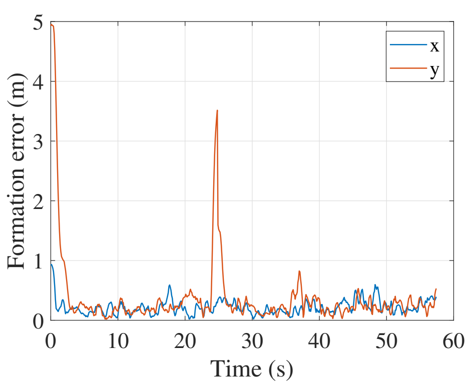
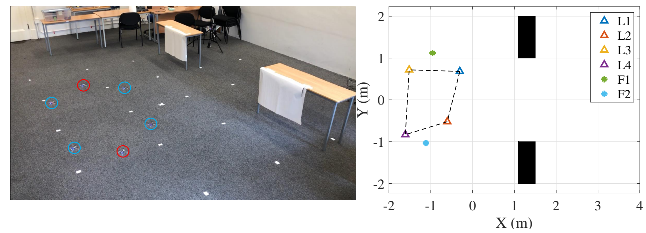


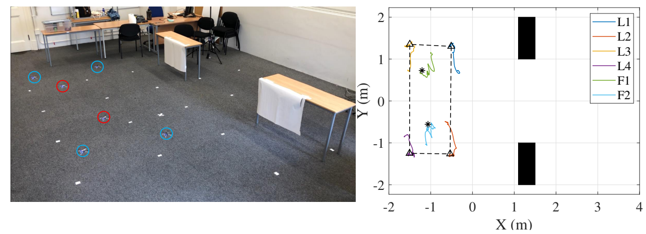
Fig. 7. Formation error in x and y directions. The formation error in each direction is defined as the summation of $|\xi_i|$ for all $i \in E$. Note that $t = 23$ s, the leaders detect the choke-point and update the scaling variable in the y-direction.

V. Conclusion

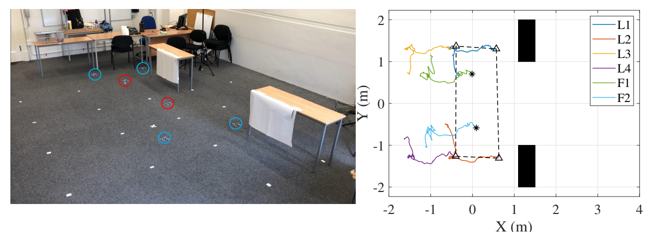
This paper proposes cooperative control laws which solve the formation-containment tracking and scaling problems for multiple quadcopters. In addition, we demonstrate that a team of quadcopters safely and cooperatively navigates through a choke-point when only the subgroup is equipped with obstacle detecting sensors. Finally, we provide two flight experiments to validate the effectiveness of the proposed cooperative control laws.



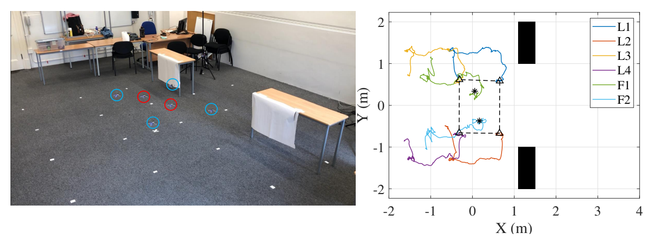
(a) $t = 0$ s



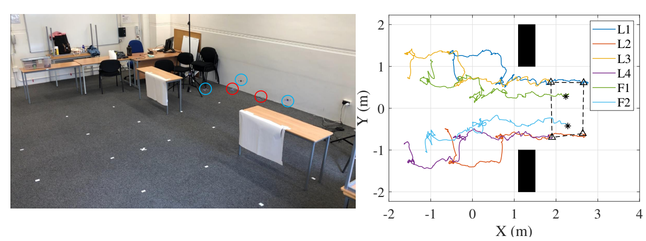
(b) $t = 10$ s



(c) $t = 23$ s



(d) $t = 33$ s



(e) $t = 58$ s

Fig. 8. Snapshots of the flight experiment and the associated trajectories of all quadcopters in the x-y plane at time instants $t = 0$ s, $t = 10$ s, $t = 23$ s, $t = 33$ s and $t = 58$ s. The red circles in the snapshots mark the two follower quadcopters whereas the blue circles mark the four leader quadcopters. The two tables in the snapshots and the two black rectangles in the trajectory figures represent the choke-point. (a) At $t = 0$ s, the leaders start from random positions while the followers are outside the convex hull. (b) At $t = 10$ s, the leaders achieve the prescribed rectangular formation and the followers converge inside the convex hull spanned by those leaders. (c) At $t = 23$ s, the leaders detect the choke-point and update the scaling variables. It is noted that the initial prescribed formation is bigger than the choke-point. (d) At $t = 33$ s, the leaders reduce the size of the formation such that the whole team can pass through the choke-point. (e) At $t = 58$ s, all quadcopters safely navigate through the choke-point and the mission is complete.

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