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A Subspace System Identification Algorithm Guaranteeing the Negative Imaginary Property

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***Index Terms*—Negative imaginary systems, flexible structures, subspace system identification.**

***Abstract*—The negative imaginary (NI) property occurs in many important applications. For instance, flexible structure systems with collocated force actuators and position sensors can be modeled as negative imaginary systems. Obtaining a mathematical model for this class of systems using system identification methods may result into inaccurate models that poorly reflect the negative imaginary property. In this paper, a modified subspace system identification algorithm that ensures the negative imaginary property is presented. As an application of these results, an example of modeling a flexible system with a piezoelectric actuator and position sensor is presented.**

I. Introduction

The negative imaginary (NI) property is defined, in the single input single output (SISO), by considering the properties of the imaginary part of the system frequency response $G(j\omega)$ and requiring the condition $j\left(G(j\omega) - G(j\omega)^*\right) \geq 0$ for all $\omega \in (0, \infty)$. Often, systems with colocated force actuators and position measurements are NI systems [1], [2]. For instance, systems with flexible structure dynamics such as flexible robot manipulators [3], ground and aerospace vehicles [4], atomic force microscopes (AFMs) [5], [6] often have colocated force actuators and position sensors and hence, can be modeled as NI systems. Another area where the underlying system dynamics are NI, are nano-positioning systems; see e.g., [5], [7]–[15]. Also, the positive-position feedback control scheme in [16], [17], can be considered using the NI framework. Furthermore, other control methodologies in the literature such as integral resonant control (IRC) [18] and resonant feedback control [19], [20], fit into the NI framework and their stability robustness properties can be explained by NI systems theory.

One important property of NI systems is the stability robustness of interconnected NI systems. This property has been studied in [1], [2]. It has been shown that a necessary and sufficient condition for the internal stability of a positive-feedback control system (see Fig. 1) consisting of an NI plant with a transfer function matrix of $G(s)$ and a strictly negative imaginary (SNI) controller with a transfer function matrix of $\bar{G}(s)$ is given by the DC gain condition

$$\lambda_{max}(G(0)\bar{G}(0)) < 1, \quad (1)$$

where the notation $\lambda_{max}(\cdot)$ denotes the maximum eigenvalue of a matrix with only real eigenvalues. This stability result has been used in a number of practical applications [5], [6], [15], [21]–[23].

In [5], [6], the NI stability result is applied to nano-positioning in an atomic force microscope. In [15], an IRC scheme based on the stability results provided in [1], [2] is used to design an active vibration control system for the mitigation of human induced vibrations in light-weight civil engineering structures, such as floors and footbridges via proof-mass actuators. In [21], the NI stability result is applied to the problem of decentralized control of large vehicle platoons. A positive position feedback control scheme based on the NI stability result provided in [1], [2] is used to design a novel compensation method for a coupled fuselage-rotor mode of a rotary wing unmanned aerial vehicle in [22]. In addition, it is shown in [24] that the class of linear systems having NI transfer function matrices is closely related to the class of linear Hamiltonian input-output systems. Also, an extension of the NI systems theory to infinite-dimensional systems is presented in [25].

Obtaining a system model for such flexible structural dynamics by constructing differential equations from first principles is often difficult. An alternative method for obtaining a mathematical model for the system is by the means of system identification. System identification methods are used to build mathematical models of dynamical systems from measured data using statistical methods. System identification methods

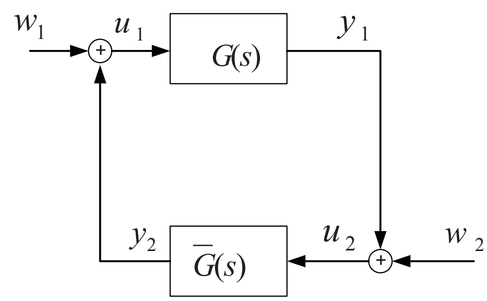

Fig. 1. A negative-imaginary feedback control system. If the plant transfer function matrix $G(s)$ is NI and the controller transfer function matrix $\bar{G}(s)$ is SNI, then the positive-feedback interconnection is internally stable if and only if the DC gain condition, $\lambda_{max}(G(0)\bar{G}(0)) < 1$, is satisfied.

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are widely used in the field of control system design, signal processing and many other areas [26]–[28]. However, the resulting model may not exactly describe the true dynamics of the underlying system, especially under noisy measurements due to sensitivity to boundary conditions and environmental effects. In such cases, the identified system models can sometimes lead to mathematical models that do not reflect the actual characteristics of the underlying system. For example, the process of system identification when applied to linear time-invariant (LTI) systems which are known to be NI from physical conditions might lead to a model which is not NI. In such cases, the system model should be perturbed to enforce the underling NI dynamics.

This problem has been addressed in [23], [29]. In [29] a first-order perturbation method is proposed for iteratively collapsing the frequency bands where the negative imaginary property is violated and finally displacing the eigenvalues of the Hamiltonian matrix away from the imaginary axis, thus restoring the negative imaginary dynamics. However, the technique presented in [29] is found to be only successful where the NI violated region in very small. A system identification algorithm which enforces the NI constraint is proposed in [23] for estimating model parameters. However, in [23], the NI enforcement comes only in identifying the matrices B and D in a state space model. In other words, no stability constraint is enforced, which is one of the requirements of a system to be NI.

In this paper, we impose constraints that guarantee negative imaginariness and stability of the identified model. This has been achieved by modifying the subspace system identification algorithm by adding two constraints that guarantee the NI property. Unlike the method presented in [23], the NI property is considered when estimating the full state space model at the beginning of the estimation process. Also, [23] considers the frequency domain approach to a subspace system identification whereas we consider the time domain approach.

This paper is further organized as follows: Section II recalls the definitions of NI systems. In Section III, introduces the subspace system identification method. In Section IV, we present a modified subspace system identification method that guarantees the NI property. In Section V-A, an example is presented to illustrate the results of the paper. Also, a practical example of modeling a flexible system with collocated force actuator and position sensor is given in Section V-B. Section VI concludes the paper.

II. Negative Imaginary Systems

In this section, we present the definition of negative imaginary systems and also the negative imaginary lemma.

Consider the LTI system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t) + Du(t), \end{aligned} \tag{2}$$

where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{m\times n}$, $D \in \mathbb{R}^{m\times m}$, and with the square transfer function matrix $G(s) = C(sI - A)^{-1}B + D$.

The notion of negative imaginary transfer functions and several related results developed in [1], [30], are now recalled.

Definition 1: (see [30]) A square transfer function matrix $G(s)$ is NI if the following conditions are satisfied:

1. $G(s)$ has no pole at the origin and in $\mathrm{Re}[s] > 0$,
2. For all $\omega > 0$ such that $j\omega$ is not a pole of $G(s)$, $j\left(G(j\omega) - G(j\omega)^*\right) \geq 0$,
3. If $j\omega_0$ with $\omega_0 > 0$ is a pole of $G(s)$, it is at most a simple pole and the residue matrix $K_0 = \lim_{s\to j\omega_0}(s - j\omega_0)sG(s)$ is positive semidefinite Hermitian.

A state-space characterization of NI systems in terms of a pair of linear matrix inequalities (LMIs) has been given in [1], [31]. This result is analogous to the positive-real lemma [32], [33] and thus is referred to as the negative imaginary lemma. This result is also generalized in [30] to include poles on the imaginary axis except at the origin.

Lemma 1: (See [30]) Let $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ be a minimal state space realization of a transfer function matrix $G(s)$. Then $G(s)$ is NI if and only if $det(A) \neq 0$, $D = D^T$ and there exists a real matrix $P > 0$ such that

$$AP + PA^* \leq 0, \tag{3}$$

and

$$B = -APC^*. \tag{4}$$

Definition 2: A square transfer function matrix $G(s)$ is SNI if the following conditions are satisfied:

1) $G(s)$ has no pole in $Re[s] \geq 0$;
2) For all $\omega > 0$, $j\left(G(j\omega) - G(j\omega)^*\right) > 0$.

A linear time-invariant system is SNI if its transfer function matrix is SNI.

Lemma 2: (See [34]) Let $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ a state space realization of a transfer function matrix $G(s)$. Suppose $G(s) + G(-s)^T$ has normal rank m and (C, A) is observable. Then, A is Hurwitz and $G(s)$ is SNI with

$$\lim_{j\omega\to\infty} j\omega(G(j\omega) - G(j\omega)^*) > 0$$

and

$$\lim_{j\omega\to 0} j\frac{1}{\omega}(G(j\omega) - G(j\omega)^*) > 0$$

if and only if $D = D^T$ and there exists a matrix $Y > 0$ such that

$$AY + YA^* < 0 \text{ and } B = -AYC^*. \tag{5}$$

III. Subspace System Identification Methods

In this section, we will give a brief introduction to the subspace system identification method.

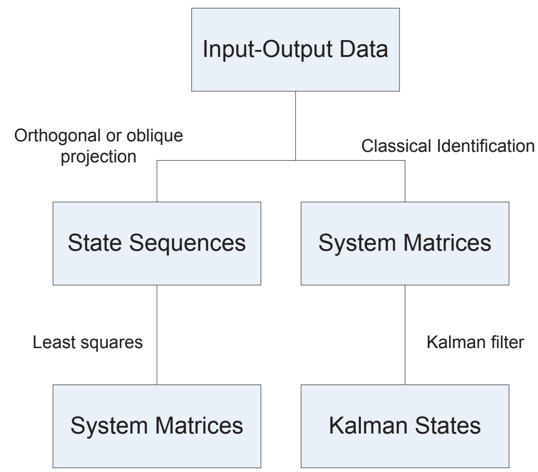

Fig. 2. Two types of system identification methods, the left hand side explains the subspace system identification method. The right hand side explains the classical method of system identification.

The discrete time system can be described as follows,

$$x(k+1) = A^d x(k) + B^d u(k), \tag{6}$$

$$y(k) = C^d x(k) + D^d u(k), \tag{7}$$

where $A^d \in \mathbb{R}^{n\times n}, B^d \in \mathbb{R}^{n\times m}, C^d \in \mathbb{R}^{m\times n}$, and $D^d \in \mathbb{R}^{m\times m}$. The main idea of the time domain system identification method uses set of time domain input $U \in \mathbb{R}^{m\times q}$ and output $Y \in \mathbb{R}^{m\times q}$ measurements to estimate the system matrices $\left[\begin{array}{c|c} A^d & B^d \\ \hline C^d & D^d \end{array}\right]$, where m is the number of the inputs and outputs and q is the number of measurements. According to [28], there are two main methods of estimating the system matrices $\left[\begin{array}{c|c} A^d & B^d \\ \hline C^d & D^d \end{array}\right]$. These two methods are illustrated in Fig. 2.

In this paper, we focus in the subspace system identification approach. In the subspace system identification, there are two major steps.

Step one: In this step, the input $u(k:k+q)$ and output $y(k:k+q)$ measurements are used to estimate the states of the system $X(k:k+q) \in \mathbb{R}^{n\times q}$. This is done using block Hankel matrices formed from the input $u(k:k+q)$ and output $y(k:k+q)$.

Step two: In this step, the estimated states $X(k:k+q)$ are used to setup a least squares problem to obtain the state space matrices $\left[\begin{array}{c|c} A^d & B^d \\ \hline C^d & D^d \end{array}\right]$. This least squares problem can be formed as follows;

$$\min_{A^d,B^d,C^d,D^d} \left\| W\left(\begin{bmatrix} X(k+1:k+q) \\ y(k:k+q-1)\end{bmatrix} - \begin{bmatrix} A^d & B^d \\ C^d & D^d\end{bmatrix}\begin{bmatrix} X(k:k+q-1) \\ u(k:k+q-1)\end{bmatrix}\right)\hat{W}\right\|^2, \tag{8}$$

where W and $\hat{W}$ are weighting matrices. Note that $u(k:k+q)$ and $y(k:k+q)$ are the given input and output and $X(k:k+q)$ is the estimated states from step one.

IV. Negative Imaginary Subspace System Identification

In this section, we will introduce a subspace system identification algorithm with NI property constraints as given in the NI lemma, Lemma 1. The NI lemma is formulated in continuous time. Therefore, a bilinear transformation [35] in the following form,

$$\begin{aligned} A &= \frac{1}{T}(I + A^d)^{-1}(A^d - 1) \\ B &= \frac{1}{\sqrt{T}}(I + A^d)^{-1}B^d \\ C &= \frac{1}{\sqrt{T}}C^d(I + A^d)^{-1} \\ D &= D^d - C^d(I + A^d)^{-1}B^d, \end{aligned} \tag{9}$$

will be used to transform the conditions (3) and (4) into corresponding discrete time conditions. The LMI (3) is transformed from continuous to a discrete time form as follows:

$$\begin{aligned} &AP + PA^T \le 0 \\ \Leftrightarrow &(I + A^d)^{-1}(A^d - 1)P + P(A^{d^T} - 1)(I + A^{d^T})^{-1} \le 0, \\ \Leftrightarrow &(A^dP - P)(I + A^{d^T}) + (P + A^dP)(A^{d^T} - I) \le 0, \\ \Leftrightarrow &A^dP + A^dPA^{d^T} - P - PA^{d^T} + PA^{d^T} + A^dPA^{d^T} \\ &\quad - P - A^dP \le 0, \\ \Leftrightarrow &A^dPA^{d^T} - P + A^dPA^{d^T} - P \le 0, \\ \Leftrightarrow &A^dPA^{d^T} - P \le 0. \end{aligned} \tag{10}$$

Also, the equality (4) can be transformed to the following corresponding discrete time form:

$$\begin{aligned} &B = -APC^T \\ \Leftrightarrow\ &B^d = -\frac{1}{T}(A^d - I)P(I + A^{d^T})^{-1}C^{d^T}, \end{aligned} \tag{11}$$

where the Lyapunov inequality in the discrete form (10) and the equality (11) are non-convex conditions in the state space matrices and P, which is a computational issue. We start by replacing the matrix inequality (10) by a strict inequality. In fact, we restrict the identified model to be strict stable NI system. In other words, we use the SNI lemma, Lemma 2, where Lyapunov inequality is $AP + PA^T < 0$ rather than Lemma 1. As in [36], we assume that there is exist an $\alpha > 0$ such that;

$$P \ge \alpha I \tag{12}$$

$$A^dPA^{d^T} - P \le -\alpha I. \tag{13}$$

Using Schur complements, (12) and (13) can be written as follows;

$$\begin{bmatrix} P - \alpha I & A^dP \\ PA^{d^T} & P \end{bmatrix} \ge 0$$

$$\left[\Leftrightarrow \begin{matrix} P - \alpha I & Q \\ Q^T & P \end{matrix}\right] \ge 0, \tag{14}$$

where $Q = A^d P$. The LMI (14) is now convex in the variables Q and P. Furthermore, the matrix A^d can be recovered from $A^d = QP^{-1}$.

Now, the optimization problem (8) can be separated into two sub-optimization problems (see [36] for more explanations why this is possible). The first sub-optimization problem is to solve for the matrices (A^d, B^d) as follows;

$$\min_{A^d,B^d} W_1 \Bigg\| \Big(\big[X(k+1:k+q)\big] - \begin{bmatrix} A^d & B^d \end{bmatrix} \begin{bmatrix} X(k:k+q-1) \\ u(k:k+q-1) \end{bmatrix} \Big) \hat{W} \Bigg\|^2. \quad (15)$$

The second sub-optimization problem is to solve for the matrices (C^d, D^d) as follows;

$$\min_{C^d,D^d} \Bigg\| W_2 \Big(\big[y(k:k+q-1)\big] - \begin{bmatrix} C^d & D^d \end{bmatrix} \begin{bmatrix} X(k:k+q-1) \\ u(k:k+q-1) \end{bmatrix} \Big) \hat{W} \Bigg\|^2. \quad (16)$$

By choosing the weighting matrix W in (15) as follows;

$$\hat{W} = \begin{bmatrix} X(k:k+q-1) \\ u(k:k+q-1) \end{bmatrix}^T \times \left(\begin{bmatrix} X(k:k+q-1) \\ u(k:k+q-1) \end{bmatrix} \begin{bmatrix} X(k:k+q-1) \\ u(k:k+q-1) \end{bmatrix}^T \right)^{\dagger} \times \begin{bmatrix} PR_1 & 0 \\ 0 & R_2 \end{bmatrix}. \quad (17)$$

Note that $\begin{bmatrix} X(k:k+q-1) \\ u(k:k+q-1) \end{bmatrix}$ is assumed full row rank (as $q > n+1$) and this assumption reasonable when sufficient data is collected. Also, R_1 and R_2 are new weighting matrices that can be chosen by the designers to tune the optimisation. This implies that the sub-optimization problem (15) can be combined with the LMI (14) constraint as follows;

$$\min_{P,Q,B} \Bigg\| \Big(W_1 \big[X(k+1:k+q)\big] \hat{W} - \begin{bmatrix} QR_1 & B^d R_2 \end{bmatrix} \Big) \Bigg\|^2, \quad (18)$$

subject to

$$\begin{bmatrix} P - \alpha I & Q \\ Q^T & P \end{bmatrix} \geq 0. \quad (19)$$

By solving this optimization problem, we obtain the matrices (A^d, B^d, P). Then, using (11), we can calculate the matrix C^d. The matrix D^d, can be obtained by solving the optimization problem (16). Then the corresponding continuous time state space models are obtained using (9).

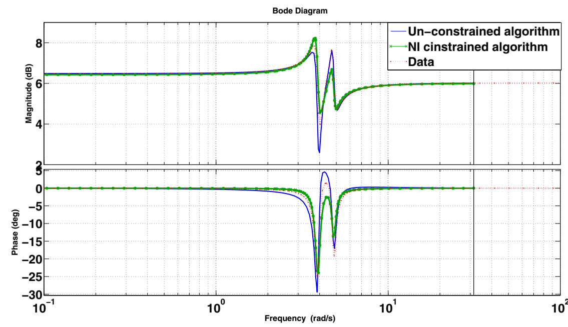

Fig. 3. Bode plot of models obtained using the NI constrained subspace system identification algorithm (green line), standard subspace system identification algorithm [35] (blue line) and the original transfer function given in (20).

V. Examples

A. Example 1

In this example, a fourth order transfer function in the form;

$$\frac{2s^4 + 1.2s^3 + 78.18s^2 + 22.6s + 728}{s^4 + 0.6s^3 + 38.09s^2 + 11.4s + 345}, \quad (20)$$

is used to generate a set of outputs $y(t_k)$ corresponding to a random set of inputs $u(t_k)$. Then, the inputs $u(t_k)$ and the outputs $y(t_k)$ is used to test our proposed NI subspace system identification algorithm and compare the algorithm with the standard subspace system identification algorithm presented in [35]. It is shown in Fig. 3 that the NI constrained subspace system identification algorithm (green line) gives a better fit to the data generated from the transfer function (20) (red dashed line) compared with standard subspace system identification algorithm [35] (blue line) for the same model order. Also, the NI model obtained from the constrained subspace method (green line) satisfies the NI property. This fact can be easily verified from the phase plot in Fig. 3, since the phase of this model lies between $[0, -180^o]$, see, [1], [2]. However the system model obtained from the standard subspace method (blue line) does not satisfy the NI property.

B. Example 2

This section presents an application of modeling a flexible system with two colocated piezoelectric patches, see Fig. 4 and Fig. 5. Here, one piezoelectric patches acts as an actuator while the other acts as a sensor. The system has one input and one output: the input is the voltage V_a applied to the piezoelectric actuator, whereas the output is the voltage V_s produced by the piezoelectric sensor. As this system involves collocated force actuators and position sensors, its transfer function should satisfy the NI property; e.g., see [2].

A DSPACE system is used to generate a random signal as an input $u(t)$ signal to the PZT-actuator through a high voltage amplifier. Then, the PZT-sensor is used to measure the output $y(t)$ signal as shown in Fig. 4. The signals $(u(t), y(t))$ are used in the identification process. In this

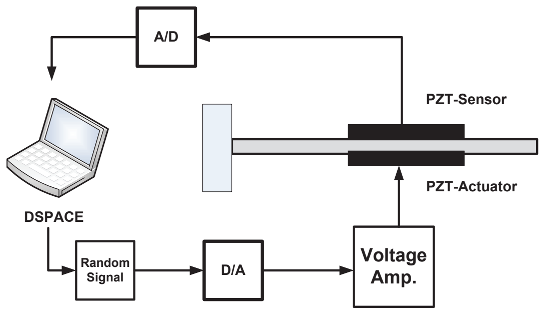

Fig. 4. Flexible beam with piezoelectric actuator and piezoelectric sensor.

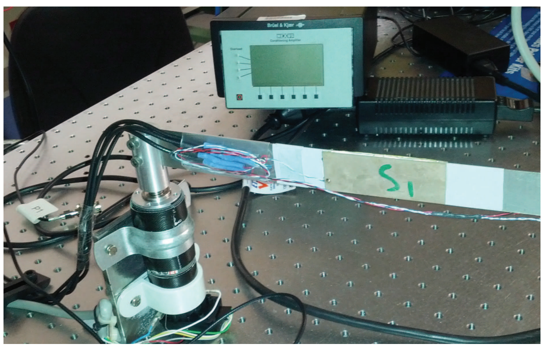

Fig. 5. Flexible beam with piezoelectric actuator and piezoelectric sensor.

example, we employ both the standard and the NI constrained subspace system identification algorithms to identify the transfer function of the given system. The weighting matrices W_1, R_1, R_2 are chosen to be identity matrices . A fifth order model is identified using both algorithms. It is shown in Fig. 6 that the NI constrained subspace system identification algorithm (green line) gives a better fit to the data (red dashed line) compared with the standard subspace system identification algorithm [35] (blue line) for the same model order. Also, the NI constrained subspace method (green line) satisfies the NI property. This fact can be easily verified using the phase plot in Fig. 6, since the phase of this model lies between $[0, -180^o]$, see, [1], [2], whereas the standard subspace system method (blue line) does not satisfy NI property.

VI. Conclusion

In this paper, a modified subspace system identification algorithm is provided. The new algorithm guarantees the negative imaginary property in the identified model. This subspace system identification algorithm can be used in the systems where the dynamics are known to be negative imaginary. An example of modeling a system with collocated force actuator and position sensor is presented. In this example, it has been shown that the modified algorithm gives a closer fit compared with the standard subspace system identification algorithm.

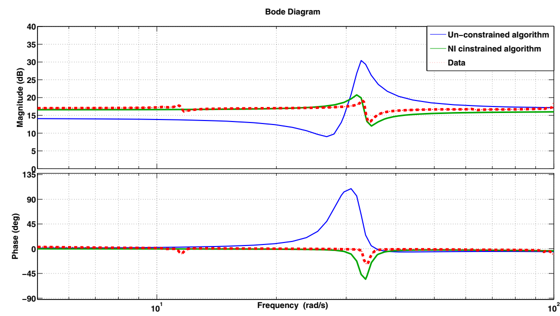

Fig. 6. Bode plot of models obtained by NI constrained subspace system identification algorithm (green line), standard subspace system identification algorithm [35] (blue line) and the measured data.

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