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Robust Formation Control of Train Platoons for Interval Maintaining

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***Abstract*— Train coordination technology in moving block signaling (MBS) systems has a huge potential in maximizing line utilization and railways safe operation [1]. However, the realization of reliable cooperative control is a nontrivial task, facing challenges such as environmental uncertainties, unpredictable and time-varying disturbances. This paper investigates the robust cooperative control problem of networked homogeneous trains with physically connected carriages using the cooperative control theories for Negative Imaginary (NI) systems. The coupling between trains within the overall system is described by a network topology. A local robust Strictly Negative Imaginary (SNI) controller is designed to track a prescribed reference and maintain a pre-defined formation. Numerical simulations are provided to demonstrate the effectiveness of the proposed controller.**

***Index Terms*— Multiple train coordination, Robust cooperative control, Negative Imaginary systems, Moving block signaling system**

I. INTRODUCTION

Driven by the demand of increasing railways' efficiency and safety, trains coordination control is playing a more and more important role in modern railway systems. However, the traditional centralized railway system can hardly satisfy the growing demands for higher line capacity while ensuring safe operation. Based on the communication-based train control (CBTC) system, the successful deployments of moving block signaling systems in urban railway systems have been proved to be capable of improving railway operations effectively. Compared with the traditional fixed block signaling (FBS) system, the speed and distance profile are calculated according to real-time motion information of other trains, which provides a basis for improving line capacity and realizing more sophisticated operations [1]. Therefore, the cooperative trains control under MBS has received attentions from scholars around the world. Virtual coupling of train platoons is studied in [2]. The Safe interval tracking problem under various kinds of disturbances and uncertainties is studied in [3]–[5] using different methods, where Lyapunov theory is used to guarantee the stability of the overall system. The malfunction problem in cooperative train operation is considered in [6]. Energy consumption under moving block signaling is optimized in [7]. Taking broadcast delays into consideration, the control of homogeneous vehicle platoons is studied in [8].

However, there has been a very limited amount of research considering the consensus between carriages while designing a cooperative controller for train platoons. The International Organization for Standardization (ISO) uses the root-mean-square (r.m.s) value of vibrational accelerations to evaluate the effect of vibration on ride comfort [9]. The consensus can effectively eliminate the oscillation between carriages, which is essential to improve the passengers' comfort. Moreover, considering the carriages is more suitable for the direct implementation of the control laws to the electric multiple unit (EMU) configuration, which has been widely used in suburban and high-speed trains around the world.

In this work, cooperative control of multiple Negative Imaginary (NI) plants based on consensus theory is utilized for the tracking problem of train platoons while maintaining pre-defined formations. NI theory is introduced in [10] and then applied in many practical systems including nano-positioning [11] and light-damped structure [12]. The stability property of positive feedback interconnected systems is studied in [10], [13]: The overall system with a NI system and a positive-feedback connected strictly negative imaginary (SNI) system is internally stable if and only if the DC loop gain is less than unity [10]. In [14], the stability conditions for NI framework are extended to capture the systems with poles on the origin, allowing application to flexible structures with free body motion like aerial or ground vehicles. In [15], stability for a string of coupled subsystems with NI frequency response is analyzed and the derived results are illustrated by applying them to train platoon control. Combined with graph theory, the robust output feedback control consensus problem for networked NI subsystems is studied in [16], [17].

Compared with existing related studies, our work considers the virtual communication among trains and physical connection couplers. The main contributions of this paper are summarized as follows:

(i) An alternative representation of the real train platoon system's physical architecture is realized. Carriages in a train along with couplers are also taken into consideration in the modelling.
(ii) Based on the theory of multiple NI systems, more sophisticated robust control for train platoons is achieved. Consensus is achieved not only among trains, but also among carriages via the proposed control law.
(iii) Improvement of railway utilization is achieved under moving block signaling system by maintaining safety

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intervals between trains, which is illustrated by numerical simulation.

The remainder of this paper is organized as follows. Preliminaries on grapy theory and NI theory are covered in Section II. The main results including the modelling process and homogeneous train platoon controller design are provided in Section III along with stability and consensus analysis. Simulation experiments for a reference tracking scenario under a pre-defined formation are designed and illustrated in Section IV, and the results are also analyzed. Conclusions are summarized in Section V.

II. Preliminaries on Graph Theory and Negative Imaginary Systems Theory

A. Graph Theory

Graph theory can be utilized to describe the interactive relationship among a group of agents. A graph $\mathscr{G}=(\mathscr{V},\varepsilon)$ consists of a set of vertices $\mathscr{V}=\{v_1,v_2,\ldots,v_n\}$ with n nodes, and a set of edges $\varepsilon=\{(v_i,v_j)\in\mathscr{V}\times\mathscr{V}\}$ with l edges. $\mathscr{A}=\{a_{ij}\}\in\mathbb{R}^{N\times N}$ is the adjacency matrix where the adjacency a_{ij} from j to i equals to 0 if $j=i$ and 1 if $(v_i,v_j)\in\varepsilon$. For an undirected graph, $a_{ij}=a_{ji}$. Define the in-degree matrix $\Delta=diag_{i=1}^{N}\{\Delta_i\}$ where $\Delta_i=\sum_{j=1}^{N}a_{ij}$ is the in-degree of node i. The Laplacian matrix of the graph is then defined as $\mathscr{L}_n=\Delta-\mathscr{A}$.Define the $n\times l$ incidence matrix $\mathscr{Q}$ of $\mathscr{G}$ as follows:

$$\mathscr{Q}=\begin{cases} q_{ij}=1 & \text{if } i \text{ is the initial vertex of edge } j,\\ q_{ij}=-1 & \text{if } i \text{ is the end vertex of edge } j,\\ q_{ij}=0 & \text{if } i \text{ and } j \text{ is not connected}\end{cases}$$

It is well-known that the relationship between the incidence matrix and the Laplacian matrix of a undirected and connected graph $\mathscr{G}$ is: $\mathscr{L}_n=\mathscr{Q}\mathscr{Q},^T$ and $\text{Ker}(\mathscr{L}_n)=\text{Ker}(\mathscr{Q}^T)=\text{span}\{1_n\}$.

B. Negative Imaginary Systems Theory

Definition 2.1 ([14]): Given a square, real and proper transfer function $G(s)$, it is NI if the following conditions are satisfied:

1) $G(s)$ has no pole in $\text{Re}[s]>0$;
2) $\forall\omega>0$ such that $j\omega$ is not a pole of $G(s)$, $j(P(j\omega)-P(j\omega)^*)\geq 0$;
3) If $s=j\omega_0$ where $\omega_0>0$ is a pole of $G(s)$, then it is a simple pole and the residue matrix $K=\lim\limits_{s\to j\omega_0}(s-j\omega_0)jG(s)$ is Hermitian and positive semi-definite;
4) If $s=0$ is a pole of $G(s)$, then $\lim\limits_{s\to 0}s^kG(s)=0\ \forall k\geq 3$ and $G_2=\lim\limits_{s\to 0}s^2G(s)$ is Hermitian and positive semi-definite.

Definition 2.2: [10] Given a square, real, rational, proper transfer function $G_s(s)$, it is SNI if the following conditions are satisfied:

1) $G_s(s)$ has no pole in $\text{Re}[s]>0$;
2) $\forall\omega>0, j(G_s(j\omega)-G_s(j\omega)^*)>0$.

For the NI transfer function $G(s)$ with pole(s) at the origin, the stability conditions are summarized below, where

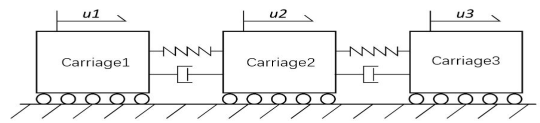


Fig. 1. Structure of a train composed of three physically connected carriages with couplers regarded as spring-damper systems

the constant matrices are initially calculated following these definitions: $G_2=\lim_{s\to 0}s^2G(s), G_1=\lim_{s\to 0}s(G(s)-G_2/s^2), G_0=\lim_{s\to 0}(G(s)-G_2/s^2-G_1/s)$. For $G_2\neq 0$, define a decomposed full column rank matrix satisfying $G_2=JJ^T$. Suppose $G_1=0$ and $J^TG_s(0)J$ is non-singular, then define the matrix N_2 as:

$$N_2=G_s(0)-G_s(0)J(J^TG_s(0)J)^{-1}J^TG_s(0). \tag{1}$$

Lemma 2.1 ([14]): Given a strictly proper NI plant $G(s)$ with $G_2\neq 0$ and a connected SNI transfer matrix $G_s(s)$ with $J^TG_s(0)J$ non-singular. Suppose $G_1=0$, then the interconnection system $[G(s),G_s(s)]$ is internally stable if and only if

$$J^TG_s(0)J<0 \tag{2}$$

and either:

$$I-N_2^{\frac{1}{2}}G_0N_2^{\frac{1}{2}}>0 \tag{3}$$

when $N_2\geq 0$ or

$$det(I+\widetilde{N}_2G_0\widetilde{N}_2)\neq 0 \tag{4}$$

when $N_2\leq 0$. Here, $\widetilde{N}_2=(-N_2)^{\frac{1}{2}}$.

Lemma 2.2 ([16]): For a undirected and connected graph $\mathscr{G}$, if the subsystem with transfer function $G(s)$ is NI, then the plant $\bar{G}(s)=\mathscr{L}_n\otimes G(s)$ is also NI, where $\mathscr{L}_n$ is the Laplacian matrix of $\mathscr{G}$.

III. Main Results

A. Train Dynamics

Each train in the platoon is assumed to have three carriages connected by string and damper couplers. According to Newton's second law, each train can be described by the following equations:

$$\begin{cases} m\ddot{q}_1=-k(q_1-q_2)-c(\dot{q}_1-\dot{q}_2)+u_1\\ m\ddot{q}_2=k(q_1-q_2)-k(q_2-q_3)+c(\dot{q}_1-\dot{q}_2)+c(\dot{q}_2-\dot{q}_3)\\ m\ddot{q}_3=-k(q_2-q_3)-c(\dot{q}_2-\dot{q}_3)+u_3\end{cases} \tag{5}$$

where m denotes the mass of each carriage, k and c are the stiffness coefficient and damping coefficient respectively, and u_i is the driving force of the i th carriage where $i\in\{1,2,3\}$. It is worth mentioning that q_2 and q_3 are defined as follows:

$$q_2=\mathbf{q}_2-d_f, q_3=\mathbf{q}_3-d_f \tag{6}$$

where $\mathbf{q}_2$ and $\mathbf{q}_3$ are the positions that maintain the connected springs in free length d_f. Define the state vector as: $x=[x_1,x_2,x_3,x_4,x_5,x_6]^T=[q_1,\dot{q}_1,q_2,\dot{q}_2,q_3,\dot{q}_3]^T$, input vector as

$u=[u_1,u_2,u_3]^T$ and output vector as $y=[q_1,q_2,q_3]^T$, then the state space of each train can be derived according to (5):

$$\begin{cases}\dot{x}=Ax+Bu\\ y=Cx+Du.\end{cases}$$

It is found that the train system with transfer function $G(s)=C(sI-A)^{-1}B+D$ is NI, which will be proved below. Take a three-carriage train as illustrated in Fig. 1 for example, the transfer function matrix can be calculated:

$$G(s)=\begin{bmatrix}g_1(s) & g_2(s) & g_3(s)\\ g_2(s) & g_4(s) & g_2(s)\\ g_3(s) & g_2(s) & g_1(s)\end{bmatrix} \tag{7}$$

where

$$g_1(s)=\frac{m^2s^4+3cms^3+3kms^2+c^2s^2+2cks+k^2}{m^2s^6+4cms^5+3c^2s^4+4kms^4+6cks^3+3k^2s^2}$$

$$g_2(s)=\frac{k+cs}{ms^4+3cs^3+3ks^2}$$

$$g_3(s)=\frac{c^2s^2+2cks+k^2}{m^2s^6+4cms^5+3c^2s^4+4kms^4+6cks^3+3k^2s^2}$$

$$g_4(s)=\frac{ms^2+cs+k}{ms^4+3cs^3+3ks^2}.$$

$G(s)$ needs to satisfy all the four conditions in Definition 2.1 to be a NI system: (1) $G(s)$ has no poles in $Re[s]>0$; (2) The matrix $j[G(j\omega)-G(j\omega)^*]$ has the following three eigenvalues: $0, \frac{2cm\omega}{c^2\omega^2+(k-m\omega^2)^2}, \frac{6cm\omega}{c^2\omega^2+(k-m\omega^2)^2}$, which are greater than or equal to 0, and $j[G(j\omega)-G(j\omega)^*]$ matrix is positive semi-definite; (3) There is no pole where $\omega_0>0$; (4)$\forall k>0, lim_{s\to 0}s^kG(s)=0$, $G_2=\lim_{s\to 0}s^2G(s)>0$ is a positive semi-definite and Hermitian matrix. Therefore, it can be concluded that the train with three carriages is NI following the above modelling method.

In fact, we found that the generic transfer function of the train system with multiple carriages can be expressed by $G(s)=[M(s)-T(s)\mathscr{L}_n]^{-1}$, where $M(s)=\mathrm{diag}\{ms^2\}$ and $T(s)=-(k+cs)$ represent the mass matrix and couplers respectively. According to Theorem 6 in [18], $G(s)$ is a NI system. Therefore, a train with arbitrary amount of carriages is a NI system.

B. Train Platoon System

In this part, we discuss the construction of train platooning via networked NI systems. First, the platoon is regarded as an augmented NI plant $\tilde{G}(s)=I_n\otimes G(s)$ composed of multiple homogeneous NI subsystems. The total networked system is:

$$y(s)=\tilde{G}(s)u(s) \tag{8}$$

where $y=[y_1^T,y_2^T,\ldots,y_n^T]^T\in\mathbb{R}_{nm\times 1}$, $u=[u_1^T,u_2^T,\ldots,u_n^T]^T\in\mathbb{R}_{nm\times 1}$ denote the augmented output and input respectively. $\tilde{G}(s)=\overset{n}{\underset{i=1}{diag}}\{G_i(s)\}\in\mathbb{R}_{nm\times nm}$ is the augmented transfer function.

By introducing graph theory into the tracking control of a multi-NI system, a cooperative control framework is proposed for solving the rendezvous problem [16], which motivates the controller design in this work. Since we only consider the homogeneous situation, the network topology

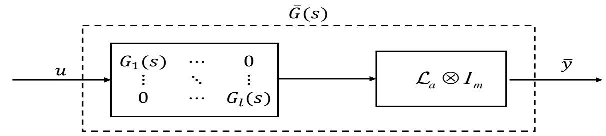


Fig. 2. Overall network system

described by incidence matrix is not suitable in this case. Referring to the method in [17], a new Laplacian matrix defined as $\mathscr{L}_a=\mathscr{L}_n+\mathscr{D}$ is introduced for additive communication links to external reference where $\mathscr{D}=\overset{n}{\underset{i=1}{diag}}\{d_i\}$ denotes the connections to the reference and $d_i=1$ indicates access to the reference. Applying the augmented Laplacian matrix $\mathscr{L}_a$, the overall system as shown in Fig. 2 is derived as follows:

$$\bar{\mathbf{y}}=\bar{G}(s)\bar{u}=(\mathscr{L}_a\otimes I_m)(I_n\otimes G(s))u=(\mathscr{L}_a\otimes G(s))u \tag{9}$$

where $\bar{\mathbf{y}}=[\mathbf{y}_1^T,\mathbf{y}_2^T,\ldots,\mathbf{y}_l^T]^T\in\mathbb{R}_{lm\times 1}$ and $\bar{u}=[u_1^T,u_2^T,\ldots,u_l^T]^T\in\mathbb{R}_{lm\times 1}$. According to Lemma 2.2, the augmented system is also an NI system, which implies that the robust output feedback theory can also be applied to it.

C. Controller Design

In the traditional FBS system, the limit-of-movement authority (LMA) of the following trains in the same line is constrained to the fixed boundary of a block where the preceding train is running until it leaves the block, indicating further space in reducing separation distances. Unlike FBS, the LMA of the following trains can continuously move forward in MBS, which can effectively improve the line capacity while maintaining the distances between trains. For this purpose, a cooperative control scheme suitable for MBS system is developed in this part.

First, referring to the robust output feedback consensus defined in [17], we define the output consensus in formation control for train platooning.

Definition 3.1: A formation for a platoon with n trains can be defined by a constant offset vector

$$X_f=[X_{f_1}^T,\ldots,X_{f_n}^T]^T \tag{10}$$

Then the formation is claimed to achieve robust output feedback consensus when $(y_i-X_{f_i})\to y_f, \forall i\in\{1,\ldots,n\}$ and $v_i\to v_f$ under additive disturbances in $L_2[0,\infty)$. The disturbances are applied to both output and input, and y_f, v_f denote the converged position and velocity trajectories respectively.

To demonstrate the effectiveness of the controller, the headway as defined in [19] is used to evaluate the efficiency of line utilization. It is actually the time interval between two subsequent trains of a defined point, and a short headway means higher utilization efficiency. The objectives of the proposed controller then can be listed as follows: i) The minimum safety distance between trains is maintained for the purpose of reducing the headway between heading and

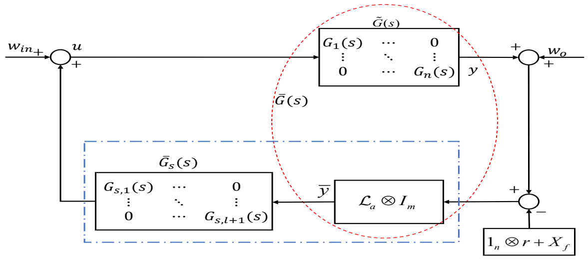


Fig. 3. The formation control structure for multi-trains through information graph

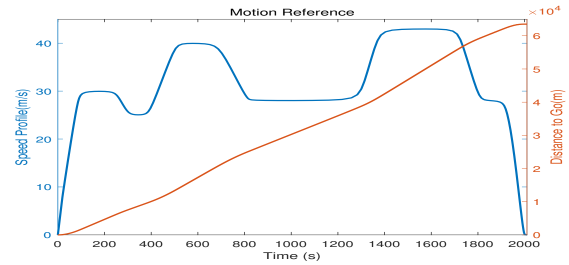


Fig. 4. Speed Profile and distance to go (DTG) curve for the train platoons

last trains under the MBS system; ii) The formation of the train platoon achieves robust output feedback consensus as described above for the purpose of reducing the force applied to couplers; iii) The scheme is robust to additive operational disturbances applied to both outputs and inputs.

Theorem 3.1: Consider a platoon of homogeneous trains with dynamics given in (5). Define a motion reference $r \in \mathbb{R}_{m\times 1}$ for tracking and a formation vector $X_f \in \mathbb{R}_{mn\times 1}$ for describing the agents' relative positions. Given a undirected and connected graph $\mathscr{G}$ with Laplacian matrix $\mathscr{L}_n$ and matrix $\mathscr{D}$ that models the communication links among agents and connection to the external reference respectively, as well as a interconnected SNI control law $\bar{G}_s(s)$ as shown in Fig. 3, the control objectives can be achieved via the following output feedback control law:

$$\begin{aligned} u =& (\mathscr{L}_n + \mathscr{D}) \otimes G_s(s) e \\ e =& y - 1_n \otimes r - X_f \end{aligned} \tag{11}$$

under any external disturbances $w_{in}, w_o \in L_2[0,\infty)$ if and only if the stability conditions of Lemma 2.1 are satisfied for interconnected system $[G(s), G_s(s)]$.

Proof: Consider the positive feedback interaction system shown in Fig. 3. Since the null space property of incidence matrix resulting in $(\mathscr{L}_n^T \otimes I_m)(1_n \otimes r) = 0$, the system output can be split:

$$\begin{aligned} \bar{y} =& ((\mathscr{L}_n + \mathscr{D}) \otimes I_m) e \\ =& (\mathscr{L}_n \otimes I_m) e + (\mathscr{D} \otimes I_m) e \\ =& (\mathscr{L}_n \otimes I_m)(y - X_f) + (\mathscr{D} \otimes I_m)(y - 1_n \otimes r - X_f). \end{aligned}$$

The stability of the overall system drives the system outputs $\bar{y}$ to converge to 0.The result of $(\mathscr{L}_n \otimes I_m)(y - X_f) \to 0$ denotes the relative outputs among agents eventually converge to the formation vector. The result of $(\mathscr{D} \otimes I_m)(y - 1_n \otimes r - X_f) \to 0$ indicates that the outputs of the agents connecting to the reference converge to the prescribed reference. ■

IV. Simulation Experiments

Simulation experiments are provided in this section to illustrate the effectiveness of the proposed control law. The DTG shown in Fig. 5 is used as the motion reference curve for the leading train to follow, which covers three accelerating phases, five cruise phases and four braking phases. There are four trains in total in the train platoons to be controlled, and each train is composed of three physically connected

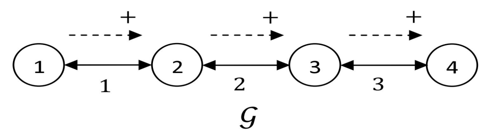


Fig. 5. Graph for Platoons with Four Trains

carriages. The minimum safe distance between trains is assumed to be 500m, corresponding to a formation vector of $[0,0,0,500,500,500,1000,1000,1000,1500,1500,1500]$. The d_f in (6) corresponding to the desired interval distance between carriages is set to be 10m, at which the force applied to the coupler is minimal. An SNI controller that is capable of stabilizing the three-carriage NI plant will be designed for applying Theorem 3.1. The constant matrices of the NI transfer function in Equation (12) are derived as follows:

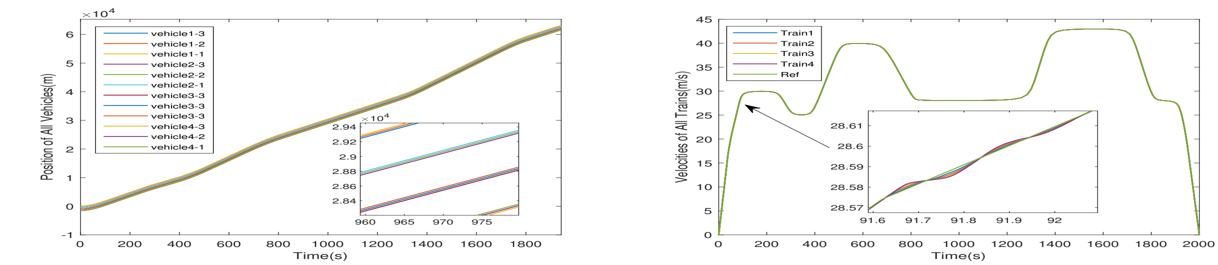
$$G_2(s) = \begin{bmatrix} \frac{1}{3m} & \frac{1}{3m} & \frac{1}{3m} \\ \frac{1}{3m} & \frac{1}{3m} & \frac{1}{3m} \\ \frac{1}{3m} & \frac{1}{3m} & \frac{1}{3m} \end{bmatrix}, G_1(s) = \mathbf{0}, G_0 = \begin{bmatrix} \frac{5m}{9k} & \frac{-m}{9k} & \frac{5m}{9k} \\ \frac{-m}{9k} & \frac{2m}{9k} & \frac{-m}{9k} \\ \frac{-4m}{9k} & \frac{-m}{9k} & \frac{5m}{9k} \end{bmatrix}.$$

An integral resonant controller (IRC), which is utilized to stabilize lightly damped flexible structures in [13], [14], is considered in this paper as the controller for single train. It can be proved that an IRC of the form $(sI + \Gamma\Phi)^{-1} - \Delta$ is SNI if $\Gamma > 0, \Phi > 0$ and Δ is a symmetric matrix [13]. The constant matrices of the chosen 3×3 IRC controller are designed in an adhoc manner as follows:

$$\Gamma = \begin{bmatrix} 40 & 10 & 10 \\ 10 & 30 & 10 \\ 10 & 10 & 20 \end{bmatrix}, \Phi = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 1 \\ 1 & 1 & 4 \end{bmatrix}, \Delta = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.75 \end{bmatrix}.$$

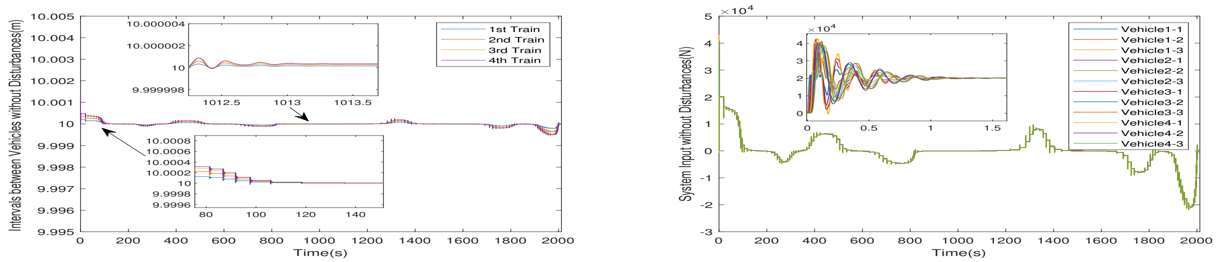
To obtain better performance, the initial IRC was multiplied by a constant of 5000, which does not change the controller's SNI property. It can be proved that condition (2) and (4) are satisfied, so the interconnection $[G(s), G_s(s)]$ is internally stable according to Lemma 2.1.

The information typology of the trains grouping architecture is depicted in Fig. (5) indicating that each train is using information about neighboring trains. Suppose every train has access to the reference, then $D = I_{4\times 4}$, which results in $Q_b = diag\{1,1,1,1\}$. The Laplacian matrix is calculated for the application of Theorem 3.1: $\mathscr{L}_a = \begin{bmatrix} 2 & 0 & 1 & 1 \\ 0 & 3 & 0 & 1 \\ 1 & 0 & 3 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}$.



(a) (b)

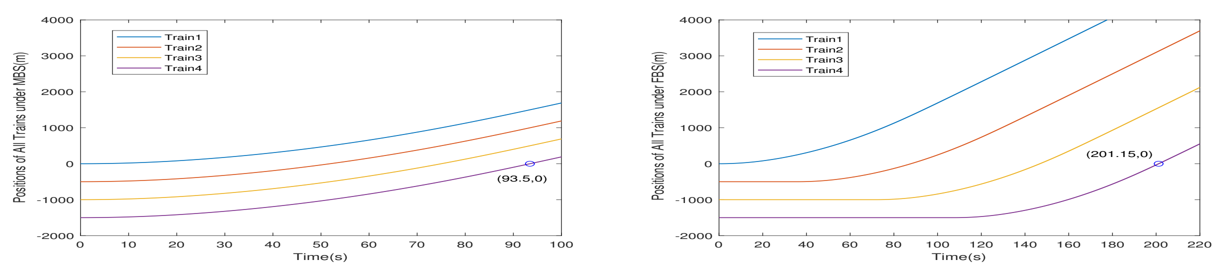
Fig. 6. Simulation Results of trains coordination. (a) Position trajectories of 4 trains with 12 carriages in total.(b) Velocity trajectories of 4 trains.



(a) (b)

(c) (d)

Fig. 7. Results with and without disturbances. (a) Intervals between 1st and 2nd carriages without disturbances. (b) Control input signals without disturbances.(c) Intervals between 1st and 2nd carriages with disturbances.(d) Control input signals with disturbances.



(a) (b)

Fig. 8. Departure headway of the platoon in different signaling system (a) Position trajectories under MBS.(b) Position trajectories under FBS.

Although the nonlinear operation resistance forces of the four trains are not included in train dynamics model, they are regarded as bounded external disturbances to the system and the robustness of SNI controller can guarantee the stability of the overall system. We referred to [5] to set the operational resistance acceleration of the four trains as $0.3 + 0.004v_1 + 0.00016v_1^2$, $0.2 + 0.0025v_2 + 0.0002v_2^2$, $0.4 + 0.0025v_3 + 0.0001v_3^2$, $0.32 + 0.0025v_4 + 0.0002v_4^2$. Additionally, external disturbances to both the inputs and outputs of the system have also been considered. Unpredictable acceleration caused by tunnel, curve, slope, etc. of the four trains are set as $0.015\sin(0.2t)$, $0.01\sin(0.3t)$, $0.015\sin(0.1t)$ and $0.018\sin(0.35t)$ respectively and applied to the inputs of the system. Disturbances applied to the system outputs of the four trains are defined as $0.05\sin(0.1t)$, $0.1\sin(0.2t)$, $0.08\sin(0.4t)$, $0.12\sin(0.3t)$.

Fig. 6 illustrates the results of coordination among trains, where position trajectory and speed trajectory are depicted respectively in (a) and (b). It is evident from Fig. 6 (a) that the train platoon operates with constant safety intervals as per the prescribed formation vector. Speed tracking is also achieved as shown in Fig. 6 (b). Intervals between the heading carriage and the second carriage in every train are provided in Fig. 7 (a) and (c) presenting the results with and without disturbances respectively. It can be concluded that the safety intervals converge to the predefined value when fluctuation of speed occurs and the tendency stills exists under disturbances. The input signals to the system with and without disturbances are also presented in Fig. 7 (d) and (b). It is worth mentioning that the maximum speed and position errors between carriages are 0.018m/s and 0.002m respectively, which can maintain a very small force applied to the couplers (within 100N).

To illustrate the potential ability of the proposed control scheme in improving line utilization, a scenario of simultaneous departure of the train platoon as described above is simulated under the MBS and FBS systems respectively. The departure headway is used to evaluate the efficiency of line utilization. Suppose that the heading train is located in the station platform, then the departure headway under MBS can be calculated as 93.5s via the proposed formation control method, which is presented in Fig. 8 (a). As a comparison, the departure headway under FBS is also calculated. Suppose that the trains are located in the middle of their corresponding blocks with length of 500m, and the interval distance is the same as it is defined under MBS. Since the following train can only depart after the preceding train leaves the front fixed block, the position trajectories of the trains in the platoon can be described in Fig. 8 (b), and the departure headway can be calculated as 201.2s. The departure headway of the proposed controller under MBS is shorter than that under FBS, which illustrates a better line utilization efficiency.

V. CONCLUSIONS

In this paper, we firstly construct the model of a train with multiple carriages as a NI system, then build an augmented NI system representing the train platoon. With the help of cooperative control theories involving networked NI systems, we introduced a robust tracking controller for train platoons. Since the transfer function matrix of the train model is proved to be NI, the proposed IRC controller satisfying the SNI definition can guarantee the internal stability of the interconnected system. A robust cooperative control is then designed to realize formation maintaining and trajectory tracking with a Laplacian matrix describing network information topology among trains. Internal stability can be guaranteed using existing NI theory, and the realization of formation and tracking is also rigorously proven. Simulation results illustrate that the controller satisfies the requirements of intervals maintained between trains and carriages, which can effectively improve line utilization. Since the current NI property verification method is only suitable for trains with a certain number of carriages, a more general proof of the NI nature for trains needs further study. Moreover, more complicated scenarios will be covered in the future work.

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