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A New Stability Result for the Feedback Interconnection of Negative Imaginary Systems with a Pole at the Origin

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Abstract— **This paper is concerned with stability conditions for the positive feedback interconnection of negative imaginary systems. A generalization of the negative imaginary lemma is derived, which remains true even if the transfer function has poles on the imaginary axis including the origin. A sufficient condition for the internal stability of a feedback interconnection for NI systems including a pole at the origin is given and an illustrative example is presented to support the result.**

I. INTRODUCTION

Structural modes in machines and robots, ground and aerospace vehicles, and precision instrumentation, such as atomic force microscopes and optical systems, can limit the ability of control systems to achieve the desired performance [1]. This problem is simplified to some extent by using force actuators combined with collocated measurements of velocity, position, or acceleration.

The use of force actuators combined with velocity measurements has been studied using the positive real (PR) theory for linear time invariant (LTI) systems; e.g., see [2], [3]. PR systems, in the single-input single-output (SISO) case, can be defined as systems where the real part of the transfer function is nonnegative. Many systems that dissipate energy fall under the category of PR systems. For instance, they can arise in electric circuits with linear passive components and magnetic couplings. In spite of its success, a drawback of the PR theory is the requirement for the relative degree of the underlying system transfer function to be either zero or one [3]. Hence, the control of flexible structures with force actuators combined with position measurements, cannot use the theory of PR systems.

Lanzon and Petersen introduce a new class of systems in [4], [5] called negative imaginary (NI) systems, which has fewer restrictions on the relative degree of the system transfer function than in the PR case. In the SISO case, such systems are defined by considering the properties of the imaginary part of the transfer function $G(j\omega) = D + C(j\omega I - A)^{-1}B$, and requiring the condition $j\left(G(j\omega) - G(j\omega)^*\right) \geq 0$ for all $\omega \in (0, \infty)$.

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In general, NI systems are stable systems having a phase lag between 0 and $-\pi$ for all $\omega > 0$. That is, their Nyquist plot lies below the real axis when the frequency varies in the open interval $(0, \infty)$ (for strictly negative-imaginary systems, the Nyquist plot should not touch the real axis except at zero frequency or at infinity). This is similar to PR systems where the Nyquist plot is constrained to lie in the right half of the complex plane [2], [3]. However, in contrast to PR systems, transfer functions for NI systems can have relative degree more than unity.

NI systems can be transformed into PR systems and vice versa under some technical assumptions. However, this equivalence is not complete. For instance, such a transformation applied to a strictly negative imaginary (SNI) system always leads to a non-strict PR system. Hence, the passivity theorem [2], [3] cannot capture the stability of the closed-loop interconnection of an NI and an SNI system. In addition, any controller design approach based on strictly PR synthesis cannot be used for the control of an NI system irrespective of whether it is strict or non-strict. Also, transformations of NI systems to bounded-real systems for application of the small-gain theorem suffers from the exact same difficulty of giving a non-strict bounded real system despite the original system being SNI; see [6] for details.

Many practical systems can be consider as NI systems. For example, when considering the transfer function from a force actuator to a corresponding collocated position sensor (for instance, piezoelectric sensor) in a lightly damped structure [1], [4], [5], [7]–[9]. Also, stability results for interconnecting systems with an NI frequency response have been applied to decentralized control of large vehicle platoons in [10]. Here, the authors discuss the availability of various designs to enhance the robust stability of the system with respect to small variations in neighbor-coupling gains.

NI systems theory has been extended by Xiong et. al. in [11]–[13] by allowing for simple poles on the imaginary axis of the complex plane except at the origin. Furthermore, NI controller synthesis has also been discussed in [4], [5]. In addition, it has been shown in [4], [5] that a necessary and sufficient condition for the internal stability of a positive-feedback interconnection of an NI system with transfer function matrix $M(s)$ and an SNI system with transfer function matrix $N(s)$ is given by the DC gain condition $\lambda_{max}(M(0)N(0)) < 1$. Here, the notation $\lambda_{max}(\cdot)$ denotes the maximum eigenvalue of a matrix with only real eigenvalues.

A generalization of the NI lemma in [12], [13] to include a simple pole at the origin was presented in [14]. In [14], stability analysis for a spacial class of generalized NI systems with the inclusion of an integrator connected in parallel with an NI system was discussed. The assumption in [14] restricts the application of the proposed stability result to NI systems which can be decomposed into the parallel connection of an NI system and an integrator.

In this paper, we extend the results in [1], [4], [5], [11]–[14] for NI systems to allow for the existence of a pole at the origin with a more general structure than allowed in the result of [14]. This extension allows us to stabilize any NI system with a pole at the origin without any parallel decomposition assumption. Also, stabilizing NI systems with a pole at the origin can be used for controller design with integral action.

This paper is further organized as follows: Section II introduces the concept of PR and NI systems and presents a relationship between them. The main results of this paper are presented in Section III. Section IV provides a numerical example and the paper is concluded with a summary and remarks on future work in Section V.

II. Preliminaries

In this section, we introduce the definitions of PR and NI systems. We also present a lemma describing the transformation between PR and NI systems, and some technical results which will be used in deriving the main results of the paper.

The definition of PR systems has been motivated by the study of linear electric circuits composed of resistors, capacitors, and inductors. For a detailed discussion of PR systems, see [2], [3] and references therein.

Definition 1: A square transfer function matrix $F(s)$ is positive real if:

1) $F(s)$ has no pole in $Re[s] > 0$.
2) $F(j\omega) + F(j\omega)^* \geq 0$ for all positive real $j\omega$ such that $j\omega$ is not a pole of $F(j\omega)$.
3) If $j\omega_0$, finite or infinite, is a pole of $F(j\omega)$, it is a simple pole and the corresponding residual matrix $K_0 = \lim_{s \longrightarrow j\omega_0} (s - j\omega_0)F(s)$ is positive semidefinite Hermitian.

To establish the main results of this paper, we consider a generalized definition for NI systems which allows for a simple pole at the origin as follows:

Definition 2: A square transfer function matrix $G(s)$ is NI if the following conditions are satisfied:

1) $G(s)$ has no pole in $Re[s] > 0$.
2) For all $\omega \geq 0$ such that $j\omega$ is not a pole of $G(s)$, $j\left(G(j\omega) - G(j\omega)^*\right) \geq 0$.
3) if $s = j\omega_0$ is a pole of $G(s)$ then it is a simple pole. Furthermore if $\omega_0 > 0$, the residual matrix $K_0 = \lim_{s \longrightarrow j\omega_0} (s - j\omega_0)jG(s)$ is positive semidefinite Hermitian.

Definition 3: A square transfer function matrix $G(s)$ is SNI if the following conditions are satisfied:

1) $G(s)$ has no pole in $Re[s] \geq 0$.
2) For all $\omega > 0$, $j\left(G(j\omega) - G(j\omega)^*\right) > 0$.

Due to advances in the theory of PR systems and the complementary definitions of PR and NI systems, it is useful to establish a lemma which considers the relationship between these notions to further develop the theory of NI systems. In order to do so, we consider the possibility of having a simple pole at the origin, and relax the condition $\det(A) \neq 0$ considered in [5], [11], [15]. This leads to a modification of the relationship between PR and NI systems as follows:

Lemma 1: (see also [14]) Given a real rational proper transfer function matrix $G(s)$ with state space realization $\left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ and the transfer function matrix $\tilde{G}(s) = G(s) - D$, the transfer function matrix $G(s)$ is NI if and only if the transfer function matrix $F(s) = s\tilde{G}(s)$ is PR. Here, we assume that any pole zero cancellation which occurs in $s\tilde{G}(s)$ has been carried out to obtain $F(s)$.

Proof: (Necessity) It is straightforward to show that if $\tilde{G}(s)$ is NI then $G(s)$ is NI and vice-versa. Suppose that $j\left(\tilde{G}(j\omega) - \tilde{G}(j\omega)^*\right) \geq 0$, for all $\omega > 0$ such that $j\omega$ is not a pole of $G(s)$. Then given any such $\omega > 0$, $F(j\omega) + F(j\omega)^* = j\omega\left(\tilde{G}(j\omega) - \tilde{G}(j\omega)^*\right) \geq 0$, and $\overline{(F(j\omega) + F(j\omega)^*)} \geq 0$. This means that $F(-j\omega) + F(-j\omega)^* \geq 0$ for all $\omega > 0$ which implies that $F(j\omega) + F(j\omega)^* \geq 0$ for all $\omega < 0$ such that $j\omega$ is not a pole of $G(s)$. Hence, $(F(j\omega) + F(j\omega)^*) \geq 0$ for all $\omega \in (-\infty, \infty)$ such that $j\omega$ is not a pole of $\tilde{G}(j\omega)$.

Now, consider the case where $j\omega_0$ is a pole of $\tilde{G}(s)$ and $\omega_0 = 0$. Since $\tilde{G}(s)$ has only a simple pole at the origin, $F(s) = s\tilde{G}(s)$ will have no pole at the origin because of the pole zero cancellation. This implies that $F(0)$ is finite. Since $F(j\omega) + F(j\omega)^* \geq 0$ for all $\omega > 0$ and $F(j\omega)$ is continuous, this implies that $F(0) + F(0)^* \geq 0$. Also, if $j\omega_0$ is a pole of $\tilde{G}(s)$ and $\omega_0 > 0$, then $\tilde{G}(s)$ can be factored as $\frac{1}{s^2+\omega_0^2}R(s)$, which according to the definition for NI systems implies that the residual matrix $K_0 = \frac{1}{2\omega_0}R(j\omega_0)$ is positive semidefinite Hermitian. This implies that $R(j\omega_0) = R(j\omega_0)^* \geq 0$. Now, the residual matrix of $F(s)$ at $j\omega_0$ with $\omega_0 > 0$ is given by,

$$\begin{aligned} \lim_{s \longrightarrow j\omega_0} (s - j\omega_0)F(s) &= \lim_{s \longrightarrow j\omega_0} (s - j\omega_0)s\tilde{G}(s), \\ &= \lim_{s \longrightarrow j\omega_0} (s - j\omega_0)s\frac{1}{s^2+\omega_0^2}R(s), \\ &= \frac{1}{2}R(j\omega_0) \end{aligned}$$

which is positive semidefinite Hermitian. Hence, $F(s)$ is positive real.

(Sufficiency) Suppose that $F(s)$ is positive real. Then, $F(j\omega) + F(j\omega)^* \geq 0$ for all $\omega \in (-\infty, \infty)$ such that $j\omega$ is not a pole of $F(s)$. This implies $j\omega\left(\tilde{G}(j\omega) - \tilde{G}(j\omega)^*\right) \geq 0$ for all $\omega \geq 0$ such that $j\omega$ is not a pole of $G(s)$. Then $\tilde{G}(j\omega) - \tilde{G}(j\omega)^* \geq 0$ for all such $\omega \in [0, \infty)$. In addition, if $j\omega_0$ is a pole of $F(s)$, then it follows from the definition of PR systems that the residual matrix $\lim_{s \longrightarrow j\omega_0} (s - j\omega_0)F(s)$ is positive semidefinite Hermitian. Also,

$$\begin{aligned}\lim_{s\longrightarrow j\omega_0}(s-j\omega_0)F(s) &= \lim_{s\longrightarrow j\omega_0}(s-j\omega_0)s\tilde{G}(s),\\ &= \omega_0\lim_{s\longrightarrow j\omega_0}(s-j\omega_0)j\tilde{G}(s).\end{aligned}$$

Then using Definition 2, we can conclude that $\tilde{G}(s)$ is NI. ■

Remark 1: Note that a pole zero cancellation at the origin in $F(s)=s\tilde{G}(s)$ will not affect the use of the PR lemma when applied to $F(s)$ since the minimality condition is relaxed in the generalized version of the PR lemma [16], [17].

Now, we present a generalized NI lemma, which allows for a pole at the origin.

Consider the following LTI system,

$$\dot{x}(t)=Ax(t)+Bu(t), \quad (1)$$

$$y(t)=Cx(t)+Du(t), \quad (2)$$

where, $A\in\mathbb{R}^{n\times n}, B\in\mathbb{R}^{n\times m}, C\in\mathbb{R}^{m\times n}$, and $D\in\mathbb{R}^{m\times m}$.

Lemma 2: (see also [14]) Let $\left[\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right]$ be a minimal realization of the transfer function matrix $G(s)\in R^{m\times m}$ for the system in (1)-(2). Then, $G(s)$ is NI if and only if there exist matrices $P=P^T\geq 0$, $W\in\mathbb{R}^{m\times m}$, and $L\in\mathbb{R}^{m\times n}$ such that the following LMI is satisfied:

$$\begin{bmatrix}PA+A^TP & PB-A^TC^T\\ B^TP-CA & -(CB+B^TC^T)\end{bmatrix}=\begin{bmatrix}-L^TL & -L^TW\\ -W^TL & -W^TW\end{bmatrix}\leq 0. \quad (3)$$

Proof: Suppose that $G(s)$ is NI, which implies from Lemma 1 that $F(s)=s\tilde{G}(s)$ with state space realization $\left[\begin{array}{c|c}A & B\\ \hline CA & CB\end{array}\right]$ is PR. It follows from Corollary 2 and Corollary 3 in [17] that there exists a matrix $P=P^T\geq 0$, such that the LMI in (3) is satisfied.

On the other hand, suppose that LMI in (3) is satisfied, then $F(s)$ is PR via Corollary 1 and Corollary 3 in [17], which implies from Lemma 1 that $G(s)$ is NI. ■

In studying the internal stability of an interconnection of NI and SNI systems, we shall use the following SNI lemma:

Lemma 3: [5], [11], [15] Suppose that the proper transfer function matrix $G(s)=C(sI-A)^{-1}B+D$ with a minimal realization $\left[\begin{array}{c|c}A & B\\ \hline C & D\end{array}\right]$ is SNI, then the following conditions are satisfied:

1) $\det(A)\neq 0$, $D=D^T$.
2) There exists a square matrix $P=P^T>0$, $W\in\mathbb{R}^{m\times m}$ and $L\in\mathbb{R}^{m\times n}$ such that the following LMI is satisfied:

$$\begin{bmatrix}PA+A^TP & PB-A^TC^T\\ B^TP-CA & -(CB+B^TC^T)\end{bmatrix}=\begin{bmatrix}-L^TL & -L^TW\\ -W^TL & -W^TW\end{bmatrix}. \quad (4)$$

Also, consider the following lemma, which will be used to derive the main results of this paper in Section III,

Lemma 4: [5] Given $A\in\mathbb{C}^{n\times n}$ with $j(A-A^*)\geq 0$ and $B\in\mathbb{C}^{n\times n}$ with $j(B-B^*)>0$, then $\det(I-AB)\neq 0$.

III. Main results

The key result of this paper is a generalization of the result in [14], which gives stability conditions for an interconnection between an NI system (which may contain a simple pole at the origin) and an SNI system. The generalization is stated in Theorem 1. Now, suppose the transfer function matrix $G_1(s)$ with a minimal realization $\left[\begin{array}{c|c}A_1 & B_1\\ \hline C_1 & D_1\end{array}\right]$ is NI, and $G_2(s)$ with a minimal realization $\left[\begin{array}{c|c}A_2 & B_2\\ \hline C_2 & D_2\end{array}\right]$ is SNI. According to Lemma 2 and Lemma 3, we have,

$$\begin{aligned}P_1A_1+A_1^TP_1&=-L_1^TL_1, & P_2A_2+A_2^TP_2&=-L_2^TL_2,\\ P_1B_1-A_1^TC_1^T&=-L_1^TW_1, & P_2B_2-A_2^TC_2^T&=-L_2^TW_2,\\ C_1B_1+B_1^TC_1^T&=W_1^TW_1, & C_2B_2+B_2^TC_2^T&=W_2^TW_2,\end{aligned} \quad (5)$$

where $P_1\geq 0$ and $P_2>0$. The internal stability of the closed-loop positive-feedback interconnection of $G_1(s)$ and $G_2(s)$ can be guaranteed by considering the stability of the transfer function matrix,

$$(I-G_1(s)G_2(s))^{-1}=\breve{D}+\breve{C}(sI-\breve{A})^{-1}\breve{B},$$

where,

$$\begin{aligned}\breve{A}&=\begin{bmatrix}A_1 & B_1C_2\\ 0 & A_2\end{bmatrix}+\begin{bmatrix}B_1D_2\\ B_2\end{bmatrix}(I-D_1D_2)^{-1}\begin{bmatrix}C_1 & D_1C_2\end{bmatrix}\\ \breve{B}&=\begin{bmatrix}B_1D_2\\ B_2\end{bmatrix}(I-D_1D_2)^{-1},\\ \breve{C}&=(I-D_1D_2)^{-1}\begin{bmatrix}C_1 & D_1C_2\end{bmatrix},\\ \breve{D}&=(I-D_1D_2)^{-1}.\end{aligned} \quad (6)$$

Now, consider the following result, which is the main result of this paper:

Theorem 1: Suppose that $G_1(s)$ is strictly proper and NI and $G_2(s)$ is SNI. Then the closed-loop positive feedback interconnection between $G_1(s)$ and $G_2(s)$ is internally stable if $G_2(0)<0$ and the matrix $A_1+B_1G_2(0)C_1$ is not singular.

Proof: To prove this theorem, we prove that the matrix $\breve{A}$ in (6) is Hurwitz; i.e., all of its poles lie in the left-half of the complex plane.

Let $T=\begin{bmatrix}P_1-C_1^TD_2C_1 & -C_1^TC_2\\ -C_2^TC_1 & P_2\end{bmatrix}$ be a candidate Lyapunov matrix. Since $G_2(0)<0$, $P_1\geq 0$, we claim that

$$P_1-C_1^TG_2(0)C_1>0. \quad (7)$$

In order to prove this claim, consider $M=P_1-C_1^TG_2(0)C_1\geq 0$ and $\mathcal{N}(M)=\{x: Mx=0\}$, where

$\mathcal{N}(\cdot)$ denotes the null space. Also, given any $x \in \mathcal{N}$ we have $P_1 x = 0$ and $C_1 x = 0$. Now, consider the equations

$$P_1 A_1 + A_1^T P_1 = -L_1^T L_1, \tag{8}$$
$$B_1^T P_1 - C_1 A_1 = -W_1^T L_1 \tag{9}$$

outlined in (5). Now pre-multiplying and post-multiplying (8) by x^T and x respectively, we get,

$$L_1 x = 0. \tag{10}$$

Also, post-multiplying (8) by x results in

$$P_1 A_1 x = 0. \tag{11}$$

Subsequently, post-multiplying (9) by x, gives

$$C_1 A_1 x = 0. \tag{12}$$

Now, let $y = A_1 x$, which from (11) and (12) gives

$$P_1 y = 0, \qquad C_1 y = 0 \tag{13}$$

which implies $y \in \mathcal{N}(M)$. Thus, we have established that

$$A_1\ \mathcal{N}(M) \subset \mathcal{N}(M) \text{ and } \mathcal{N}(M) \subset \mathcal{N}(C_1) \tag{14}$$

which leads to the fact that $\mathcal{N}(M)$ is a subset of the unobservable subspace of (A_1, C_1); e.g., see Chapter 18 of [18]. It now follows from the minimality of (A_1, B_1, C_1, D_1) that $\mathcal{N}(M) = \{0\}$. Hence, $M = P_1 - C_1^T G_2(0) C_1 > 0$. This completes the proof of the claim.

Now, using this claim, we have

$$\begin{aligned} & P_2 > 0 \text{ and} \\ & P_1 - C_1^T(D_2 + G_2(0) - D_2)C_1 > 0, \\ \Rightarrow & P_2 > 0 \text{ and} \\ & P_1 - C_1^T D_2 C_1 - C_1^T C_2 P_2^{-1} C_2^T C_1 > 0, \\ \Rightarrow & \begin{bmatrix} P_1 - C_1^T D_2 C_1 & -C_1^T C_2 \\ -C_2^T C_1 & P_2 \end{bmatrix} > 0. \end{aligned}$$

That is, $T > 0$.

Now, the corresponding Lyapunov inequality is given by,

$$\begin{aligned} T\breve{A} + \breve{A}^T T = & \begin{bmatrix} P_1 - C_1^T D_2 C_1 & -C_1^T C_2 \\ -C_2^T C_1 & P_2 \end{bmatrix} \\ & \times \begin{bmatrix} A_1 + B_1 D_2 C_1 & B_1 C_2 \\ B_2 C_1 & A_2 \end{bmatrix} \\ & + \begin{bmatrix} A_1 + B_1 D_2 C_1 & B_1 C_2 \\ B_2 C_1 & A_2 \end{bmatrix}^T \\ & \times \begin{bmatrix} P_1 - C_1^T D_2 C_1 & -C_1^T C_2 \\ -C_2^T C_1 & P_2 \end{bmatrix}, \\ = & - \begin{bmatrix} (C_1^T D_2 W_1^T + L_1^T) & C_1^T W_2^T \\ C_2^T W_1^T & (L_2^T) \end{bmatrix} \\ & \times \begin{bmatrix} (W_1 D_2 C_1 + L_1) & W_1 C_2 \\ W_2 C_1 & (L_2) \end{bmatrix} \\ \leq & 0. \end{aligned}$$

This implies that $\breve{A}$ has all its poles in the closed left half of the complex plane. We now show that $\det(\breve{A}) \neq 0$. Indeed, using the assumption $(A_1 + B_1 G_2(0) C_1)$, we obtain

$$\begin{aligned} & \det(\breve{A}) \\ & = \det(A_2)\det((A_1 + B_1 D_2 C_1 - B_1 C_2 (A_2)^{-1} B_2 C_1) \\ & = \det(A_2)\det(A_1 + B_1 G_2(0) C_1) \\ & = \det(A_2)\det(A_1 + B_1 G_2(0) C_1) \\ & \neq 0 \end{aligned}$$

since $(A_1 + B_1 G_2(0) C_1)$ is non singular and $\det(A_2) \neq 0$. Also, using Lemma 4 and the fact that $G_1(s)$ is NI and $G_2(s)$ is SNI, we conclude that $\det(I - G_1(j\omega)G_2(j\omega)) \neq 0$. This implies that $\breve{A}$ has no eigenvalues on the imaginary axis for $\omega > 0$. Hence, the matrix $\breve{A}$ is Hurwitz. This completes the proof of the theorem. ∎

IV. Illustrative Example

To illustrate the main result of this paper, consider the SNI transfer function $G_2(s) = \frac{1}{s+3} - 1$, which satisfies $G_2(0) = -\frac{2}{3} < 0$ and the strictly proper NI transfer function $G_1(s) = \frac{1}{s(s+1)}$, which has a pole at the origin. Thus, the assumptions in Theorem 1 are satisfied and we can conclude that the closed-loop system is stable. Also, the poles of the closed-loop transfer function corresponding to $G_2(s)$ and $G_1(s)$ are the roots of the polynomial $(1 - G_1(s)G_2(s)) = s^3 + 4s^2 + 4s + 2$ which are $\{-2.84, -0.58 \pm 0.61i\}$. This verifies that the closed-loop transfer function is indeed asymptotically stable.

V. Conclusion

In this paper, stability results for a positive-feedback interconnection of NI systems have been derived. A generalization of the NI lemma, allowing for a simple pole at the origin, has been used in deriving these results. This work can be used in the controller design to allow for a broader class of NI systems than considered previously. Also, the stability result for an NI system with a pole at the origin connected with an SNI system can be used for controller design including integral action. The validity of the main results in this paper have been illustrated via a numerical example.

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