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Unfalsified Adaptive Control: A New Controller Implementation and Some Remarks

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Abstract— **The concept of unfalsified adaptive control using multiple controllers and switching ideas has been developed and extensively investigated over the past decade [1]–[16]. In this literature of unfalsified adaptive control, it is required that the controller set only contains linear, bi-proper minimum-phase controllers. In this note, we propose a controller implementation scheme which overcomes this restrictive assumption on the controller set though retaining linearity. Furthermore, we advocate caution in some circumstances where the unfalsified adaptive control algorithm actually connects up a destabilizing controller in the closed-loop for a long period of time, and explain how this phenomenon can arise.**

Index Terms— ***Multiple Model Adaptive Control, Multi-Controller Adaptive Switching, Robust Control.***

I. INTRODUCTION

A strikingly different approach to adaptive control was proposed in [1]–[3] and has been termed Unfalsified Adaptive Control. In this real-time data-driven adaptive control approach, an active controller selection algorithm aims to decide at time intervals which controllers in a given set (with the assumption that it contains at least one stabilizing controller) are falsified. Roughly speaking, a controller is said to be falsified by measurement data when it has the following property: if it were in the feedback loop, a nominated performance requirement (which includes stability) on the closed-loop would fail. Otherwise the control law is said to be unfalsified [3], [4]. A more precise definition of controller falsification will be given in Section II-A.

The innovative ideas of unfalsified adaptive control offer appealing features and intuitive advantages [12], [13], and the possibility of overcoming some of the difficulties (one of which is discussed in the next paragraph) associated with adaptive control switching algorithms [17], [18]. A celebrated property is the fact that the algorithm does not require "any prior assumption on the plant" [12], [13], and there is no need for identifying a model of the true plant. The plant can be of any order, stable or unstable, non-minimum phase, linear or non-linear; see e.g. [13]. Indeed it is stated that all one would need to use the algorithm are (i) a given cost function which satisfies certain prescribed properties, reviewed in Section II, (ii) a set of bi-proper minimum-phase linear controllers (even if the plant is nonlinear), which includes at least one stabilizing controller, and (iii) a switching mechanism (see Section II).

The potential problem with the Multiple Model Adaptive Control (MMAC) algorithms, see [19]–[21] and the references therein—which require one to identify a good approximation of the true plant from a set of candidate plant models at instants of time [19], [20], [22], [23]—is that model-plant mismatch in the closed-loop may cause a new controller, chosen on the basis of its suitability for use with the model, to actually destabilize the true plant [17], probably if the new controller is dissimilar to the controller which it replaces. The unfalsified adaptive control approach can potentially address this problem as it does not require explicit identification of a plant model. Indeed, references [11]–[15], assert that the approach exploits information in the real-time measurement data to evaluate performance levels of all candidate controllers simultaneously *before* insertion of any one of them into the actual closed-loop to replace the existing controller. Put another way, the algorithm evaluates what the performance levels of all candidate controllers would have been if they had been placed in the closed-loop.

The stability and convergence of an unfalsified adaptive control system using multiple controllers and switching are studied in [10]–[13], in which it is stated that if the candidate controller set contains linear, finite dimensional, bi-proper and minimum-phase controllers (with a stabilizing one among them), and if the cost function is continuous and monotone increasing in time and has a certain cost-detectability property (see Section II), then the unfalsified adaptive control algorithm can "reliably" [10], [11], [13] identify controllers that can achieve closed-loop stability and performance specifications based on cost-minimization.

In this paper we first show by way of a simple academic example that the unfalsified adaptive control approach unfortunately gives no guarantee of protection against inserting a destabilizing controller in the closed-loop; moreover, such a destabilizing controller can remain in the closed-loop (before being replaced by a stabilizing controller) for a long period of time resulting in very large control signals. Indeed, one cannot even put a global upper bound on the time during which the destabilizing controller is attached. Further, we use the coprime factorization of the controllers in the controller set and implement the controllers in a specialized way to remove the restrictive assumption on the controller set, see

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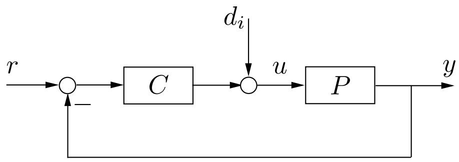

Fig. 1. Standard Feedback Configuration

e.g. [12], [13], which requires all the controllers in the set to be have bi-proper and minimum-phase controllers.

Section II collects the required and necessary definitions and notations and briefly reviews the latest unfalsified adaptive control ideas of [12], [13]. This is followed by Section III which discusses some open questions and make a few critically important remarks and observations on the unfalsified adaptive control approach. In Section IV we discuss different aspects of implementing controllers in the forward path or in the feedback path of a feedback interconnection. The proposed controller implementation is laid out in Section IV-A which provides intuition and describes advantages of such implementation. Section V contains concluding remarks.

II. BACKGROUND

In this section we first define the notation used throughout this manuscript and then outline the unfalsified adaptive control algorithm for ease of reference. We shall restrict our attention to linear plant case.

We shall denote by $\mathbb{R}_+$ the set of strictly positive real numbers, by $\mathscr{H}_\infty$ the Hilbert space of functions bounded along the $j\omega$-axis and analytic in the right-half plane, and the same function spaces with prefix $\mathscr{R}$ their real-rational proper subspaces. The plant and the controller are denoted by P and C respectively. We will use coprime factor representations of C, and adopt as an assumption that all controller transfer functions are always proper. We shall collect standard definitions related to this representation in the sequel.

Definition 1: Consider the unity feedback LTI system in Fig. 1. The system is internally stable if the closed-loop mapping $\left[\begin{smallmatrix} r \\ d_i \end{smallmatrix}\right] \mapsto \left[\begin{smallmatrix} y \\ u \end{smallmatrix}\right]$ is stable [24].

Definition 2: The ordered pair $\{\tilde{U}, \tilde{V}\}$, with $\tilde{U}, \tilde{V} \in \mathscr{RH}_\infty$, is a left-coprime factorization (*lcf*) of $C \in \mathscr{R}$ if $\tilde{V}$ is invertible in $\mathscr{R}$, $C = \tilde{V}^{-1}\tilde{U}$, and $\tilde{U}$ and $\tilde{V}$ are left-coprime over $\mathscr{RH}_\infty$. Furthermore, the ordered pair $\{\tilde{U}, \tilde{V}\}$ is a normalized *lcf* of C if $\{\tilde{U}, \tilde{V}\}$ is a *lcf* and $\tilde{V}\tilde{V}^* + \tilde{U}\tilde{U}^* = I$.

A. An Overview of Unfalsified Adaptive Control

The unfalsified adaptive control scheme using multiple controllers and switching has been presented in [1]–[15] and we shall collect necessary materials here from the latest related publications, i.e. [11]–[13].

Consider the configuration of Fig. 2 and let P be an unknown plant with (u, y) the measurable plant input/output signals and r the reference signal. The signals r, u, y are all assumed to be square integrable over any bounded interval $[0, \tau] \in \mathbb{R}_+$. The time-invariant controllers $C_1, C_2, \cdots, C_N$ belong to the given finite element candidate controller set $\boldsymbol{C}$.

At any instant of time, the controller is chosen to be one of the controllers in the controller set $\boldsymbol{C}$, and it can switch among the controllers of the set. The switching occurs in response to the measured data (u, y) and at discrete instants of time with a minimum interval (dwell-time) between successive switches. More precisely, with $\boldsymbol{Z}_\tau$ denoting the truncated space of measured signals (u, y) from $t = 0$ to the current time τ, and with some abuse of notation, we denote the adaptive control law at time t as $\hat{C}(t, \boldsymbol{Z}_t)$; the law $\hat{C}(\cdot, \boldsymbol{Z}.)$ maps the function $\left[\begin{smallmatrix} r(\cdot) \\ y(\cdot) \end{smallmatrix}\right]$ (as in Fig. 2 below $r - y = e$) to the function $u(\cdot)$. It is assumed that the switching stops after some final switching time t^f and the final controller is denoted by C^f.

Using the concept of a multi-controller [25]–[27] it is possible, though not essential in the linear controller case (see e.g. [19], [25], [26]), to require that the internal state variable of the controller just before switching is also that for the controller just after switching. Thus, some rule is required which determines the controller state just after a switching. Also note that for "bumpless" transfer [25], one needs to ensure that the output of the switched system remains continuous across switching instants provided its input is reasonably well-behaved and is piecewise continuous.

The so-called fictitious reference signals $\tilde{r}_1, \tilde{r}_2, \cdots, \tilde{r}_i$, which are defined below, are constructed using (u, y) data. At any instance of time, a switching mechanism (discussed in the sequel) *decides* which controller should be placed in the closed-loop. The set of experimental data (u, y) measured during $t \in [0, \infty)$ is denoted by $\boldsymbol{Z}$ and its truncated space $\boldsymbol{Z}_\tau$ denotes the set of (u, y) measured during $t \in [0, \tau]$.

Definition 3: A truncated signal $x(t)$ is defined as $x_\tau(t) = \left\{\begin{smallmatrix} x(t), & t \in [0,\tau] \\ 0, & \text{otherwise} \end{smallmatrix}\right.$ and its truncated norm

$$\|x\|_\tau = \left(\int_0^\tau x^2(t)\, dt \right)^{1/2} .$$

It is said that $x \in \mathscr{L}_{2e}$ if $\|x\|_\tau$ exists for all $\tau < \infty$.

Definition 4: The system H with input w and output z is said to be stable if there exist constants β, $\alpha \geq 0$ such that for every input $w \in \mathscr{L}_{2e}$

$$\|z\|_\tau < \beta \|w\|_\tau + \alpha, \quad \forall \tau \in \mathbb{R}_+$$

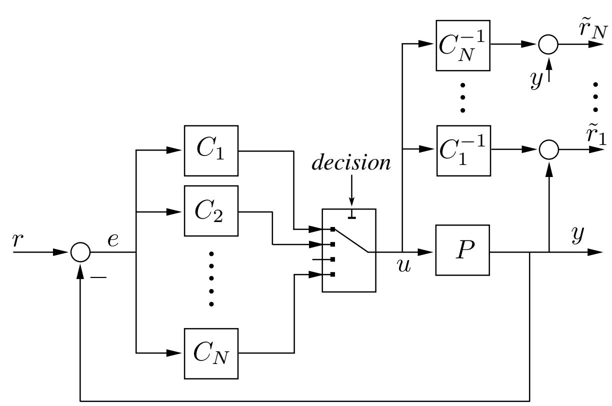

Fig. 2. An Adaptive Control Setup—Unfalsified Approach: Forward-path Controller Implementation

Note that the definition of stability above (Definition 4) admits non-zero value for α and allows consideration of systems with non-zero initial condition [13].

Definition 5: Stability of the system $H : w \mapsto z$ is said to be falsified by the input-output data (w, z) if

$$\sup_{\tau \in \mathbb{R}_+, \|w\|_\tau \neq 0} \frac{\|z\|_\tau}{\|w\|_\tau} \to \infty\,.$$

Otherwise, stability of the system H is said to be unfalsified by data.

Given Definitions 4 and 5, it can be easily shown that if a system is stable, its stability cannot be falsified by input/output data, and if stability of a system is falsified by input/output data, then stability of that system does not hold.

Assumption 1: The given finite set of candidate controllers $\boldsymbol{C} = \{C_i\,, \quad i = 1, \cdots, N\}$ is assumed to contain bi-proper minimum-phase controllers only.

Definition 6: The adaptive control problem is said to be 'feasible' if there is at least one controller $C_i \in \boldsymbol{C}$ for P that achieves stability and performance objectives.

Definition 7: Given a current time τ, a set of past measured data (u, y) obtained with a fixed plant over $[0, \tau]$, and a new candidate controller $C_i \in \boldsymbol{C}$ for future use, a fictitious reference signal $\tilde{r}_i$ for this candidate controller is a hypothetical reference signal defined over $[0, \tau]$ that would have produced exactly the measured data (u, y) over $[0, \tau]$ had C_i been in the feedback loop (noise free) with the unknown plant during the entire time period $[0, \tau]$. This fictitious reference signal can be constructed by using $\tilde{r}_i = C_i^{-1} u + y$ for the configuration in Fig. 2, in which it is assumed for simplicity that noise, disturbance and initial conditions x_0 are all zero.

As is analyzed in more detail in Section IV, difficulties will occur in obtaining $\tilde{r}_i$ if C_i is strictly proper or non minimum-phase.

To complete our review of the unfalsified adaptive control approach, we shall need to explore the switching mechanism in Fig. 2, and to discuss the choice of a performance index and the switching algorithm.

For a candidate controller $C_i \in \boldsymbol{C}$, the performance index $V(C_i, u, y, \tau)$ is defined to be a mapping $V : \boldsymbol{C} \times \boldsymbol{Z}_\tau \times \mathbb{R}_+ \to \mathbb{R}_+$, where $\boldsymbol{Z}_\tau$ is the truncated space of the measured signals (u, y) from initial to the current time τ. This is a measure of performance of the closed loop system over the interval $[0, \tau]$, had the controller C_i been in the closed-loop with the corresponding $\tilde{r}_i$ as the reference signal and (u, y) as the measured data. Example of such indices will be given below. High values correspond to poor performance.

Definition 8: Given the cost function V, the candidate controller set $\boldsymbol{C}$ and a scalar $\gamma \in \mathbb{R}$, the controller $C_i \in \boldsymbol{C}$ is said to be falsified at time τ with respect to cost level γ by past measurement data $(u, y)_\tau$ if $V(C_i, u, y, \tau) > \gamma$. Otherwise the controller is said to be unfalsified by collected data up to time τ. The set of unfalsified controllers with an unfalsified cost level of γ or less at time τ is defined as the unfalsified controller set [13].

Definition 9: Given the switched adaptive control system in Fig. 2, the input r, the collected plant input/output data (u, y), the cost function $V(C_i, u, y, \tau)$ and the candidate controller set $\boldsymbol{C}$, suppose the adaptive control law $\hat{C}(\cdot, \boldsymbol{Z}.) \in \boldsymbol{C}$—which maps $\left[\begin{smallmatrix} r(\cdot) \\ y(\cdot) \end{smallmatrix}\right]$ to $u(\cdot)$ and is piecewise constant—converges after finitely many switching times to a controller $C^f \in \boldsymbol{C}\ \forall \tau > t^f$. The pair $(V, \boldsymbol{C})$ is said to be cost-detectable if the following two statements are equivalent:

a. $V(C^f, \boldsymbol{Z}_\tau, \tau)$ is monotone increasing and bounded as $\tau \to \infty$;
b. The stability of the closed-loop system in Fig. 2 with C^f is unfalsified by $(r, \boldsymbol{Z}_\tau)$ as $\tau \to \infty$.

Several performance indices have been investigated in [11]–[13], [15] and the references therein, but the following performance index[1]

$$V(C_i, u, y, t) = \max_{\tau\,\in\,[0,t]} \frac{\|u\|_\tau^2 + \|\tilde{r}_i - y\|_\tau^2}{\|\tilde{r}_i\|_\tau^2 + \alpha},\ \alpha \in \mathbb{R}_+ \quad (1)$$

stems from the ideas discussed in [3] and is considered as a 'good' cost function [12], [13], [15]. It is evident that the cost function is monotone increasing in time and with very mild assumptions on u and y or external excitation is continuous with time. Thus, a sufficient condition for it to satisfy the cost-detectability property, see Definition 9, is that there is at least one stabilizing controller in the controller set $\boldsymbol{C}$.

We shall now discuss the switching algorithm. The hysteresis switching algorithm utilized in the literature of unfalsified adaptive control algorithms (see e.g. [12], [13]) and other related references, which is the algorithm proposed in [28], is given below.

Algorithm 1: The switching algorithm, which is copied directly from the literature of unfalsified adaptive control e.g. [12], [13], is as follows:

Step 1. Let ε be a small positive constant, τ be the current time (initialized to $\tau = 0$), $\Delta\tau$ be the time increment, and $\tilde{C} \in \boldsymbol{C}$ be the controller in the feedback loop of Fig. 2;
Step 2. $\tau \leftarrow \tau + \Delta\tau$;
Step 3. If $V(\tilde{C}, u, y, \tau) > \min_{C_i \in \boldsymbol{C}} V(C_i, u, y, \tau) + \varepsilon$, then $\tilde{C} \leftarrow \arg\min_{C_i \in \boldsymbol{C}} V(C_i, u, y, \tau)$;
Step 4. Go to Step 2.

We shall discuss a modification to the above algorithm and the choice of cost level γ and the use of unfalsified controller set as in Definition 8 in Section III-D.

Although the references [1]–[4], [6]–[9], [12]–[15] make a clear case for the value of and advocate the unfalsified adaptive control algorithm, we shall discuss below some severe situations where caution must be exercised.

III. A Critique of the Unfalsified Adaptive Control Approach

In the literature of the unfalsified adaptive control, see e.g. [13] and the references therein and the discussions

[1]The performance index here is similar to the $\mathscr{H}_\infty$ mixed-sensitivity *performance* criterion for linear time-invariant systems [24] and are used in the literature of unfalsified adaptive control; see e.g. [3], [13] and the references therein.

of Section II-A, it is stated that the switching algorithm converges to a stabilizing controller after a "finite number of switches". One may ask the question which kind of bound can be found for the number of switches. Another question is whether there is a protection against inserting a destabilizing controller into the closed-loop, and if a destabilizing controller is inserted, whether there is a bound on the time during which the destabilizing controller is attached.

In an attempt to answer the posed questions and to study the unfalsified adaptive control approach in some detail, we shall consider a simple academic example below.

Suppose that the unknown SISO plant has a simple transfer function of the form $P = 1/(s-1)$ and let the controller set consist of $\boldsymbol{C} = \{C_1 = 2, C_2 = 0.5\}$ which satisfies the assumptions: both controllers are obviously minimum-phase and bi-proper, see Assumption 1, and using Definition 1, one can easily verify that C_1 is stabilizing and C_2 is destabilizing.

Let the stabilizing controller $C_1 = 2$ be initially placed in the closed-loop and assume that the reference signal is of the form $r(t) = \sin(t)\,\mathbf{1}(t)$, where $\mathbf{1}(t)$ is unit step function. We shall follow the switching algorithm in Algorithm 1, copied directly from [13] and the references therein. For the sake of clarity and exactness in following the switching algorithm experimental simulations are carried out, however, the motivation and insight for the choice of experimental setting and the observations in the following stem from some transfer function analysis.

A. Different dwell-times

Let us run the switching algorithm of Section II-A with zero initial condition, $x(0) = 0$, and the switching be allowed after 5 seconds; i.e. there is a dwell-time of 5 seconds. Following the discussion and in particular the switching algorithm in Section II, the algorithm decides to switch the stabilizing controller C_1 out and connects up the destabilizing controller C_2 as $V(C_2, u, y, \tau) < V(C_1, u, y, \tau)$, $\tau \in [0, 5]$. However, at the end of the second dwell-time the cost functions (as in Equation (1)) satisfy $V(C_2, u, y, \tau) > V(C_1, u, y, \tau)$ which leads to the algorithm decision of switching from C_2 to C_1. The stabilizing controller C_1 remains in the closed-loop as time proceeds and is the final controller. Note that in this case, i.e. dwell-time of 5 seconds, the algorithm switches twice and the destabilizing controller remains in the closed-loop for 5 seconds.

If one chooses a dwell-time of 1 second and runs the algorithm with the identical experimental set-up and initial conditions as above, the number of switches between stabilizing controller C_1 and destabilizing controller C_2 increases to four times before it finally, $\forall \tau > 9$, converges to C_1. In this case the total time the destabilizing controller C_2 was placed in the closed-loop is 7 seconds since C_2 was connected up during $1 \leq \tau \leq 2$ and for $3 \leq \tau \leq 9$. And if one sets the dwell-time to 0.1 and runs the algorithm with $x(0) = 0$, the number of switches between C_1 and C_2 increases to six times before $C^f = C_1\ \forall \tau > 3.4$ and the sum of all time intervals during which the destabilizing controller C_2 was placed in the closed-loop totals 2.5 seconds. Table III-A summarizes the results discussed above.

The aforementioned situations of this section show that the unfalsified adaptive control algorithm of Section II-A is not as efficacious as it seems. It can repeatedly return to the destabilizing controller and the number of switchings can be arbitrary large (albeit with arbitrarily small dwell times). Also the time interval during which the destabilizing controller is connected before switching occurs can be arbitrarily long. It is obviously not the case that it switches n times where n is the number of state variables or the dimension of the plant, which might be considered acceptable.

B. Different initial conditions

Another point about the algorithm is that the total sum of all time intervals during which the destabilizing controllers are connected up can be arbitrarily long. Let us run the algorithm with the identical experimental set-up to the above with a dwell-time of 5 seconds but with different initial conditions $x(0)$. Table. III-B shows the total time durations during which the destabilizing controllers are connected up for different initial conditions and the maximum value the control signal $u(t)$ takes up during the interval the destabilizing controller is connected. For example, with the initial condition $x(0) = 10$, the algorithm switches out the stabilizing controller C_1 and connects up the destabilizing controller C_2 after 5 seconds but the destabilizing controller C_2 remains in the closed-loop for 20 seconds before it is switched out. This will result in the control signal $u(t)$ reaching a value of more than 7.483×10^3.

The discussion above gives answers to the questions raised at the beginning of this section. It is evident that there is no universal bound on the number of switches to a destabilizing controller if the bounded set of all possible initial conditions and all possible dwell-times are considered. If one fixes dwell-time and considers all possible initial conditions within a bounded set then there is a bound. It is the case that for a bounded set of initial conditions and a fixed dwell-time and a finite set of controllers, the number of switchings to a destabilizing controller is bounded. However, the bigger the set of initial conditions the greater will be the total time intervals during which the destabilizing controller C_2 was placed in the closed-loop.

Dwell-Time (seconds)	Number of switchings between C_1 and C_2
10	2
5	2
1	4
0.1	6
0.01	8
0.001	10
$\vdots$	$\vdots$

TABLE I

DIFFERENT DWELL-TIMES AND THE NUMBER OF SWITCHINGS BETWEEN THE STABILIZING CONTROLLER C_1 AND THE DESTABILIZING C_2 WITH ZERO INITIAL CONDITION $x(0) = 0$

Note that the finiteness of the overall number of switchings is discussed in [10], [12] and a bound is developed which depends on the data. However, having the bound on the number of switchings vary with the data is to say that the number of switchings assumes finite values and hence there exist a set of data for which the number of switchings can be arbitrarily large (though finite). It is not the case that the number of switchings to destabilizing controller is bounded[2].

C. Detection of Instability

Another noticeable difficulty with the unfalsified adaptive control algorithm is that the closed-loop system can actually turn unstable but the instability need not be revealed by the value of the cost function $V(C_i, u, y, t)$ in Equation (1). Note that Definition 5 defines falsified stability of a system over all non-zero inputs to the system whereas the cost function $V(C_i, u, y, t)$ in Equation (1) only includes a particular input, viz the fictitious input $\tilde{r}_i$, rather than all possible inputs. Notice also that the aim here is to prevent connecting a destabilizing controller in order to avoid large signals. A large value for the cost function in Equation (1) would normally be expected if the control signal or the tracking error were large. There, however, can be situations where the control signal $u(t)$ was growing large but the fictitious input $\tilde{r}_i$ could grow large too resulting in a cost function with no reflection of instability.

To explore further this deficiency in detecting instability, let us recall Definition 7 for generating the fictitious input $\tilde{r}_i$. Notice that here we have a system driven by (u, y) in which $\tilde{r}_i$ is the response rather than having a system driven by the reference and u and y being the response. Ideally, what one would like to be able to do with different C_i is to compare performance indices which have a common reference signal but different u and y for each C_i. Unfortunately, here we are comparing performance indices which have the same signals u and y but different input signals, and although the mapping $(u, y) \mapsto \tilde{r}_i$ is stable, the mapping $r \mapsto (u, y)$ could be unstable. Thus we unfortunately cannot expect to have instability reflected in the cost function if we never actually connect the destabilizing controller. We can formalize the argument as follows.

Observation 1: Suppose u and y are obtained with a bounded actual external input r. Suppose also that C_i^{-1} is stable for all i. Even if C_j is destabilizing $V(C_j, u, y, \tau)$ need not diverge as $\tau \to \infty$.

Proof: Because u and y are obtained with a bounded actual external input r, we will have $\|u\|_\tau \le k_1\|r\|_\tau$ and $\|y\|_\tau \le k_2\|r\|_\tau$ for all τ and some k_1 and k_2. Also, because C_i^{-1} is stable and $\tilde{r}_i = C_i^{-1}u + y$, $\|\tilde{r}_i\|_\tau \le k_3\|r\|_\tau$ and hence $\|u\|_\tau^2 + \|\tilde{r}_i - y\|_\tau^2 \le k_4\|r\|_\tau^2$. Thus

$$\begin{aligned} V(C_j, u, y, t) &= \max_{\tau \in [0,t]} \frac{\|u\|_\tau^2 + \|\tilde{r}_j - y\|_\tau^2}{\|\tilde{r}_j\|_\tau^2 + \alpha} \\ &\le \max_{\tau \in [0,t]} \frac{k_4\|r\|_\tau^2}{\|\tilde{r}_j\|_\tau^2 + \alpha} \\ &\le \frac{k_4\|r\|_\tau^2}{\alpha} \end{aligned}$$

This is bounded. ■

Note that if there is a constant controller C in use over all time, there will hold $\tilde{r}_j = ((C_j^{-1}C + PC)/(1 + PC))r$ and $\|r\|_\tau^2 \le \max_\omega |\frac{1+PC}{C_j^{-1}C+PC}(j\omega)|\ \ \|\tilde{r}_j\|_\tau^2$. Consequently, if $|(1 + PC)/(C_j^{-1}C + PC)\,(j\omega)|$ is bounded, $V(C_j, u, y, t)$ cannot go unbounded. Even if $|(1 + PC)/(C_j^{-1}C + PC)\,(j\omega)|$ is unbounded, if r is a linear combination of sinusoids, we will have $\|r\|_\tau^2 \le k\|\tilde{r}_j\|_\tau^2$ for some k depending only on the value of $|(1 + PC)/(C_j^{-1}C + PC)|$ at the frequencies in question.

D. Choice of Cost Level γ and Modification of Switching Algorithm

As discussed earlier, the cost level γ and the unfalsified controller set in Definition 8 are not incorporated in the switching algorithm in Algorithm 1 of Section II-A. The literature of unfalsified adaptive control approach, see e.g. [13], is silent about *how* to choose this critical cost level γ and how to utilize it[3]. Evidently, if γ were to be chosen in advance and were not chosen wisely, the algorithm could lead to a null set of falsified controllers.

For the choice of γ, we assert, and one could easily verify, that by considering the cost function in Equation (1), one could conclude that a controller was not a "nice" controller if $\|u\|_\tau^2/\|\tilde{r}_i\|_\tau^2$ was, let us say, 100 and this could be used as a basis for choosing an acceptable cost level γ. That is to choose γ simply on the basis of observing the performance index and decide what one would be uncomfortable with as an outcome in terms of the ratio $\|u\|_\tau^2/\|\tilde{r}_i\|_\tau^2$. Note that the above choice of γ needs however special tuning for non-zero initial conditions.

As for the switching algorithm and the inclusion of the unfalsified controller set, Step 3 of the switching algorithm in Algorithm 1 needs to be changed such that the controllers

[2]Here *bounded* is taken to mean that there is a data-independent fixed number which is never exceeded

Initial Condition $x(0)$	Time Duration (sec)	Maximum of $u(t)$
0	5	4.05
2	10	54.65
5	15	645.3
10	20	7.4832×10^3
50	25	5.50×10^4
100	30	1.194×10^5
120	35	1.228×10^6
$\vdots$	$\vdots$	$\vdots$

TABLE II

TIME DURATIONS DURING WHICH THE DESTABILIZING CONTROLLER C_2 IS CONNECTED UP AND THE MAXIMUM VALUE OF CONTROL SIGNAL $u(t)$ FOR DIFFERENT INITIAL CONDITIONS $x(0)$ BUT FIXED DWELL-TIME OF 5 SECONDS

[3]In a recent private communications with Professor Safonov, he mentioned that the cost level γ is not chosen beforehand. Instead, the cost level γ is determined after the fact from data as the limiting final value of the cost for the final controller.

are taken from the unfalsified controller set discussed in Definition 8. This replacement is implemented by:

$$\text{If } V(\tilde{C}, u, y, \tau) > \min_{C_i \in \boldsymbol{C}^{unf}} V(C_i, u, y, \tau) + \varepsilon, \text{ then}$$
$$\tilde{C} \leftarrow \arg \min_{C_i \in \boldsymbol{C}^{unf}} V(C_i, u, y, \tau)$$

where $\boldsymbol{C}^{unf}$ denotes the unfalsified controller set and $\boldsymbol{C}^{unf}(\gamma, \tau)$ denotes the same set at each time τ for a given cost level γ.

Another potentially negative feature of the unfalsified adaptive control algorithm is that the controller C_i must be bi-proper and minimum phase. This restrictive assumption is removable. In the sequel, we first discuss different aspects of implementing controllers in the forward path or in the feedback path of a feedback interconnection and pave the way for the introduction of the proposed structure adjustment and implementation of the controller using its coprime factors discussed in Section IV-A.

IV. Controller Implementation

Let us again consider the configuration of Fig. 2 which shows forward-path implementation of candidate controllers.

With reference to Definition 7, the fictitious reference signal for the configuration in Fig. 2 is constructed by using $\tilde{r}_i = C_i^{-1}u + y$. However, notice that the mapping $(u, y) \mapsto \tilde{r}_i(C_i, u, y)$ in Fig. 2 is stable and causal if C_i^{-1} is causal and has no right half-plane poles. This is satisfied if C_i is **minimum-phase and bi-proper**.

If these constraints on the controllers in the set are not satisfied, there will be implications on the fictitious reference signal, Definition 7, and the cost detectability property of the cost function, see Definition 9. To examine that, let us suppose that the controller set contains only one controller, i.e. $\boldsymbol{C} = \{C_1\}$, which is bi-proper but *not* minimum-phase, i.e. one of the constraints is not satisfied. Let C_1 be placed in the feedback loop of Fig. 2 to control the plant, $u = C_1(r - y)$, and let the cost function be chosen as in Equation (1). The corresponding fictitious reference signal can be calculated using $\tilde{r}_1 = C_1^{-1}u + y$. Given that C_1 is the only controller in the set $\boldsymbol{C}$ and is in the loop for the entire time during which the plant data (u, y) were collected, $\tilde{r}_1$ should be the same as the actual reference signal r; see Definition 7. However, given that C_1 in this case is non-minimum-phase, even a very small disturbance or noise or initial condition mismatch would make $\tilde{r}_1$ grow unbounded. Thus, as $t \to \infty$ the fictitious reference signal $\tilde{r}_1 \to \infty$ irrespective of the collected measurement data (u, y). This will also result in the denominator of the cost function in Equation (1) growing unboundedly as $t \to \infty$ regardless of the collected data (u, y). The same is also true for the numerator of the cost function. Thus, given one or more non minimum-phase zeros, one will not be able to find an algorithm which computes the cost function V in Equation (1) which is numerically acceptable because the calculation of $\tilde{r}_i$ will not be numerically acceptable; rather, we will find $V \to 1$ (at least when (u, y) are bounded).

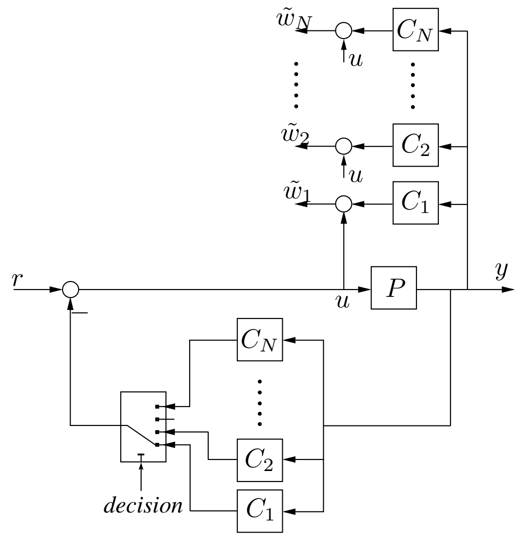

Fig. 3. Feedback-path Controller Implementation

Now let us consider the feedback-path implementation of the candidate controllers shown in Fig. 3.

Given Definition 7, but with the new implementation structure in Fig. 3, the fictitious reference signal for controller C_i is now constructed by using $\tilde{w}_i = u + C_i y$. Clearly, the mapping $(u, y) \mapsto \tilde{w}_i(C_i, u, y)$ in Fig. 3 is stable and causal if C_i has **no right half-plane poles**.

Given the argument discussed above for the forward-path implementation case, it is now straightforward to verify that in the setup of Fig. 3 if any of the controllers C_i have right half-plane poles, the corresponding fictitious reference signal $\tilde{w}_i$ will grow unbounded with time, and the cost detectability property of the cost function in Equation (1) will also be lost. Thus, a necessary condition for a performance index of the form in Equation (1) to be cost detectable is that the candidate controllers in the set $\boldsymbol{C}$ do not have any right half-plane poles.

The above discussion shows that right half-plane poles of the controller in one case (forward-path controller implementation) and non-minimum phase zeros of the controller in another case (feedback-path controller implementation) can impose restrictions on fictitious reference signal and the cost detectability property (see Definition 9) of the cost function will be lost. If these constraints are not met, the fictitious reference signals will be unstable and will grow unbounded as time proceeds to infinity irrespective of the collected closed-loop data (u, y). To address the above-mentioned difficulties and to remove the above mentioned constraints on the controller set, in the following section we make adjustments to the structure and implement controllers using their coprime factorizations in a specialized way. The ideas of [29] also shed some light in the direction of finding causal inversion of non-minimum phase controllers so that they can be used in Fig. 2.

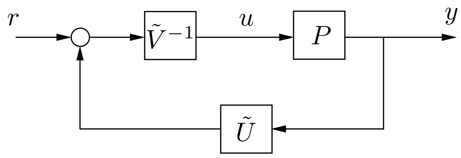

Fig. 4. Controller Implementation with $C = \tilde{V}^{-1}\tilde{U}$

A. Controller Implementation Adjustments

In order for the poles and zeros of the controller not to impose restrictions on the construction of the fictitious reference signal and for the unfalsified adaptive control scheme discussed in Section II-A to be utilized in a wider class of applications, we will make some structural adjustments.

In this work we will refer to the so-called "observer-form implementation" of the controller, see [30] and [31, Chapter 5], where the factor $\tilde{V}^{-1}$ of C, see Definition 2, is implemented in the feed-forward path and the factor $\tilde{U}$ of C is implemented in the feedback path as depicted in Fig. 4.

Simple manipulations show that the controller equation can also be rewritten as

$$u = \begin{bmatrix} -\tilde{U} & I + \tilde{V} \end{bmatrix} \begin{bmatrix} y \\ u \end{bmatrix} - r \tag{2}$$

which is depicted in Fig. 5 and clearly justifies why this configuration is referred to as the observer-form. Similar controller implementation is also utilized in [32], [33]. The reader is referred to [24], [34] for further discussion on the observer-based controllers and the link to the controller implementation in Fig. 5.

In the context of the unfalsified adaptive control scheme discussed in Section II-A, the above-mentioned observer-form controller implementation can be utilized to avoid the situations discussed in Section IV as shown in Fig. 6.

With reference to Definition 7, the fictitious reference signal for the configuration in Fig. 6 is constructed using

$$\tilde{z}_i = [\tilde{V}_i \quad -\tilde{U}_i] \begin{bmatrix} u \\ y \end{bmatrix}$$

where $\tilde{U}_i$, $\tilde{V}_i \in \mathscr{RH}_\infty$ are coprime factors of the controllers in the set $\boldsymbol{C}$. Notice that the mapping $(u, y) \mapsto \tilde{z}_i(C_i, u, y)$ in Fig. 6 is stable and causal as $\tilde{U}_i$ and $\tilde{V}_i$ are both stable operators; see Definition 2.

The associated performance index here can also be obtained in a similar manner and can be chosen to be the same as the one in Equation (1) with $\tilde{z}_i$ replacing $\tilde{w}_i$. Since the mapping $(u, y) \mapsto \tilde{z}_i(C_i, u, y)$ is stable, the fictitious reference signal $\tilde{z}_i$ will not grow unbounded irrespective of

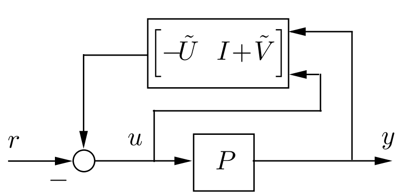

Fig. 5. Alternative Implementation of $C = \tilde{V}^{-1}\tilde{U}$

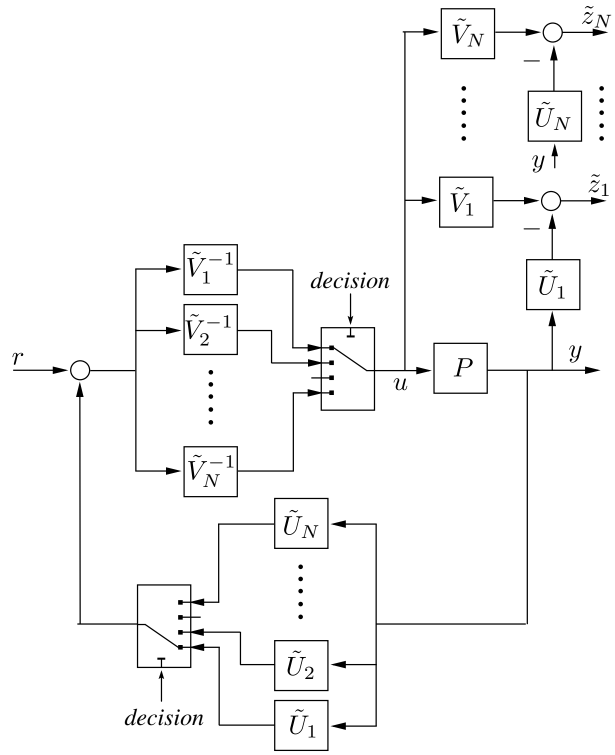

Fig. 6. Proposed Unfalsified Adaptive Control Setup

the collected data (u, y) and hence the the cost detectability property of the cost function in (1) will not be lost.

It is evident that the difficulties mentioned above have now been addressed and there is no constraint on the controllers in the candidate controller set. The controllers can have right half-plane poles or non-minimum-phase zeros, and do not need to be necessarily bi-proper.

V. Conclusions

We have proposed a structural adjustment for implementing controllers in the area of the unfalsified adaptive control of [1]–[16] where it is assumed that the controller set contains bi-proper and minimum-phase controllers only or else the cost detectability property will not hold; see [13, Thm. 1]. The proposed controller implementation adjustments removes these restrictive constraints on the controllers in the controller set. Thus, the controller set is now assumption free and controllers can have right half-plane poles or zeros, and do not need to be bi-proper.

Although the references make a clear case for the value of the unfalsified adaptive control algorithm, the discussion of Section III simply advocated caution in some circumstances. We showed by way of an academic example that in the vast literature of the unfalsified adaptive control approach, see [13] and the references therein, no guarantee of protection against actually inserting a destabilizing controller in the closed-loop is given and one cannot even put a global upper bound on the time during which the destabilizing controller is attached. Though this should not be misinterpreted as unstable behavior, Table III-B shows that the destabilizing controller can remain in the closed-loop (before the algorithm replaces it by a stabilizing controller) for as short as a dwell-time period, but with different initial conditions (e.g. $x(0) = 120$) the destabilizing controller can remain in the closed-loop for longer time duration (35 seconds) resulting in the closed-loop signals to take very large values (e.g. the plant input signal $u(t)$ can grow to 1.228×10^6).

It should also be noted that there may not exist other adaptive control algorithms which overcome the situations discussed in the previous paragraph and in Section III if these algorithms also assume no ‘a priori’ information about the plant. The unfalsified adaptive control algorithm, e.g. [13], states that no information about the plant is known ‘a priori’, and all one would need to use the algorithm are a given cost function which satisfies certain prescribed properties and a set of bi-proper minimum-phase linear controllers (even if the plant is nonlinear), which is guaranteed to include at least one stabilizing controller among them. However, in many situations, unlike the purist view on the prior knowledge of the plant, one would know something about the plant [22] and there are some preassumptions about the plant (like it is a linear system). There may even exist some qualitative or semi-quantitative data that mounts to ‘a priori’ of some aspects of the plant which can be exploited. The fact that the unfalsified adaptive control algorithm postulates that at least one controller is stabilizing would make sense if the designer knew something about the plant.

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