Robust Performance Improvement Test for Stabilizing Controllers Using Closed-Loop Data ⋆

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Abstract: In this paper, novel test methods are proposed to check whether a new stabilizing controller improves robust performance or not when the existing controller is replaced by this new controller in the closed-loop. For the proposed tests, the plant dynamics is assumed to be unknown whereas the existing and new controller transfer function matrices are known to the designer. The proposed tests are based on closed-loop data and can be used for both the SISO and MIMO systems. The test methods in this paper build on the experimental set-up of Dehghani et al. (2009), however, the proposed results in this paper for robust performance improvement test cannot be obtained from the test results of Dehghani et al. (2009).

Keywords: Robust adaptive control and Iterative Identification, Robust control, Robust stability margin, Robust performance, Positive-real.

1. INTRODUCTION

In robust adaptive control and iterative identification and control redesign techniques, a control systems engineer starts to identify the plant model based on the closed-loop data to design more attractive controller so that the robustness of the closed-loop system is improved while the existing known controller is replaced by the new designed controller Hjalmarsson et al. (1998), Gevers et al. (2003), Gevers (2000). The plant model identification and controller redesign method progresses iteratively until a satisfactory level of robustness is achieved Gevers (2002), Gevers (2000), Schrama and Van Den Hof (1992), Bitmead (1993). This method needs a 'safe' controller change, however, it is not always possible to ensure the robust performance improvement a priori Dehghani et al. (2009), Gevers (2002), Anderson and Gevers (1998), Lecchini et al. (2006), Lanzon et al. (2006), Dehghani et al. (2007), Bitmead (1993), Anderson (2004), Dehghani et al. (2007). Insertion of a destabilizing controller in the closed-loop is avoided at all costs and hence, for 'safe adaptive control' it is always very important to check that the newly designed controller that seems to be attractive before inserting into the closed-loop system is guaranteed to at least stabilize the unknown plant Hjalmarsson et al. (1998), Hildebrand et al. (2005), Anderson (2004), Callafon and Van Den Hof (1997), Kammer et al. (2000). A novel set of experiments were recently proposed in Dehghani et al. (2009) to test internal stability of an apparently attractive controller based on data-only experiments which do not require the full frequency spectrum which prevent the possibility of inserting a destabilizing controller in the closed-loop system.

Although guaranteeing internal stability of a newly designed controller on the unknown physical plant is a necessary prerequisite to a good robust adaptive algorithm, it is not sufficient as it is important to ensure monotonic robust performance improvement when the designer has one or a set of attractive stabilizing controllers at hand. In this scenario, although the available controllers are all stabilizing, the following is an important question: which of these stabilizing controllers will improve performance when the existing controller is replaced by the newly chosen stabilizing controller? The present paper gives answer to this question by proposing novel test methods based on closed-loop data. For the tests, the physical plant is assumed to be unknown whereas all the controllers are known to the designer.

The proposed tests build on the experimental set-up proposed in Dehghani et al. (2009) where the tested controller is implemented in coprime factorization form. In this paper, we use the same experimental set-up as in Dehghani et al. (2009) to propose new additional test methods to ascertain robust performance improvement of the closed-loop system.

2. NOTATIONS AND DEFINITIONS

Let $\mathscr{R}$ denote the set of all real rational transfer function matrices and $\mathscr{RH}_\infty^{m\times n}$ be the set of all real rational stable transfer function matrices with m rows and n columns. Let a transfer function matrix $G \in \mathscr{R}$, then $\mathscr{L}_2$-adjoint system $G^*(s)$ denotes $G(-s)^T$. Let $\mathbb{R}$ and $\mathbb{C}$ denote the fields of real and complex numbers respectively. Also let $\mathbb{C}_-$ and $\bar{\mathbb{C}}_-$, respectively, denote the open and closed left-half planes. Let A^* and $\rho(A)$ denote the complex conjugate transpose and spectral radius of a matrix A, respectively. Let $\bar{\sigma}(A)$ and $\underline{\sigma}(A)$, respectively, denote the largest and smallest singular value of matrix A. Let $\|P\|_\infty$ denote the $\mathscr{H}_\infty$-norm of $P \in \mathscr{RH}_\infty$. The number wno(.) indicates the winding number of a scalar transfer function evaluated on a standard $D-$contour indented to the right around any imaginary axis poles Vinnicombe (2000). Consider the standard feedback interconnection of systems as shown in Figure 1. From $\begin{bmatrix} \omega \\ d \end{bmatrix}$

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to $\begin{bmatrix} y \\ u \end{bmatrix}$, the transfer function matrix is $H(P,C) = \begin{bmatrix} P \\ I \end{bmatrix}(I - CP)^{-1}\begin{bmatrix} -C & I \end{bmatrix}$.

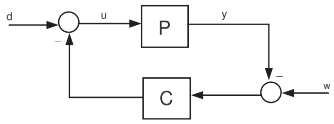

Fig. 1. Feedback interconnection of systems

Definition 1. (Vinnicombe (2000)) The interconnection $[P,C]$ as depicted in Figure 1 is well-posed if $H(P,C)$ exists, and furthermore $[P,C]$ is said to be internally stable if it is well-posed and $H(P,C) \in \mathscr{RH}_\infty$.

Let $P = NM^{-1}$ be a right coprime factorization (rcf) of $P \in \mathscr{R}$ and $C = \tilde{V}^{-1}\tilde{U}$ be a left-coprime factorization (lcf) of $C \in \mathscr{R}$.

Definition 2. (Vinnicombe (2000)) Given $P, C \in \mathscr{R}$. Let $\{N, M\}$ be a rcf of P and $\{\tilde{U}, \tilde{V}\}$ be a lcf of C. Then $G := \begin{bmatrix} N \\ M \end{bmatrix}$ and $\tilde{K} := \begin{bmatrix} -\tilde{U} & \tilde{V} \end{bmatrix}$ where G is referred to as the right graph symbol of P, and $\tilde{K}$ is the inverse left graph symbol of C.

If the coprime factors are normalized, the graph symbols G and $\tilde{K}$ are normalized to satisfy $G^*G = I$ and $\tilde{K}\tilde{K}^* = I$.

Definition 3. (McGowan and Kuc (1982)) The unwrapped phase of a transfer function is denoted by unwarg and refers to the phase of the frequency response when it is in the form of a continuous function of frequency.

3. INTERNAL STABILITY TEST FOR CONTROLLERS AND THE EXPERIMENTAL SET-UP

In this section, the test methods of Dehghani et al. (2009) are described briefly for checking internal stability of an attractive new controller on the unknown plant. The same experimental set-up described in this section will also be used for testing robust performance improvement which is the main concern and key proposition of this paper. For testing, the controller C is implemented in 'observer-form', depicted in Figure 2(a) Vinnicombe (2000), where a left coprime factorization of the controller $C = \tilde{V}^{-1}\tilde{U}$ and the factor $\tilde{V}^{-1}$ is implemented in forward path and $\tilde{U}$ is placed in feedback path of the closed-loop system.

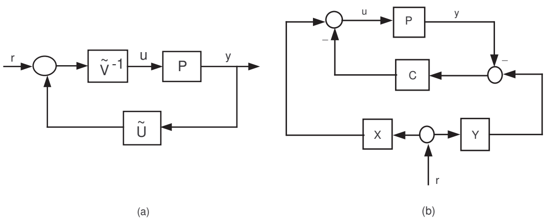

Fig. 2. Controller implementation

If a controller is not already implemented in this observer form, but simply implemented as in Figure 1, then one could use the injection of exogenous signals ω and d before and after the controller, as shown in Figure 2(b), to produce an equivalent observer form implementation- see Dehghani et al. (2009) for details.

Theorem 1. (Vinnicombe (2000)) Given $P \in \mathscr{R}$ and $C \in \mathscr{R}$ connected in a feedback interconnection as shown in Figure 1. Define graph symbols G and $\tilde{K}$ for P and C as given in Definition 2. Then the followings are equivalent:

(1) $[P,C]$ is internally stable;
(2) $(\tilde{K}G)^{-1} \in \mathscr{RH}_\infty$;
(3) $\det(\tilde{K}G)(j\omega) \neq 0\ \forall\omega$ and wno $\det(\tilde{K}G) = 0$.

This theorem underpinned the development in Dehghani et al. (2009). Interestingly the proposed tests are performed on the existing closed-loop (which is already internally stable), and a new candidate controller whose stabilizability will be checked is implemented in its stable coprime factorization form as shown in Figure 3 as post-filtration of signals u and y.

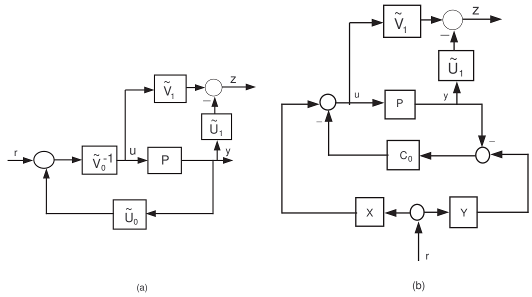

Fig. 3. Experimental set-up

For these tests, the existing controller transfer function matrix is assumed to be known to the designer and has a left coprime factorization framework $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$. This controller is realized in observer form as described in Figure 2(a) or its equivalent implementation of Figure 2(b). Since $\tilde{U}_1, \tilde{V}_1 \in \mathscr{RH}_\infty$, the map from signal r to z is always stable (even when C_1 is destabilizing) as the existing closed-loop system is assumed to be internally stable system and hence this set-up allows for safe experiments before inserting the new controller into the closed-loop. The plant transfer function P is assumed to be unknown but available for data collection onto the physical closed-loop.

Theorem 2. (Dehghani et al. (2009) Lanzon et al. (2006)) Given controllers $C_0, C_1 \in \mathscr{R}$ and assume $[P,C_0]$ is internally stable on a physical plant $P \in \mathscr{R}$. Let $C_0 = \tilde{V}_0^{-1}\tilde{U}_0$ and $C_1 = \tilde{V}_1^{-1}\tilde{U}_1$ be left coprime factorizations over $\mathscr{RH}_\infty$. Using the stable mapping $T : r \mapsto z$ in Figure 3(a) or Figure 3(b), then the following statements are equivalent:

(1) $[P,C_1]$ is internally stable;
(2) $T^{-1} \in \mathscr{RH}_\infty$;
(3) $\det T(j\omega) \neq 0\ \forall\omega$ and wno $\det T = 0$;
(4) $\det T(j\omega) \neq 0\ \forall\omega$ and unwrag $\det T(j\infty) =$ unwarg $\det T(j0)$

where unwarg(.) denotes the unwrapped phase of a scalar transfer function as in Definition 3.

The proof of this theorem is given in Dehghani et al. (2009). Because of P being unknown to the designer, T is also not known to the designer. Using graph symbols for P, C_0 and C_1, T can formally be written as $T = (\tilde{K}_1 G)(\tilde{K}_0 G)^{-1}$ from which it is easy to see that T is always stable since $[P,C_0]$ is internally stable (via Theorem 1). Hence the input-output map T from r to z is always stable even if C_1 is a destabilizing controller for the closed-loop system. In Dehghani et al. (2009), using the experimental set-up shown in Figure 3(a) or Figure 3(b) the input-output data from r to z are collected and based on these closed-loop data, the internal stability condition for $[P,C_1]$ is checked. For checking Condition 4 of Theorem 2, two separate experiments have been proposed in Dehghani et al. (2009). The first experiment provided an easily measurable necessary condition for internal stability. The second experiment required more experimental effort and provided a necessary and sufficient condition for internal stability. The first experiment is repeated here because these results will be used to develop the main results of this paper.

For the closed-loop data-based stability tests, the following two assumptions were made: *Assumption 1:* The factors $\tilde{V}_0$ and $\tilde{V}_1$ are chosen such that $\tilde{V}_0(j\infty) = \tilde{V}_1(j\infty) = I$. *Assumption 2:* The transfer functions PC_0 and PC_1 are strictly proper.

Assumption 1 is without loss of generality and Assumption 2 is very mild and can be easily satisfied in practise.

Theorem 3. (Dehghani et al. (2009) Lanzon et al. (2006)) Let the suppositions of Theorem 2 and Assumptions 1 and 2 hold. Let e_i denote a reference signal where a step is applied at the i-th input while the other inputs are kept as 0. Perform n experiments with reference signal $r(t) = e_i(t), i = 1,\ldots,n$ and let $\bar{z}_i$ be the steady state output of the map $T : r \mapsto z$ recorded in each experiment. Define $\bar{Z} = [\bar{z}_1,\ldots,\bar{z}_n]$. Then $[P,C_1]$ is internally stable implies $\det \bar{Z} > 0$. Thus if $\det \bar{Z} \leq 0$, stability of $[P,C_1]$ is not internally stable.

This is a falsification test and this theorem was proved by using the final value theorem (see Dehghani et al. (2009) for detail). Note that, $\bar{Z} = [\bar{z},\ldots,\bar{z}_n] = \lim_{s\to 0} s\left[T(s)\frac{1}{s}\right] = T(j0)$. Hence $\det T(j0) = \det \bar{Z}$. The following formal relation (see Lemma 11 in Dehghani et al. (2009)) underpins the main results of this paper. Since

$$T = (\tilde{K}_1 G)(\tilde{K}_0 G)^{-1} \qquad (1)$$

then $T' = T - I = \left[\, -(\tilde{U}_1 - \tilde{U}_0) \;\; (\tilde{V}_1 - \tilde{V}_0) \,\right] \begin{bmatrix} P(I-C_0P)^{-1} \\ (I-C_0P)^{-1} \end{bmatrix} \tilde{V}_0^{-1}$

$$= (\tilde{K}_1 - \tilde{K}_0) G (\tilde{K}_0 G)^{-1} \qquad (2)$$

From (1), we can rewrite $T = \tilde{V}_1(I - C_1P)(I - C_0P)^{-1}\tilde{V}_0^{-1}$. Then by Assumptions 1 and 2, it is evident that at high frequency T tends to I, i.e. from (2), T' is strictly proper. This trick simplifies the experiment significantly and indicates that experiments need not be performed on the whole frequency range to characterize the closed-loop system T, but only up to some finite frequency (i.e. bandwidth) ω_0.

4. TEST FOR ROBUST PERFORMANCE IMPROVEMENT

Once the stability conditions are satisfied, an immediate subsequent important question is raised: Does this stabilizing controller improve robust performance of the closed-loop system or not? In this section, new experiments are proposed to answer this last question. We will use the same experimental set-up shown in Figure 3(a) and Figure 3(b) to test for robust performance improvement.

We now define the robust stability margin Vinnicombe (2000), Lanzon and Papageorgiou (2009) for the interconnected systems shown in Figure 1 as follows:

$$b(P,C) = \begin{cases} \left\| \begin{bmatrix} P \\ I \end{bmatrix} (I - CP)^{-1} \left[\, -C \;\; I \,\right] \right\|_\infty^{-1} & \text{when } [P,C] \text{ is internally stable,} \\ 0 & \text{otherwise.} \end{cases} \qquad (3)$$

Using normalized graph symbols, we can rewrite $b(P,C)$ when $[P,C]$ is internally stable as $b(P,C) = \|G(\tilde{K}G)^{-1}\tilde{K}\|_\infty^{-1} = \|(\tilde{K}G)^{-1}\|_\infty^{-1}$. Hence the generalized robust stability margin $b(P,C)$ can now be equivalently represented as $b(P,C) = \inf_\omega \underline{\sigma}\left(\tilde{K}G(j\omega)\right)$ when $[P,C]$ is internally stable. Define also $\rho(P(j\omega),C(j\omega)) = \underline{\sigma}(\tilde{K}(j\omega)G(j\omega))$ to be the pointwise in frequency generalized robust stability margin.

The generalized robust stability margin $b(P,C)$ is a measure of robust performance, not just robust stability, of the closed-loop system Vinnicombe (2000), Zhou et al. (1996), Lanzon and Papageorgiou (2009), McFarlane and Glover (1992). Higher value of $b(P,C)$ indicates higher level of robust performance. This means that when an existing controller is replaced by a new attractive stabilizing controller in the closed-loop system, an increase in $b(P,C)$ implies an improvement in robust performance. In this section, new test methods will be proposed to check whether $b(P,C)$ increases or not so that we can ascertain whether robust performance improves or not for the closed-loop system. Throughout this paper, we denote the existing controller and the new attractive stabilizing controller which we would like to test by, C_0 and C_1 respectively. The performance improvement conditions are given here both pointwise in frequency and over all frequencies.

The transfer function matrix $T : r \mapsto z$ in Figure 3(a) or Figure 3(b) is formally given by $T = (\tilde{K}_1 G)(\tilde{K}_0 G)^{-1}$ and $T' : r \mapsto (z - r)$ is formally given by $T' = (\tilde{K}_1 - \tilde{K}_0)G(\tilde{K}_0 G)^{-1}$ though neither of these two transfer functions can be computed explicitly during the experiment as P is not known to the designer. Here, $\tilde{K}_1$ and $\tilde{K}_0$ are respectively the normalized inverse left graph symbols of C_1 and C_0 and G is the normalized right graph symbol of P (see Definition 2). From above relation, we have $(\tilde{K}_1 G) = T(\tilde{K}_0 G)$. Using singular value inequalities, the above expression can be rewritten pointwise in frequency as follows:

$$\underline{\sigma}(T(j\omega))\underline{\sigma}(\tilde{K}_0 G(j\omega)) \leq \underline{\sigma}(\tilde{K}_1 G(j\omega)) \leq \bar{\sigma}(T(j\omega))\underline{\sigma}(\tilde{K}_0 G(j\omega)). \qquad (4)$$

4.1 Performance improvement pointwise in frequency

Sufficient condition If $\underline{\sigma}(T(j\omega)) > 1\ \forall\omega$, then $\underline{\sigma}(\tilde{K}_1 G(j\omega)) > \underline{\sigma}(\tilde{K}_0 G(j\omega))\ \forall\omega$, which means we have pointwise improvement in $\rho(P(j\omega),C(j\omega))$.

Checking this sufficient condition pointwise in frequency is equivalent to check the following condition: $\forall\omega, \forall 0 \neq r_\omega \in \mathbb{C}^m, z_\omega^* z_\omega > r_\omega^* r_\omega$ where at each frequency $T(j\omega) : \begin{smallmatrix}\mathbb{C}^m \to \mathbb{C}^m \\ r_\omega \mapsto z_\omega\end{smallmatrix}$ as $\underline{\sigma}(A) = \inf_{x\neq 0} \frac{\|Ax\|_2}{\|x\|_2}$ and $\|x\|_2^2 = x^*x$.

Necessary condition In order to have any hope for the desired pointwise in frequency robust performance improvement to be achieved via an increase in $\rho(P(j\omega),C(j\omega))$, we necessarily need $\bar{\sigma}(T(j\omega)) > 1\ \forall\omega$.

Checking the above condition is equivalent to check the following condition: $\forall\omega, \exists 0 \neq r_\omega \in \mathbb{C}^m : z_\omega^* z_\omega > r_\omega^* r_\omega$ where at each frequency $T(j\omega) : \begin{smallmatrix}\mathbb{C}^m \to \mathbb{C}^m \\ r_\omega \mapsto z_\omega\end{smallmatrix}$ as $\bar{\sigma}(A) = \sup_{x\neq 0} \frac{\|Ax\|_2}{\|x\|_2}$ and $\|x\|_2^2 = x^*x$.

Remark 1. The pointwise in frequency necessary condition is easier to test than the pointwise in frequency sufficient condition because for the necessary condition, one need to find only one pointwise signal $0 \neq r_\omega \in \mathbb{C}^m$ that results in amplification of signal norms, whereas for the sufficient condition one needs to check that all signals [1] $0 \neq r_\omega \in \mathbb{C}^m$ result in amplification of signal norms.

Inequality (4) implies (by taking the appropriate infimum and supremum in the correct orders)

$$\left[\inf_\omega \underline{\sigma}(T(j\omega))\right] b(P,C_0) \leq b(P,C_1) \leq \left[\sup_\omega \bar{\sigma}(T(j\omega))\right] b(P,C_0)$$

$$\Rightarrow \frac{b(P,C_0)}{\|T^{-1}\|_\infty} \leq b(P,C_1) \leq \|T\|_\infty b(P,C_0) \tag{5}$$

Since $[P,C_0]$ and $[P,C_1]$ are both internally stable, from Theorem 2 we have $T, T^{-1} \in \mathscr{RH}_\infty$ where $T : \mathscr{L}_2[0,\infty) \underset{r \mapsto z}{\rightarrow} \mathscr{L}_2[0,\infty)$ and then equivalently $T^{-1} : z \mapsto r$, then noting that $\|T\|_\infty = \sup_{0 \neq r \in \mathscr{L}_2[0,\infty)} \frac{\|z\|_{\mathscr{L}_2}}{\|r\|_{\mathscr{L}_2}}$ and $\|T^{-1}\|_\infty = \sup_{0 \neq z \in \mathscr{L}_2[0,\infty)} \frac{\|r\|_{\mathscr{L}_2}}{\|z\|_{\mathscr{L}_2}} = \sup_{0 \neq r \in \mathscr{L}_2[0,\infty)} \frac{\|r\|_{\mathscr{L}_2}}{\|z\|_{\mathscr{L}_2}}$ since T is a unit in $\mathscr{RH}_\infty$ (i.e. bijective on $\mathscr{L}_2[0,\infty)$).

4.2 Performance improvement over all frequencies

Sufficient condition If $\|T^{-1}\|_\infty < 1$, then $b(P,C_1) > b(P,C_0)$ which is the desired improvement in robust performance.

Necessary condition For having any hope of achieving $b(P,C_1) > b(P,C_0)$ (i.e. achieving robust performance improvement), we need the necessary condition $\|T\|_\infty > 1$.

Similar to Remark 1, it is worth noting that the necessary condition is easy to test because it involves finding just one (any) energy bounded input r which achieves signal amplification. This is in contrast with the sufficient condition which requires checking that signal amplification occurs for all bounded-energy inputs r. [2]

The following necessary and sufficient result in a SISO setting allows us to check a priori robust performance improvement for a new stabilizing controller C_1 if it were to be inserted into the closed-loop system and all the tests performed without actually replacing C_0 by C_1.

Theorem 4. Given the experimental set-up of Figure 3(a) or Figure 3(b) with $T : \mathscr{L}_2[0,\infty) \underset{r \mapsto z}{\rightarrow} \mathscr{L}_2[0,\infty)$ and define $z' = z - r$. Then
$|T(j\omega)| > 1 \Leftrightarrow |z'(j\omega)| > 2|r(j\omega)|\cos[\pi - (\angle z'(j\omega) - \angle r(j\omega))]$.
Consequently, $\{\omega : T(j\omega) > 1\} \equiv \{\omega : |z'(j\omega)| > 2|r(j\omega)|\cos[\pi - (\angle z'(j\omega) - \angle r(j\omega))]\}$.

Proof. This theorem is proved using the cosine rule for triangle. The detailed proof will be published elsewhere.

Corollary 1. Given the suppositions of Theorem 4. Then the following two statements hold:

$$(a) |\angle z'(j\omega) - \angle r(j\omega)| \leq \frac{\pi}{2} \Rightarrow |T(j\omega)| > 1;$$
$$(b) |z'(j\omega)| > 2|r(j\omega)| \Rightarrow |T(j\omega)| > 1.$$

[1] In a practical sense, to achieve 'adequate' confidence that there will be pointwise in frequency improvement in $\rho(P(j\omega), C(j\omega))$ one needs to check 'adequately' rich signals.

[2] If one obtains signal amplification for an 'adequately' rich family of bounded-energy inputs, then one can obtain 'adequate' confidence that is likely robust performance improvement via an increase in $b(P,C)$. This may be sufficient confidence in a practical scenario.

Proof. Trivial via Theorem 4 statement.

The above theorem gives the necessary and sufficient condition for improvement in $b(P,C)$ that in turn indicates the improvement of robust performance as well as the robust stability margin. Note that the above theorem is only applicable to SISO systems. In the following theorem, sufficient conditions for robust performance improvement are given for MIMO systems.

Theorem 5. Given $T : \mathscr{L}_2[0,\infty) \underset{r \mapsto z}{\rightarrow} \mathscr{L}_2[0,\infty)$ and define $T' = T - I$. Then $\underline{\sigma}(T(j\omega)) > 1$ if $\underline{\sigma}(T'(j\omega)) > 2$ or $T'(j\omega) + T'(j\omega)^* > 0$.

Proof. Due to its length and limited space, it will be published elsewhere.

In the above theorem, two sufficient conditions are presented for pointwise in frequency improvement of $b(P,C)$ for MIMO systems. However, the first condition is impossible to satisfy when the controller change is small whereas the second condition can still be rather easily fulfilled. The second condition also has a profound philosophical implication - that as long as the controller change is in the correct direction, then the controller change does not need to be small. Indeed, it can be arbitrarily large in the correct direction of fulfilment of $T'(j\omega) + T'(j\omega)^* > 0$ and robust performance improvement is still guaranteed. Checking the second condition is equivalent to check a necessary and sufficient condition which is presented in the following theorem.

Theorem 6. Given $T = (\tilde{K}_1 G)(\tilde{K}_0 G)^{-1}$ and $T' = T - I$. Then $T'(j\omega) + T'(j\omega)^* > (\geq) 0$ if and only if

$$(\tilde{K}_1 G)^*(\tilde{K}_1 G)(j\omega) - (\tilde{K}_0 G)^*(\tilde{K}_0 G)(j\omega) > (\geq) [(\tilde{K}_1 - \tilde{K}_0)G]^*[(\tilde{K}_1 - \tilde{K}_0)G](j\omega). \tag{6}$$

Proof. This theorem is proved via sequence of equivalent steps. The detailed proof will be published elsewhere.

In inequality (6), the right hand side is related to the size of the controller change and the left hand side is the difference between the new and the old robust stability margins. This condition states that for the controller change to yield a change in the positive-real direction (i.e. $T'(j\omega) + T'(j\omega)^* > 0$) which then guarantees robust performance improvement, we need the controller change to be such that it has larger impact on the increase in $b(P,C)$ than it has on the size of the transfer function $T'(j\omega)$. This is needed so that the left hand side is greater than the right hand side in inequality (6).

In robust adaptive control, if one is close to the critical Nyquist point and also has no information on which direction to perform a controller change, it is always better to do small changes on the controller. These kind of results then can only give a lower bound on the maximum performance degradation and we often are content with this as an acceptable compromise to robust adaptive control algorithms and use this kind of argument to justify why one should do small steps so that we do not inadvertently loose stability. But if one is not completely lacking all information and can perform the tests in this paper, then there is a large set of directions where huge controller changes are perfectly acceptable and indeed yield performance and stability margin improvement. This means we are allowed to take arbitrary huge steps that satisfy condition (6), which corresponds to a step in the positive-real direction and still attain robust performance improvement. Note that this is equivalent to $< z', r >_{\mathscr{L}_2} \geq 0 \; \forall r \in \mathscr{L}_2$ where $z' = z - r$ and $z = Tr$.

5. CLOSED-LOOP DATA-BASED TESTS FOR PERFORMANCE IMPROVEMENT

To test robust performance improvement conditions, it is however practically unrealistic to perform experiments for all frequencies as well as for all signals in $\mathscr{L}_2$ space. To circumvent this difficulty, experimental procedures are proposed in this section based on the closed-loop set-up shown in Figure 3(a) and 3(b). The first experiment proposed in Dehghani et al. (2009) was a falsification test for internal stability and interestingly, the same falsification test data collected during the experiment can be reprocessed to also check for robust performance improvement of the closed-loop system. This experiment significantly reduces the experimental effort as well as utilizes an extremely simple test procedure.

From falsification test data for internal stability, we use $\bar{Z}$ to check whether this stabilizing controller will improve $b(P,C)$ of the closed-loop system. Note that, for both SISO and MIMO systems, a sufficient condition for improving $b(P,C)$ is the positive-realness of T' where $T' = T - I$. In this regard, the following falsification test for positive-realness of T' is useful. If this test is not falsified then there is a hope for improvement of the robust performance as well as the robust stability margin of the closed-loop system at all frequencies and we need to perform further experiments to obtain absolute guarantee.

Theorem 7. Given $T : \mathscr{L}_2[0,\infty) \underset{r \mapsto z}{\rightarrow} \mathscr{L}_2[0,\infty)$ and define $T' = T - I$. Let the suppositions of Theorem 2 and Assumptions 1 and 2 hold. Let e_i denote a reference signal where a step is applied at the i-th input while the other inputs are kept as 0. Perform n experiments with reference signal $r(t) = e_i(t), i = 1,\dots,n$ and let $\bar{z}_i$ be the steady state output of the map $T : r \mapsto z$ recorded in each experiment. Define $\bar{Z} = [\bar{z}_1,\dots,\bar{z}_n]$. Then $\exists \omega_1 > 0$: $T'(j\omega) + T'(j\omega)^* > 0 \; \forall \; \omega \in [0,\omega_1] \Leftrightarrow \bar{Z} + \bar{Z}^T > 2I$.

Proof. Due to limited space, it will be published elsewhere.

To check the conditions of Theorems 4 and 5, an experiment will be performed up to the frequency ω_0 such that $|T'|$ is much smaller than unity for all $\omega > \omega_0$. Since $T = I + T'$, the collected closed-loop data up to the frequency ω_0 will be sufficient to characterize the required properties onto the system T as $|T| \approx 1$ when $|T'| \ll 1$.

6. SIMULATION EXAMPLE

Although the experiments proposed in this paper are based on the unknown plant, however, for simulation purpose the plant transfer function is considered as known. We consider the same SISO example presented in Dehghani et al. (2009) to check whether the new controller improves robust performance or not when it were to be inserted into the closed-loop.

Let a SISO plant which is not known to the designer be given by

$$P = \frac{-186.66(s-5)(s+4.5)}{(s+10)^2(s+7)(s+6)}.$$

A stabilizing controller

$$C_0 = \frac{0.021(s+10.92)(s+8.87)(s+7.31)(s+5.93)}{(s^2+8.6s+19.84)(s^2-0.603s+5.34)}$$

is physically connected to the plant in closed-loop. In Dehghani et al. (2009), it is shown that an attractive new controller

$$C_1 = \frac{0.33(s+0.586)(s+2.99)(s+3.416)}{(s+2)(s^2+2.26s+3.52)}$$

also ensures internal stability of the closed-loop system if C_0 were to be replaced by C_1. We have used the same set of left coprime factors for C_0 and C_1 which were given in Dehghani et al. (2009). To check the robust performance improvement, we do the following experiments.

(a) On the experimental set-up shown in Figure 3(a) or 3(b), we apply the test method of Theorem 7 and record $\bar{Z} = 5.68$. Since $\bar{Z} + \bar{Z}^T > 2I$, this test does not falsify the necessary and sufficient condition for strictly positive-realness of T' at DC and its neighborhood frequencies. Consequently, no conclusion can be drawn whether this new controller improves robust performance or not at all other frequencies.

(b) We do a sine-sweep on the experimental set-up shown in Figure 3(a) or 3(b) starting from DC frequency and the corresponding magnitude plot of T' is shown in Figure 4. Notice that the exact plot is irrelevant as we need only some key

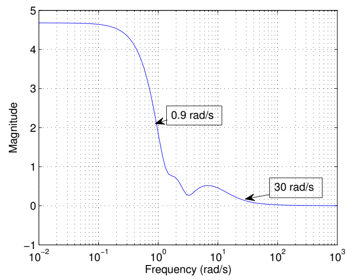

Fig. 4. Magnitude plot of T'

properties and points on this curve to characterize the transfer function T. Beyond 30 rad/s, $|T'| \approx 0$ and so $|T| \approx 1$ as $T = I + T'$. This means we only need to test up to approximately 30 rad/s. Also, if the sine-sweep confirms confidently that $|T'| > 2$ up to approximately 0.9 rad/s, as indeed depicted in Figure 4, then we know via Corollary 1 that up to 0.9 rad/s we have robust performance improvement.

(c) From the sine-sweep data beyond 0.9 rad/s, compute also $\mathrm{Re}[z'(j\omega)^* r(j\omega)]$. This is plotted in Figure 5. The exact frequency plot is irrelevant as the information that needs to be extracted only uses a few key data points. From this experiment,

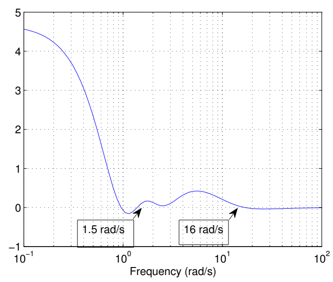

Fig. 5. $\mathrm{Re}[z'(j\omega)^* r(j\omega)]$ vs. frequency

we can confidently conclude that there is robust performance improvement also in the frequency range (1.5 rad/s, 16 rad/s) as $\mathrm{Re}[z'(j\omega)^* r(j\omega)] \geq 0$ in this frequency range implying $T'(j\omega) + T'(j\omega)^* \geq 0$.

(d) Consequently, robust performance improvement is guaranteed in [0, 0.9 rad/s) and (1.5 rad/s, 16 rad/s) via the preceding tests. The question of whether robust performance improvement happens also in the frequency intervals [0.9 rad/s, 1.5

rad/s] and [16 rad/s, 30 rad/s] cannot be answered without the precise data having experimental effort related to Theorem 4 as in Figure 6.

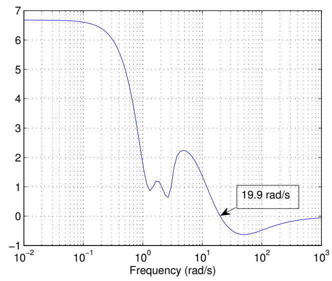

Fig. 6. $|T'(j\omega)| - 2cos(\pi - \angle T'(j\omega))$ vs. frequency

7. CONCLUSIONS

In this paper, conditions are derived for robust performance improvement based on the closed-loop data when an existing controller is replaced by an attractive new stabilizing controller in the feedback loop. For the proposed tests, the plant model is assumed to be unknown which is a common assumption in robust adaptive control. A sufficient condition is derived for robust performance improvement that shows that as long as the controller change is done in the positive-real direction, such a controller change can be of an arbitrary large size. The experimental set-up used in this paper is identical to Dehghani et al. (2009), Lanzon et al. (2006), however, the proposed conditions for robust performance improvement cannot be obtained from the test results of Dehghani et al. (2009), Lanzon et al. (2006) directly.

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