Linear Equation Solvers via Robust and Optimal Control

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Solving linear systems of equations is the core problem of numerical linear algebra. In the last 50 years, there has been a huge development of very different powerful iterative algorithms and software packages. Nevertheless, at present, algorithms that combine global stability with low computational complexity are available only for restricted classes of matrices.

We present a new iterative approach which is inspired by feedback stabilization schemes from robust control and yields a control system whose parameters can be tuned to achieve prescribed convergence properties. In particular, we analyze the control system

$$\begin{bmatrix} x_{t+1} \\ u_{t+1} \end{bmatrix} = \begin{bmatrix} I & \Phi \\ -A & -\Xi \end{bmatrix} \begin{bmatrix} x_t \\ u_t \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} b$$

with output function $z_t = Ax_t - b$. We apply standard techniques from robust and optimal control to stabilize the dynamics of the system. The resulting sequence of state-vectors $(x_t)_{t\in\mathbb{N}}$ converges to the solution of $Ax = b$. We show that certain choices of the parameters Φ and Ξ yield known algorithms, such as GMRES(m), LQRES or splitting methods. On the other hand, other choices for Φ and Ξ yield new families of algorithms which are globally stable. We optimize a subfamily of those algorithms and compare their convergence properties with the classical algorithms.