Authors' Camera Ready Manuscript. To appear in the
Proceedings of the 17th European Control Conference, Limassol, Cyprus, June 2018,
Please cite using bibliographic data of the associated published version.

Robust Stabilization of Networked Multi-agent Systems with Strict Negative Imaginary Uncertainties: An LMI Approach*

Ola Skeik[1] and Alexander Lanzon[2]

Abstract— **This paper presents sufficient conditions, in a linear matrix inequality (LMI) framework, for a control protocol to robustly stabilize a networked multi-agent system in the presence of strict negative imaginary (SNI) uncertainties with certain DC size. The control protocol under consideration is based on relative state measurements of neighbouring agents and absolute state measurements of a subset of agents and therefore the network graph which models the information exchange among agents is assumed a connected undirected graph with at least one self-loop. Under such assumptions on the network graph it is shown that the negative imaginary (NI) property of the networked system is unchanged due to transformation. Hence, the sufficient conditions are derived based on multiple reduced order subsystems satisfying the NI property simultaneously. The paper also summarizes the steps required to design the control protocol parameters; which are a positive scalar and a gain matrix. It is shown that appropriately adjusting the positive scalar while leaving the gain matrix unchanged guarantees robustness of the control protocol to variations in the network topology as well as robustness to SNI uncertainties. A numerical example is given to show the usefulness of the proposed results.**

I. INTRODUCTION

Distributed control of networked multi-agent systems and negative imaginary systems theory are two distinct areas of significant importance to the control systems community.

Distributed control of networked multi-agent systems has been an active field of study over the past two decades. See, for instance, [1]–[4] for an extensive overview. Broadly speaking, agents interact locally with each other to achieve a desired collective behaviour or a global control objective such as stability of a network, synchronization, consensus, etc. The dynamics of agents and the interaction among them play a key role in analysing the problem for a certain behaviour and in protocol design. Also, robustness of the designed protocol is a key factor that needs to be considered. Robust stability of multi-agent dynamical systems was studied in [5] where three different types of multiplicative perturbations were considered. A distributed H_∞ control problem for multi-agent systems with linear dynamics was addressed in [6] and a new definition of an H_∞ performance region was introduced to evaluate the performance of a networked system subject to external disturbances. Robust synchronization of uncertain multi-agent networks was addressed in [7] and [8] with uncertainties in the form of additive perturbations in [7] and in the form of coprime factor perturbations in [8]. Robust consensus control for multi-agent systems involving gap metric uncertainties was investigated in [9].

Negative imaginary (NI) systems are systems with negative imaginary frequency response. The theory of NI systems was first introduced in [10] in the interest to develop, in a systematic framework, robust stability results for flexible structures with co-located force actuators and position sensors. Since then, significant progress in this area has been reported. For example, [10]–[13] provide robust stability analysis results for NI systems, [14]–[18] consider negative imaginary synthesis problems, [19] studies the negative imaginary property of descriptor linear systems, [20] introduces the notion of discrete-time NI systems, and [21] studies non-proper, non-rational NI systems. Robust output feedback consensus was investigated in [22], [23] for networks of homogeneous and heterogeneous negative imaginary multi-agent systems respectively under external disturbance and model uncertainties. In this paper, we consider the situation where the only thing known about each agent/system is that the perturbation belongs to the strict negative imaginary (SNI) class and we seek to address the following question: How can we design distributed controllers that can robustly stabilize the uncertain closed loop networked multi-agent system in the presence of SNI uncertainties of certain DC size, and achieve robustness to variations in the network topology? The results in [15]–[18] address the case where a system's perturbation is SNI. The results showed that if a controller is designed such that the closed loop system is negative imaginary, then the robust feedback stability result in [10]–[13] can be applied to guarantee robustness to this class of uncertainties. The aforementioned papers deal with individual systems. In particular, [16] proposed a systematic robust static state feedback synthesis method for (single) systems with SNI uncertainty in an LMI framework. In this paper we address such issue for networked systems. We consider the case in which the uncertain agents are connected over a network topology that can be modelled by an undirected graph with self-loops. Furthermore, the information available for each agent is relative state measurements and only a subset of agents can additionally have access to their own absolute state measurements. The paper presents sufficient conditions in an LMI framework that ensure the existence of a control protocol that achieves the desired

*This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) [grant number EP/R008876/1] and a University of Manchester President's Doctoral Scholar Award. All research data supporting this publication are directly available within this publication.

[1]Ola Skeik is with the School of Electrical and Electronic Engineering, University of Manchester M13 9PL, UK ola.skeik@manchester.ac.uk

[2]Alexander Lanzon is with the School of Electrical and Electronic Engineering, University of Manchester M13 9PL, UK Alexander.Lanzon@manchester.ac.uk

objectives. An algorithm for control protocol design is also provided. This study is inspired by the work in [6].

The paper is further organized as follows: The notations used in this paper and some preliminaries on negative imaginary systems and graph theory are presented in Section II. A description of the problem is stated in Section III. The problem is also addressed in this section where the main results of this paper are provided. A numerical example to support the results is provided in Section IV. The paper is then concluded in Section V.

II. Preliminaries

A. Notation

Let $\mathbb{R}^{m\times n}$ and $\mathcal{R}^{m\times n}$ denote the set of $m\times n$ real matrices and real, rational, proper transfer function matrices respectively. Given a matrix A, A^T and A^* denote the transpose and the complex conjugate transpose of A respectively. $\lambda_i(A)$ and $\lambda_{\max}(A)$ denote the ith and the largest eigenvalue (when the matrix has only real eigenvalues) of A respectively. $\Re[\cdot]$ is the real part of a complex number. I_N denotes the identity matrix of dimension N. $A\otimes B$ denotes the Kronecker product of matrices A and B. diag$(A_1,\ldots,A_N)$ represents a block-diagonal matrix with matrices A_i for $i=1,\ldots,N$ on the main diagonal. For a real symmetric matrix X, the notation $X>0$ $(X\geq 0)$ means that matrix X is positive definite (positive semidefinite).

B. Negative Imaginary Systems

Negative imaginary systems are defined as follows.

Definition 1 ([11]): A square real, rational, proper transfer function matrix $R(s)$ is termed negative imaginary if

1) $R(s)$ has no poles at the origin and in $\Re[s]>0$;
2) $j[R(j\omega)-R^*(j\omega)]\geq 0$ for all $\omega\in(0,\infty)$ except values of ω where $j\omega$ is a pole of $R(s)$;
3) if $j\omega_0$ with $\omega_0\in(0,\infty)$ is a pole of $R(s)$, then it is a simple pole and the residue matrix $K_0\triangleq\lim_{s\to j\omega_0}(s-j\omega_0)jR(s)$ is Hermitian and positive semidefinite.

Note that the aforementioned definition, which we use in this paper, is for NI systems without poles at the origin. A definition for NI systems which includes poles at the origin is given in [12].

Strict negative imaginary systems are defined as follows.

Definition 2 ([10]): A square real, rational, proper transfer function matrix $R(s)$ is termed strictly negative imaginary if

1) $R(s)$ has no poles in $\Re[s]\geq 0$;
2) $j[R(j\omega)-R^*(j\omega)]>0$ for $\omega\in(0,\infty)$.

The following lemma is used to check whether a system belongs to the class of NI or not.

Lemma 1 ([16]): Let (A,B,C,D) be a state space realization of $R(s)\in\mathcal{R}^{m\times m}$ where A $\in\mathbb{R}^{n\times n}$, B $\in\mathbb{R}^{n\times m}$, C $\in\mathbb{R}^{m\times n}$, D $\in\mathbb{R}^{m\times m}$ with $m\leq n$. If $\det(A)\neq 0$, $D=D^T$ and there exists a real matrix $Y=Y^T>0$ such that

$$AY+YA^T\leq 0 \quad \text{and} \quad B+AYC^T=0, \tag{1}$$

then $R(s)$ is negative imaginary.

The following lemma characterises robust stability for NI systems. The result we use here is not the main theorem (see [10, Th. 5] or [11, Th. 1] for the main feedback stability theorem) as stated in the literature but a corollary to the principal theorem stated in the same form as the small gain theorem in order to suit our purpose. This was first proposed in [10] for stable NI systems and later shown to be also valid for marginally stable NI systems in [11].

Lemma 2 ([10], [11]): Given $\gamma>0$ and a negative imaginary transfer function matrix $R(s)$. Then the positive feedback interconnection $[\Delta(s),R(s)]$ is internally stable for all strict negative imaginary transfer function matrices $\Delta(s)$ satisfying $\Delta(\infty)R(\infty)=0$, $\Delta(\infty)\geq 0$ and $\lambda_{\max}(\Delta(0))<(1/\gamma)$ (respectively, $\leq(1/\gamma)$) if and only if $\lambda_{\max}(R(0))\leq\gamma$ (respectively, $<\gamma$).

C. Graph theory

In this paper we focus our attention on undirected graphs. An undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E})$ consists of a non-empty finite vertex set $\mathcal{V}=\{v_1,v_2,\ldots,v_N\}$ and an edge set $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$ of unordered pairs of vertices, called edges. An edge in $\mathcal{G}$ is denoted by (v_i,v_j). If $(v_i,v_j)\in\mathcal{E}$, then vertices (i.e., agents) v_i and v_j are adjacent (or neighbours) and can obtain information from each other. The set of neighbours of vertex v_i is defined as $\mathcal{N}_i=\{v_j\in\mathcal{V}:(v_j,v_i)\in\mathcal{E}\}$. An edge (v_i,v_i) is called a self-loop. A graph is said to be simple if it contains no self-loops and no repeated edges. A loop around vertex v_i means that agent v_i has access to its own absolute measurements. A path in a graph from v_i to v_j is a sequence of edges of the form $(v_i,v_{i+1}),(v_{i+1},v_{i+2}),\ldots,(v_{j-1},v_j)$. An undirected graph is connected if there is an undirected path between every pair of distinct vertices. The adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined as $a_{ij}=a_{ji}=1$ if $(v_i,v_j)\in\mathcal{E}$, 0 otherwise and $a_{ii}=1$ if v_i has a self loop, 0 otherwise. Note that for a simple graph $a_{ii}=0$ for $i=1,\ldots,N$. The Laplacian matrix $\mathcal{L}=[l_{ij}]\in\mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined as $l_{ij}=-a_{ij}$, for $i\neq j$ and $l_{ii}=\sum_{k=1}^{N}a_{ik}$ for $i=1,\ldots,N$. Based on the adjacency matrix, this definition can fit for both simple graphs and for graphs with self-loops. The notation $\hat{\mathcal{L}}$ will hereafter be used to indicate the Laplacian matrix associated with a graph with self-loops.

Lemma 3 ([6]): For a graph with at least one self-loop, the Laplacian matrix $\hat{\mathcal{L}}$ is positive definite, if the graph is connected.

III. Main Results

A. Problem Statement

Consider a network of N linear uncertain agents. The dynamics of the ith agent are described by

$$\begin{aligned}\dot{x}_i(t)&=Ax_i(t)+B_1w_i(t)+B_2u_i(t),\\ z_i(t)&=C_1x_i(t),\\ \hat{w}_i(s)&=\Delta_i(s)\hat{z}_i(s),\end{aligned} \tag{2}$$

where $x_i(t)\in\mathbb{R}^n$, $w_i(t)\in\mathbb{R}^m$, $u_i(t)\in\mathbb{R}^p$, and $z_i(t)\in\mathbb{R}^m$ are the state, disturbance, control input and controlled output of the ith agent, respectively with $m\leq n$. The matrices $A\in\mathbb{R}^{n\times n}$, $B_1\in\mathbb{R}^{n\times m}$, $B_2\in\mathbb{R}^{n\times p}$, $C_1\in\mathbb{R}^{m\times n}$ are known constant matrices. The transfer function matrix $\Delta_i(s)$ represents the uncertainty in the dynamics of the ith agent where $\hat{w}_i(s)$ and $\hat{z}_i(s)$ are the Laplace transform of $w_i(t)$ and $z_i(t)$ respectively. The following assumption is made about the uncertainty in the agents dynamics.

Assumption 1: For all $i \in \{1,\ldots,N\}$, the uncertainty $\Delta_i(s)$ is strict negative imaginary and satisfies $\Delta_i(\infty) \geq 0$ and $\lambda_{\max}(\Delta_i(0)) \leq (1/\gamma)$, where $\gamma > 0$ is a pre-specified number.

Following [6], the control protocol for the ith agent is

$$u_i(t) = cK\left(\sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) + a_{ii}x_i(t)\right), \quad (3)$$

where $c > 0$ is the coupling strength to be selected, $K \in \mathbb{R}^{p\times n}$ is the control feedback gain matrix to be designed and a_{ij} are the elements of the adjacency matrix with $a_{ii} = 1$ $\forall i \in \{1,\ldots,q\}$, and $a_{ii} = 0$ $\forall i \in \{q+1,\ldots,N\}$ and $(q \ll N)$.
The interpretation of protocol (3) is that each agent can collect relative state measurements with respect to its neighbours. In addition, a subset of agents can collect their own absolute state measurements. It is assumed, without any loss of generality, that the subset of the agents which can collect their own absolute state measurements are the first q agents with $(q \ll N)$. Therefore, in terms of the network graph which models the interaction among the agents the following assumption can be made.

Assumption 2: The graph $\mathcal{G}$ is connected, undirected and at least one vertex has a self-loop.

Dropping time dependency and Laplace variable dependency where it is clear from the context, it is clear that agent dynamics (2) can be rewritten as

$$\begin{aligned}\dot{x} &= (I_N \otimes A)x + (I_N \otimes B_1)w + (I_N \otimes B_2)u,\\ z &= (I_N \otimes C_1)x,\\ \hat{w} &= \Delta(s)\hat{z},\end{aligned} \quad (4)$$

and control protocol (3) can be rewritten as

$$u = (c\hat{\mathcal{L}} \otimes K)x, \quad (5)$$

where $x = \left[x_1^T,\ldots,x_N^T\right]^T \in \mathbb{R}^{nN}$, $w = \left[w_1^T,\ldots,w_N^T\right]^T \in \mathbb{R}^{mN}$, $u = \left[u_1^T,\ldots,u_N^T\right]^T \in \mathbb{R}^{pN}$, $z = \left[z_1^T,\ldots,z_N^T\right]^T \in \mathbb{R}^{mN}$, $\Delta(s) = \text{diag}(\Delta_1(s),\ldots,\Delta_N(s))$, $\hat{w}$ is the Laplace of w, $\hat{z}$ is the Laplace of z and $\hat{\mathcal{L}} \in \mathbb{R}^{N\times N}$ is the Laplacian matrix associated with $\mathcal{G}$. The uncertain closed loop networked multi-agent system resulting from applying control protocol (5) (or equivalently (3) to each agent i in (2)) to the uncertain agents (4) is given by

$$\begin{aligned}\dot{x} &= \Big((I_N \otimes A) + (c\hat{\mathcal{L}} \otimes B_2K)\Big)x + (I_N \otimes B_1)w,\\ z &= (I_N \otimes C_1)x,\end{aligned} \quad (6)$$

and

$$\hat{w} = \Delta(s)\hat{z}. \quad (7)$$

Observe that $\Delta(s)$ is SNI since each $\Delta_i(s), i \in \{1,\ldots,N\}$ is SNI and satisfies $\Delta(\infty) \geq 0$. Moreover, $\lambda_{max}(\Delta(0)) \leq 1/\gamma$ since $\lambda_{max}(\Delta(0)) = \max_{i=1,\ldots,N} \lambda_{\max}(\Delta_i(0)) \leq 1/\gamma$. The transfer function matrix of the nominal closed loop networked multi-agent system from w to z is strictly proper and given by

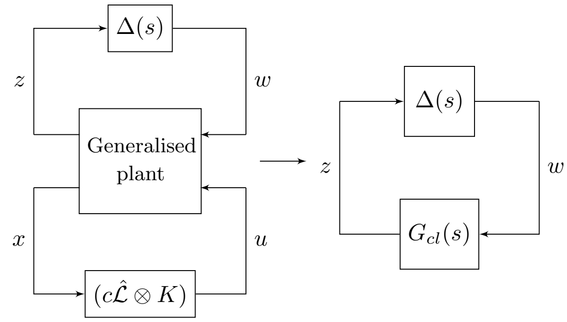

Fig. 1. Networked multi-agent system with SNI uncertainty.

$$G_{cl}(s) = C_{cl}(sI_{nN} - A_{cl})^{-1}B_{cl}, \quad (8)$$

where $A_{cl} = (I_N \otimes A) + (c\hat{\mathcal{L}} \otimes B_2K)$, $B_{cl} = (I_N \otimes B_1)$, $C_{cl} = (I_N \otimes C_1)$ and has an associated DC gain of

$$\lambda_{\max}(G_{cl}(0)) = \lambda_{\max}(C_{cl}(-A_{cl})^{-1}B_{cl}). \quad (9)$$

The uncertain networked multi-agent system is depicted in Fig. 1. Note that since each system in the network is modelled with a heterogeneous uncertainty, the networked system is considered heterogeneous from this perspective.

In this paper we are concerned with the the distributed robust stabilization problem. According to Lemma 2, this problem can be stated as follows.

Definition 3: Given $\gamma > 0$, control protocol (3) is said to robustly stabilize the networked system with agent dynamics (2) against any strict negative imaginary uncertainty satisfying Assumption 1 if it is designed such that the transfer function matrix (8) is negative imaginary and satisfies the DC gain condition $\lambda_{max}(G_{cl}(0)) < \gamma$.

B. Problem reduction and robust protocol design

To address the distributed robust stabilization problem for a class of networked systems with SNI uncertainties, we take the following steps:

1) Problem reduction: The large scale networked system is first analysed. Under Assumption 2 of the network graph, Theorem 1 below states that analysis of the transfer function matrix $G_{cl}(s)$ is equivalent to analysis of multiple reduced order subsystems each having the order of a single system.

Theorem 1: Given $\gamma > 0$ and assume that that the network topology $\mathcal{G}$ satisfies Assumption 2. Let $\hat{\mathcal{L}}$ be the Laplacian matrix of $\mathcal{G}$ and let λ_i for all $i \in \{1,\ldots,N\}$ be the eigenvalues of $\hat{\mathcal{L}}$. Then, the transfer function matrix (8) of the networked system (6) is negative imaginary and satisfies $\lambda_{\max}(G_{cl}(0)) < \gamma$ if and only if for all $i \in \{1,\ldots,N\}$, the transfer functions $\tilde{G}_i(s)$ of the following N isolated subsystems

$$\begin{aligned}\dot{\tilde{x}}_i &= (A + c\lambda_i B_2K)\tilde{x}_i + B_1\tilde{w}_i,\\ \tilde{z}_i &= C_1\tilde{x}_i,\end{aligned} \quad (10)$$

are all negative imaginary and satisfy $\lambda_{\max}(\tilde{G}_i(0)) < \gamma$ simultaneously, where $\tilde{G}_i(s) = C_1(sI - A - c\lambda_i B_2 K)^{-1} B_1$.

Proof: The proof will be published elsewhere. ■

This means that the NI property of the networked system is not affected due to transformation. Therefore, it is adequate to find a positive scalar c and a gain matrix K such that systems (10) satisfy the NI property simultaneously for networked dynamical system (6) to satisfy the NI property. The basis of Theorem 2, which is stated next, is based on Theorem 1.

2) Robust protocol design: Based on the reduced order subsystems, sufficient conditions are provided in Theorem 2 which ensure the existence of a positive scalar c and a gain matrix K such that a control protocol in the form of (3) is able to maintain stability of the networked system in face of uncertainties which belong to the SNI class.

Theorem 2: Given $\gamma > 0$, a network topology that satisfies Assumption 2 and an uncertain multi-agent system (2) with $C_1 B_2 = 0$, $m \leq n$ and (A, B_2) controllable. If there exists a matrix $Y = Y^T > 0$ and a scalar $\tau > 0$ such that

$$\begin{bmatrix} AY + YA^T - \tau B_2 B_2^T & B_1 + AYC_1^T \\ B_1^T + C_1 Y A^T & 0 \end{bmatrix} \leq 0, \quad (11)$$

$$C_1 Y C_1^T < \gamma I, \quad (12)$$

$$\det(AY - \frac{1}{2}\tau B_2 B_2^T) \neq 0, \quad (13)$$

then there exists a feedback gain matrix K and a scalar $c \geq \frac{\tau}{\min\limits_{i\in\{1,\ldots,N\}} \lambda_i}$ such that control protocol (3) robustly stabilizes the networked multi-agent system in the presence of any strict negative imaginary uncertainty satisfying Assumption 1. Moreover, a suitable feedback gain matrix K is given by $K = -0.5 B_2^T Y^{-1}$.

Proof: The proof will be published elsewhere. ■

While the determinant condition is non-convex, and appears due to the NI property excluding poles at the origin, it is not troublesome. This is because a feasible solution for Y and τ can always be obtained first by solving the LMI conditions and then checking whether the computed values satisfy the determinant condition or not. If they do not, then a small increase in τ often resolves the problem. Therefore, we can summarize the steps needed to design control protocol (3) in the following algorithm:

1) Solve the LMI conditions (11)–(12) for $Y > 0$ and $\tau > 0$. Then, check whether the determinant condition (13) is satisfied or not. If not, perturb τ and/or Y to satisfy all of (11)–(13).
2) Let the feedback gain matrix $K = -0.5 B_2^T Y^{-1}$.
3) Select the coupling strength c not less than the threshold value $c_{th} = \frac{\tau}{\min\limits_{i=1,\ldots,N} \lambda_i}$, where $\lambda_i \ \forall i \in \{1, \ldots, N\}$ are the eigenvalues of $\hat{\mathcal{L}}$.

Remark 1: The reason for control protocol (3) to have two design parameters is simply to avoid construction of a different feedback gain matrix K for different network topologies and thereby leaving the effect of the network topology to be handled by the positive scalar c. Thus, by appropriately selecting the positive scalar c robust stability in the agents dynamics and robustness to variations in the network topology can be guaranteed.

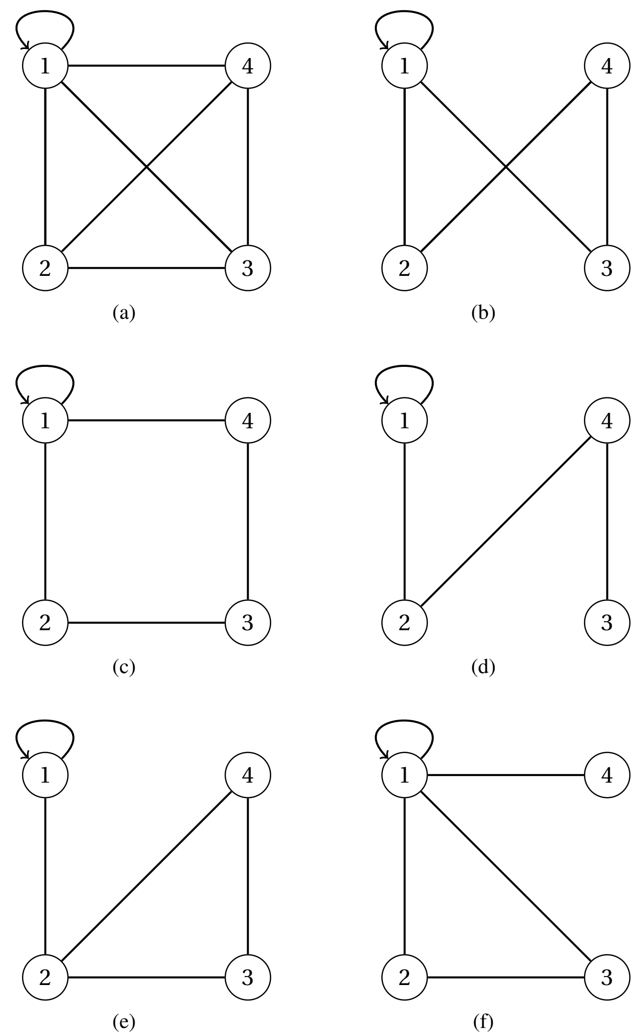

Fig. 2. Six different network topologies.

IV. Numerical Example

Consider a group of $N = 4$ uncertain agents where the dynamics of each agent are

$$A = \begin{bmatrix} -0.4 & -0.9 \\ 0.2 & -0.8 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

We can easily see that $C_1 B_2 = 0$, $m \leq n$ and (A, B_2) is controllable. Now choose $\gamma = 1$. We follow the steps of the algorithm in the preceding section to design the control protocol parameters. Using the YALMIP [24] and SeDuMi [25] toolboxes, we solve the LMI conditions (11) and (12). A feasible solution is given by

$$Y = \begin{bmatrix} 2.0816 & 0.7 \\ 0.7 & 0.8 \end{bmatrix} > 0, \quad \tau = 1.1307. \quad (14)$$

For the values of Y and τ as given in (14), condition (13) is satisfied; i.e. $\det(AY - \frac{1}{2}\tau B_2 B_2^T) = 0.8703 \neq 0$.

Consequently, the feedback gain matrix K is given by

$$K = -0.5 B_2^T Y^{-1} = \begin{bmatrix} -0.3403 & 0.2978 \end{bmatrix}.$$

Now consider the six different network topologies in Fig. 2. We select $c = 15$. Accordingly, it is guaranteed by Theorem 2 that control protocol (3) with the designed values of c and K achieves robust stability for all six networked systems (i.e. agents may be connected over any of the six network topologies in Fig. 2) in the presence of SNI uncertainties with DC gains less than or equal to unity since $c = 15$ is greater than the threshold value c_{th} corresponding to each of the network graphs in Fig. 2. These values are 5.4176, 6.0662, 6.0662, 9.3745, 8.1233, 5.4176 which correspond to the network graphs of Fig. 2a to Fig. 2f respectively.

We can easily demonstrate that for some specific uncertainties the conclusion holds. We do so only for the network topology in Fig. 2a as the conclusion can be demonstrated for the remaining network graphs in a similar manner. For instance, choose $\Delta_1(s) = 0.5/(s+1)$, $\Delta_2(s) = 1/(s+3)$, $\Delta_3(s) = (1-s)/(1+s)$, $\Delta_4(s) = 1/(s+1)^2$ which are SNI. $\Delta(s)$ in Fig. 1 has $\lambda_{max}(\Delta(0)) = 1 \leq 1/\gamma$. The poles of $G_{cl}(I - \Delta G_{cl})^{-1}$ are -0.06, -0.062, -0.28, -0.45, -1.4, $-1.8 \pm j0.11$, $-1.3 \pm j0.71$ -3.2, -21, -21, and -25. Since all closed-loop poles are in the left half plane, we conclude that the heterogeneous perturbed closed loop system of Fig. 1 is internally stable.

V. CONCLUSION

In this paper we showed how to design distributed controllers that can robustly stabilize the uncertain closed loop networked multi-agent system in the presence of strict negative imaginary uncertainties of certain DC size, and also achieve robustness to variations in the network topology. The problem under consideration was addressed by first analysing the large scale networked system where it was shown that the NI property of the networked system remains unchanged due to transformation under certain assumptions on the network graph. Consequently, sufficient conditions were provided which ensure the existence of a control protocol that satisfies the desired objectives. A numerical example was finally given to demonstrate the results.

REFERENCES

[1] R. Olfati-Saber, J. A. Fax, and R. M. Murray, "Consensus and cooperation in networked multi-agent systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 215–233, 2007.

[2] Y. Cao, W. Yu, W. Ren, and G. Chen, "An overview of recent progress in the study of distributed multi-agent coordination," *IEEE Transactions on Industrial Informatics*, vol. 9, no. 1, pp. 427–438, Feb. 2013.

[3] D. J. Hill, "Multi-agent systems with dynamical topologies: Consensus and applications," *IEEE Circuits and Systems Magazine*, vol. 13, no. 3, pp. 21–34, Jan. 2013.

[4] S. Knorn, Z. Chen, S. Member, and R. H. Middleton, "Overview : Collective Control of Multiagent Systems," *IEEE Transactions on Control of Network Systems*, vol. 3, no. 4, pp. 334–347, 2016.

[5] S. Hara and H. Tanaka, "D-stability and robust stability conditions for LTI systems with generalized frequency variables," in *49th IEEE Conference on Decision and Control (CDC)*, Dec. 2010, pp. 5738–5743.

[6] Z. Li, Z. Duan, and G. Chen, "On H_∞ and H_2 performance regions of multi-agent systems," *Automatica*, vol. 47, no. 4, pp. 797–803, Apr. 2011.

[7] H. L. Trentelman, K. Takaba, and N. Monshizadeh, "Robust synchronization of uncertain linear multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 58, no. 6, pp. 1511–1523, Jun. 2013.

[8] H. Jongsma, H. Trentelman, and M. Camlibel, "Robust synchronization of coprime factor perturbed networks," *Systems and Control Letters*, vol. 95, pp. 62–69, Sep. 2016.

[9] L. D. Alvergue, G. Gu, and L. Qiu, "Output consensus control for multi-agent systems in the presence of gap metric uncertainties," in *54th IEEE Conference on Decision and Control (CDC)*, Osaka, Japan, Dec. 2015, pp. 5581–5586.

[10] A. Lanzon and I. R. Petersen, "Stability robustness of a feedback interconnection of systems with negative imaginary frequency response," *IEEE Transactions on Automatic Control*, vol. 53, no. 4, pp. 1042–1046, May 2008.

[11] J. Xiong, I. R. Petersen, and A. Lanzon, "A negative imaginary lemma and the stability of interconnections of linear negative imaginary systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 10, pp. 2342–2347, Oct. 2010.

[12] M. A. Mabrok, A. G. Kallapur, I. R. Petersen, and A. Lanzon, "Generalizing negative imaginary systems theory to include free body dynamics: Control of highly resonant structures with free body motion," *IEEE Transactions on Automatic Control*, vol. 59, no. 10, pp. 2692–2707, Oct. 2014.

[13] A. Lanzon and H. J. Chen, "Feedback stability of negative imaginary systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5620–5633, Nov. 2017.

[14] Z. Song, A. Lanzon, S. Patra, and I. R. Petersen, "Towards controller synthesis for systems with negative imaginary frequency response," *IEEE Transactions on Automatic Control*, vol. 55, no. 6, pp. 1506–1511, Jun. 2010.

[15] I. R. Petersen and A. Lanzon, "Feedback control of negative-imaginary systems," *IEEE Control Systems Magazine*, vol. 30, no. 5, pp. 54–72, Oct. 2010.

[16] Z. Song, A. Lanzon, S. Patra, and I. R. Petersen, "A negative-imaginary lemma without minimality assumptions and robust state-feedback synthesis for uncertain negative-imaginary systems," *Systems and Control Letters*, vol. 61, no. 12, pp. 1269–1276, Dec. 2012.

[17] M. A. Mabrok, A. G. Kallapur, I. R. Petersen, and A. Lanzon, "A generalized negative imaginary lemma and Riccati-based static state-feedback negative imaginary synthesis," *Systems and Control Letters*, vol. 77, pp. 63–68, Mar. 2015.

[18] J. Xiong, J. Lam, and I. R. Petersen, "Output feedback negative imaginary synthesis under structural constraints," *Automatica*, vol. 71, pp. 222–228, Sep. 2016.

[19] J. Xiong, A. Lanzon, and I. R. Petersen, "Negative imaginary lemmas for descriptor systems," *IEEE Transactions on Automatic Control*, vol. 61, no. 2, pp. 491–496, Feb. 2016.

[20] A. Ferrante, A. Lanzon, and L. Ntogramatzidis, "Discrete-time negative imaginary systems," *Automatica*, vol. 79, pp. 1–10, May 2017.

[21] ——, "Foundations of not necessarily rational negative imaginary systems theory: Relations between classes of negative imaginary and positive real systems," *IEEE Transactions on Automatic Control*, vol. 61, no. 10, pp. 3052–3057, Oct. 2016.

[22] J. Wang, A. Lanzon, and I. R. Petersen, "Robust output feedback consensus for networked negative-imaginary systems," *IEEE Transactions on Automatic Control*, vol. 60, no. 9, pp. 2547–2552, Sep. 2015.

[23] ——, "Robust cooperative control of multiple heterogeneous negative-imaginary systems," *Automatica*, vol. 61, pp. 64–72, Nov. 2015.

[24] J. Lofberg, "YALMIP : a toolbox for modeling and optimization in MATLAB," in *2004 IEEE International Symposium on Computer Aided Control Systems Design*. Taipei, Taiwan: IEEE, 2004, pp. 284–289.

[25] J. F. Sturm, "Using SeDuMi 1.02 , A Matlab toolbox for optimization over symmetric cones," *Optimization Methods and Software*, vol. 11, no. 1, pp. 625–653, 1999.