

Linear time-varying controller design with stability guarantees*

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A time-varying plant is in practice frequently modelled by a set of LTI models identified in different operational conditions. Here we focus on the case where the operational conditions are dependent on one measured variable and where the plant is described by two LTI models identified for the limit values of the measured variable. Consider a single-input, single-output linear parameter dependent model:

$$G(s, \lambda) = \frac{\lambda n_1(s) + (1 - \lambda)n_0(s)}{\lambda d_1(s) + (1 - \lambda)d_0(s)} = \frac{n(s, \lambda)}{d(s, \lambda)} \quad (1)$$

where $n_0(s)$, $n_1(s)$, $d_0(s)$, and $d_1(s)$ are some known linear time-invariant (LTI) polynomials and $\lambda \in [0, 1]$ is the measured parameter. For time-varying parameter $\lambda(t)$ equation (1) corresponds to a linear time-varying (LTV) model. The LTI plant models for the two limit cases $\lambda = 0$ and $\lambda = 1$ are known and they are: $G(s, 0) = \frac{n_0(s)}{d_0(s)}$, $G(s, 1) = \frac{n_1(s)}{d_1(s)}$ and suppose that we have designed two closed-loop LTI controllers $C_0(s)$, $C_1(s)$ stabilizing $G(s, 0)$, $G(s, 1)$ respectively.

The goal is to find a methodology to design a closed-loop parameter-dependent controller $C(s, \lambda)$ (leading to LTV controller for $\lambda(t)$ time-varying):

$$C(s, \lambda) = \frac{\lambda^n r_n(s) + \lambda^{n-1} r_{n-1}(s) + \dots + r_0(s)}{\lambda^n s_n(s) + \lambda^{n-1} s_{n-1}(s) + \dots + s_0(s)} \quad (2)$$

(with $r_i(s)$, $s_i(s)$ some LTI polynomials) stabilizing the plant model (1) for all frozen $\lambda \in [0, 1]$, and given a bound on the average rate of variation of $\lambda(t)$ also stabilizes the given LTV plant model for all admissible time-varying $\lambda(t)$. Additionally the LTV controller should preserve in some way the performance obtained with $(G(s, 0)$ and $C_0(s))$ and $(G(s, 1)$ and $C_1(s))$ closed loops.

The main idea is that we search for a controller $C(s, \lambda)$ constraining the closed loop characteristic polynomial $P_{CL}(s, \lambda)$ to be affine in λ , i.e. in the form $P_{CL}(s, \lambda) = \lambda p_1(s) + p_0(s)$. This closed-loop property has several advantages: 1) Using conditions developed in [1], closed-loop stability for all constant values of $\lambda \in [0, 1]$ can be checked. 2) A bound on the average rate of $\lambda(t)$ variation assuring closed loop stability can be evaluated using conditions from [3]. 3) Since the system performance is given in part by its characteristic polynomial, it might be that the time-varying closed-loop performance for slow-time variations will be a kind of linear interpolation of the closed-loop performances for the two

limit LTI closed loops $(G(s, 0), C_0(s))$ and $(G(s, 1), C_1(s))$. A similar philosophy is proposed in [2], but for the more general case of multiple time-varying parameters.

We have first considered the simplest 1st-order λ -dependent controller $C = (\lambda r_1 + r_0)/(\lambda s_1 + s_0)$ for (2). We show that under certain restrictive conditions on the controllers $C_0(s)$ and $C_1(s)$, there exist a unique controller making the polynomial $P_{CL}(s, \lambda)$ affine in λ . Nevertheless, nothing guarantees that this controller provide LTV closed-loop stability. For the 2nd-order λ -dependent controller $C = (\lambda^2 r_2 + \lambda r_1 + r_0)/(\lambda^2 s_2 + \lambda s_1 + s_0)$, we show that under some mild conditions on controllers $C_0(s)$, $C_1(s)$ and models $G(s, 0)$, $G(s, 1)$, there exists a set of controllers making the polynomial $P_{CL}(s, \lambda)$ affine in λ . This set of controllers can be searched to obtain: 1) stability for all fixed $\lambda \in [0, 1]$, 2) the highest possible bound on average $\lambda(t)$ -variation guaranteeing stability, 3) a better LTV closed-loop performance. Finally we show that for higher-order controllers the existence conditions, the controller set, and the closed-loop characteristic polynomial are the same as in the case of the 2nd-order controller. The only difference is in the numerators of sensitivity functions containing higher orders of λ .

References

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* This work is supported by an Australian Research Council Discovery-Project Grant DP0342683 and by the Belgian Programme on Interuniversity Poles of Attraction.