Proceedings of the American Control Conference
San Diego, California • June 1999

Trajectory/Force Control for Robotic Manipulators using Sliding-Mode and Adaptive Control

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Abstract

This paper presents a new robust trajectory/force controller for non-redundant rigid manipulators designed using sliding-mode and adaptive control techniques. Sliding-mode control is used to take care of the uncertain robot dynamics, whereas adaptive control is used to estimate the unknown environment stiffness. Experimental results show that trajectory tracking and force regulation are achieved with bounded errors. This paper assumes known location and geometry of the environment.

Keywords: compliant motion control, constrained motion control, trajectory control, force control, sliding-mode control, adaptive control.

1 Introduction

Several researchers have investigated the problem of trajectory control of rigid robots and such control schemes are now well known [1, 4, 12]. These control techniques are adequate when the manipulator does not interaction significantly with the environment, but in applications where considerable contact is necessary, the control of both force and position is required. The task then is to exert a desired profile of force in the constrained degrees of freedom while following the reference trajectory in the unconstrained degrees of freedom. This is generally referred to as the *Compliant Motion Control* problem.

The problem is non-trivial because the location and geometry of the environment are usually not well known, the environment stiffness is also usually unknown, trajectory tracking and force regulation must occur in the presence of model uncertainty and disturbances, the controller must be capable of handling changes in constraints, and the force measurement is usually very noisy thus prohibiting force derivatives to be computed.

The problem considered here is that of controlling a general n-degree of freedom non-redundant rigid manipulator to track reference trajectories in the unconstrained directions and to regulate force at a desired value in the constrained directions, in the presence of *model uncertainty* and *unknown environment stiffness.* Furthermore, changes in constraints are also considered. Alternative ways of addressing this problem can be found in [5, 8, 9, 13] and references therein.

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2 Problem Formulation

Consider a non-redundant rigid manipulator with n-degrees of freedom. Let q be the vector of joint displacements, τ be the vector of joint torques, $H(q)$ be the manipulator inertia matrix, and $h(q,\dot{q})$ be the nonlinear term containing centrifugal, Coriolis and gravitational forces. Also, let F be the environment reaction force read with respect to the compliance frame (see Figure 2 for direction of compliance frame). Furthermore, let x be the position vector of the end-effector measured from the compliance frame and $\Upsilon(\cdot)$ be the kinematic transformation mapping *joint space* (q-coordinates) to *task space* (x-coordinates). Note that $\Upsilon : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear transformation whose Jacobian is denoted by $J(q)$. Then the manipulator dynamics are given by:

$$H(q)\ddot{q} + h(q,\dot{q}) = \tau + J^T(q)F, \tag{1}$$

and the kinematic relations are given by:

$$\begin{aligned} x &= \Upsilon(q) &&\Rightarrow\quad q = \Upsilon^{-1}(x), \\ \dot{x} &= J(q)\dot{q} &&\Rightarrow\quad \dot{q} = J(q)^{-1}\dot{x}, \\ \ddot{x} &= J(q)\ddot{q} + \dot{J}(q)\dot{q} &&\Rightarrow\quad \ddot{q} = J(q)^{-1}\big(\ddot{x} - \dot{J}(q)\dot{q}\big). \end{aligned} \tag{2}$$

Here it is assumed that the manipulator arm does not pass through a manipulator singularity. This is necessary so that inverse transformations are possible. Note that $J(q)$ is the Jacobian of the transformation $\Upsilon(\cdot)$, and is not the same as the manipulator Jacobian. Dropping the operands for clarity (they are all in joint space) and using equation (2) in (1) gives $HJ^{-1}(\ddot{x} - \dot{J}\dot{q}) + h = \tau + J^T F$. The dynamics are then inverted by letting:

$$\tau = \hat{H}\hat{J}^{-1}\big(u - \dot{\hat{J}}\dot{q}\big) + \hat{h} - \hat{J}^T F, \tag{3}$$

where $\hat{H}$, $\hat{h}$ and $\hat{J}$ are estimates of H, h and J respectively and u represents a new input to the system which is yet to be chosen. Applying this torque input τ and rearranging gives:

$$\ddot{x} = G(X)u + f(X) + M(X)F, \tag{4}$$

in which

$$\begin{aligned} G(X) &= JH^{-1}\hat{H}\hat{J}^{-1}, \\ f(X) &= JH^{-1}\Big[\big(HJ^{-1}\dot{J} - \hat{H}\hat{J}^{-1}\dot{\hat{J}}\big)\dot{q} + \big(\hat{h} - h\big)\Big], \\ M(X) &= JH^{-1}(J^T - \hat{J}^T). \end{aligned} \tag{5}$$

Note that, in the above equations, the expressions on the right-hand side are functions of q and $\dot{q}$ whereas those on the left-hand side are functions of $X = (x^T, \dot{x}^T)^T \in \mathbb{R}^{2n}$. This is valid since equations (2) provide a mapping from $(q^T, \dot{q}^T)^T$ to $(x^T, \dot{x}^T)^T$.

In equation (4), $x, u, F \in \mathbb{R}^n$, $X = (x^T, \dot{x}^T)^T \in \mathbb{R}^{2n}$, $f : \mathbb{R}^{2n} \to \mathbb{R}^n$ and $G, M : \mathbb{R}^{2n} \to \mathbb{R}^{n\times n}$. Equation (4) is now suitable for sliding-mode controller synthesis [10]. Designing a controller for this equation immediately yields a controller for the robot arm $H(q)\ddot{q} + h(q, \dot{q}) = \tau + J^T(q)F$, since equation (3) provides a mapping from the control input u to the torque input τ.

3 Controller Design

The controller is derived in two steps. In the first step, the environment stiffness is assumed to be completely known, and in the second step an adaptive algorithm is derived to relax this assumption. The reader is referred to [6] for a more detailed synthesis derivation.

3.1 Known Environment Stiffness

Consider the system described by equation (4). Let the components of $f(X)$ be denoted by $f(X)_i$ and the elements of $G(X)$ and $M(X)$ be $g(X)_{ij}$ and $m(X)_{ij}$ respectively. Here u is the control input, $X = (x^T, \dot{x}^T)^T$ is the state-vector — wherein x is the output position, and F is the reaction force exerted by an environment placed at $x = 0$ and having environment stiffness k. Thus, $F_i = -kx_i$ if $x_i \geq 0$ and $F_i = 0$ otherwise. There is no loss of generality in assuming that the environment is located at $x = 0$. Expressing equation (4) in component form yields:

$$\ddot{x}_i = g(X)_{ii}u_i + \Big[f(X)_i + c_i(X, u, F)\Big] + m(X)_{ii}F_i \quad (6)$$

where $c_i(X, u, F) = \sum_{j\neq i}\big[g(X)_{ij}u_j + m(X)_{ij}F_j\big]$.

$G(X)$, $f(X)$ and $M(X)$ are, in general, nonlinear matrix functions of the state. They are not exactly known but the extent of imprecision from their corresponding nominal functions I, 0 and 0 [see equations (5)] is bounded *component-wise* by the specified matrix functions $V : \mathbb{R}^{2n} \to \mathbb{R}^{n\times n}$, $U : \mathbb{R}^{2n} \to \mathbb{R}^n$ and $W : \mathbb{R}^{2n} \to \mathbb{R}^{n\times n}$ respectively as follows: $V(X)_{ii}^{-1} \leq g(X)_{ii} \leq V(X)_{ii}$, $|g(X)_{ij}| \leq V(X)_{ij}$ whenever $j \neq i$, $|f(X)_i| \leq U(X)_i$ and $|m(X)_{ij}| \leq W(X)_{ij}$. Here $U(X)_i$ are the components of $U(X)$ and $W(X)_{ij}$ and $V(X)_{ij}$ are the elements of $W(X)$ and $V(X)$ respectively. Above, it is assumed that $g(X)_{ii} > 0$[1] and that $V(X)_{ii} \geq 1$ for a sensible problem. $g(X)_{ii}$ is bounded differently from $g(X)_{ij}$ [when $j \neq i$] because $g(X)_{ii}$ appears multiplying u_i.

Let λ be a scalar design parameter representing the desired control bandwidth. Define $\tilde{x} = x - x_d$ as the error between the actual position and the desired position and let the desired position x_d be obtained by subtracting some internal control signal $\alpha(t)$ from the reference signal x_r (i.e. $x_d = x_r - \alpha$). Thus $x_d, x_r, \alpha \in \mathbb{R}^n$. It will be seen that $\alpha(t)$ plays an important role in force control. $\alpha(t) = 0$ in trajectory control, hence giving $x_d(t) = x_r(t)$, and varies so that force control is achieved when the system is in contact with the environment.

Let the variables of interest be $\int^t \tilde{x}_i\, dT$ and define time-varying sliding surfaces in state-spaces $\mathbb{R}^r$ by the equations $s_i(X, t) = 0$, where $s_i = \left(\frac{d}{dt} + \lambda\right)^{r-1} \int^t \tilde{x}_i\, dT$ with $r = 3$. The integral $\int^t \tilde{x}_i\, dT$ is defined to within a constant as there is no lower limit of integration. This constant is chosen so that $s_i(t=0) = 0$ *regardless of* any initial condition. Here $r = 3$ as the system described by equation (6) is third-order with respect to the variable of interest $\int^t \tilde{x}_i\, dT$. Thus s_i can be rewritten as:

$$s_i = \dot{\tilde{x}}_i + 2\lambda\tilde{x}_i + \lambda^2 \int_0^t \tilde{x}_i\, dT \underbrace{- \dot{\tilde{x}}_i(0) - 2\lambda\tilde{x}_i(0)}_{\text{so that } s_i = 0 \text{ at } t = 0}. \quad (7)$$

Let 'sat' denote the saturation function defined as $\operatorname{sat} y = y$ if $|y| \leq 1$ and $\operatorname{sat} y = y/|y|$ otherwise; and let Φ_i be the (non-constant) thickness of a boundary layer neighbouring the switching surface $s_i(t) = 0$. This boundary layer is required so that the switched control law (which causes control chattering) can be approximated by a continuous control law inside this boundary [11]. Differentiating equation (7), substituting for $\ddot{x}_i$ from equation (6) and choosing

$$u_i = \ddot{x}_{d_i} - 2\lambda\dot{\tilde{x}}_i - \lambda^2\tilde{x}_i - \Psi_i \operatorname{sat}\frac{s_i}{\Phi_i}, \quad (8)$$

where Ψ_i is the extent of nonlinearity required to guarantee that state-trajectories outside the boundary layer converge to within the boundary layer, together give:

$$\begin{aligned}\dot{s}_i = {} & (g(X)_{ii} - 1)(\ddot{x}_{d_i} - 2\lambda\dot{\tilde{x}}_i - \lambda^2\tilde{x}_i) + f(X)_i \\ & + c_i(X, u, F) + m(X)_{ii}F_i - g(X)_{ii}\Psi_i \operatorname{sat}\frac{s_i}{\Phi_i}. \quad (9)\end{aligned}$$

When $|s_i| > \Phi_i$, all state-trajectories are required to converge to within the sliding region. This is ensured by satisfying the Lyapunov Condition $\frac{1}{2}\frac{d}{dt}s_i^2 = \dot{s}_i s_i \leq (\dot{\Phi}_i - \eta)|s_i|$ [10], where η is a positive constant required to ensure that this convergence occurs in finite time. By substituting equation (9) in the Lyapunov Condition above and rearranging, one can show that this Lyapunov Condition is satisfied and boundary layer attractiveness is guaranteed if Ψ_i is chosen as:

$$\begin{aligned}\Psi_i = {} & (V(X)_{ii} - 1)\left|\ddot{x}_{d_i} - 2\lambda\dot{\tilde{x}}_i - \lambda^2\tilde{x}_i\right| + V(X)_{ii}\Big[\eta + U(X)_i \\ & + C_i(X, u, F) + W(X)_{ii}|F_i|\Big] - V(X)_{ii}^{-\operatorname{sgn}\dot{\Phi}_i}\dot{\Phi}_i \quad (10)\end{aligned}$$

where $C_i(X, u, F) = \sum_{j\neq i}\big[V(X)_{ij}|u_j| + W(X)_{ij}|F_j|\big]$.

[1] This assumption is very mild. It only means that u_i contributes to $\ddot{x}_i$ with known sign.

When $|s_i| \leq \Phi_i$, from equation (9) we see that:

$$\dot{s}_i + \left(\frac{g(X)_{ii}\Psi_i}{\Phi_i}\right) s_i = (g(X)_{ii} - 1)(\ddot{x}_{d_i} - 2\lambda\dot{\tilde{x}}_i - \lambda^2\tilde{x}_i) + f(X)_i + c_i(X,u,F) + m(X)_{ii}F_i. \quad (11)$$

By letting $\left(\frac{g(X)_{ii}\Psi_i}{\Phi_i}\right)_{\max} = \lambda$ we get:

$$V(X)_{ii}\frac{\Psi_i}{\Phi_i} = \lambda \quad \Rightarrow \quad \Psi_i = V(X)_{ii}^{-1}\lambda\Phi_i. \quad (12)$$

This condition is known as the "Balanced Condition". Using equation (12) in (10) gives:

$$\dot{\Phi}_i + V(X)_{ii}^{-1+\operatorname{sgn}\dot{\Phi}_i}\lambda\Phi_i = V(X)_{ii}^{1+\operatorname{sgn}\dot{\Phi}_i}\left[\eta + U(X)_i + C_i(X,u,F) + W(X)_{ii}|F_i|\right] + V(X)_{ii}^{\operatorname{sgn}\dot{\Phi}_i}(V(X)_{ii} - 1)\left|\ddot{x}_{d_i} - 2\lambda\dot{\tilde{x}}_i - \lambda^2\tilde{x}_i\right|. \quad (13)$$

From the above synthesis, it should be clear that the control inputs make x_i follow x_{d_i}. Thus *Trajectory Control* is achieved by setting $\alpha_i = 0$ (as in this case $x_{d_i} = x_{r_i}$) and *Force Control* can be obtained, while in contact with the environment, by varying $\alpha_i(t)$ so that F_i remains constant at the value of the desired contact force F_d (one value of F_d for all directions i).

Then during contact (i.e. $x_i \geq 0$), we have $F_i = -kx_i$ and defining $\tilde{F}_i = F_i - F_d$ gives $\dot{F}_i = \dot{\tilde{F}}_i = -k\dot{x}_i$ and $\ddot{F}_i = \ddot{\tilde{F}}_i = -k\ddot{x}_i$, as F_d is a constant. Note that due to our sign convention, the desired contact force F_d should be defined as a negative number. Substituting the above contact relations and equation (8) into equation (6) gives the following closed-loop contact error dynamics:

$$\ddot{\tilde{F}}_i + 2\lambda\dot{\tilde{F}}_i + \lambda^2\tilde{F}_i = -\lambda^2 F_d - k\Big\{(\ddot{x}_{r_i} + 2\lambda\dot{x}_{r_i} + \lambda^2 x_{r_i}) - (\ddot{\alpha}_i + 2\lambda\dot{\alpha}_i + \lambda^2\alpha_i) + (1 - g(X)_{ii}^{-1})\ddot{x}_i + g(X)_{ii}^{-1}\Big[f(X)_i + c_i(X,u,F) + m(X)_{ii}F_i\Big] - \Psi_i \operatorname{sat}\frac{s_i}{\Phi_i}\Big\}. \quad (14)$$

We now seek an equation which determines how $\alpha_i(t)$ should vary during contact so as to achieve Force Control. Treating equation (14) as our contact system dynamics and thinking of $\ddot{\alpha}_i + 2\lambda\dot{\alpha}_i + \lambda^2\alpha_i$ as our contact control input, we again use a sliding-mode control approach to get such a relation. Let the variables of interest be $\int^t \frac{F_i}{-k}\,dT$ and define time-varying sliding surfaces in state-spaces $\mathbb{R}^r$ by the scalar equations $p_i(X,t) = 0$, where $p_i = \left(\frac{d}{dt} + \lambda\right)^{r-1}\int^t \frac{\tilde{F}_i}{-k}\,dT$ with $r = 3$. Thus,

$$(-kp_i) = \dot{\tilde{F}}_i + 2\lambda\tilde{F}_i + \lambda^2\int_0^t \tilde{F}_i\,dT - \dot{\tilde{F}}_i(0) - 2\lambda\tilde{F}_i(0). \quad (15)$$

Similarly to before, let ϕ_i be the thickness of a boundary layer neighbouring the force control switching surface $p_i(t) = 0$ and let ψ_i be the extent of nonlinearity required to ensure that state-trajectories outside this boundary layer converge to within it. Then differentiating equation (15), substituting for $\ddot{\tilde{F}}_i + 2\lambda\dot{\tilde{F}}_i + \lambda^2\tilde{F}_i$ from equation (14) and choosing

$$\ddot{\alpha}_i + 2\lambda\dot{\alpha}_i + \lambda^2\alpha_i = \frac{\lambda^2 F_d}{k} + (\ddot{x}_{r_i} + 2\lambda\dot{x}_{r_i} + \lambda^2 x_{r_i}) - \Psi_i \operatorname{sat}\frac{s_i}{\Phi_i} + \psi_i \operatorname{sat}\frac{p_i}{\phi_i}, \quad (16)$$

gives (after some rearrangement):

$$\dot{p}_i = (g(X)_{ii} - 1)u_i + f(X)_i + c_i(X,u,F) + m(X)_{ii}F_i - \psi_i \operatorname{sat}\frac{p_i}{\phi_i}. \quad (17)$$

By an analogous argument to above, when $|p_i| > \phi_i$, state-trajectories are required to satisfy the Lyapunov Condition $\frac{1}{2}\frac{d}{dt}p_i^2 = \dot{p}_i p_i \leq (\dot{\phi}_i - \eta)|p_i|$ for boundary layer attractiveness, which is the case if ψ_i is chosen as:

$$\psi_i = \eta + (V(X)_{ii} - 1)|u_i| + U(X)_i + C_i(X,u,F) + W(X)_{ii}|F_i| - \dot{\phi}_i. \quad (18)$$

When $|p_i| \leq \phi_i$, equation (17) reduces to:

$$\dot{p}_i + \left(\frac{\psi_i}{\phi_i}\right)p_i = (g(X)_{ii} - 1)u_i + f(X)_i + c_i(X,u,F) + m(X)_{ii}F_i. \quad (19)$$

Hence let $\quad \frac{\psi_i}{\phi_i} = k \quad \Rightarrow \quad \psi_i = (k\phi_i). \quad (20)$

Using this in equations (18) and (19) gives:

$$\dot{\phi}_i + (k\phi_i) = (V(X)_{ii} - 1)|u_i| + \eta + U(X)_i + C_i(X,u,F) + W(X)_{ii}|F_i|, \quad (21)$$

and $$\dot{p}_i + kp_i = (g(X)_{ii} - 1)u_i + f(X)_i + c_i(X,u,F) + m(X)_{ii}F_i. \quad (22)$$

Now note that for a rigid environment k is very large (around 10^7 Nm^{-1}) and hence equations (21) and (22) become approximately algebraic. This, however, does not imply that there will be high control activity leading to control chattering since $\psi_i \operatorname{sat}\frac{p_i}{\phi_i} \equiv -\psi_i \operatorname{sat}\frac{(-kp_i)}{(k\phi_i)}$. Thus the ratio p_i/ϕ_i remains unchanged and it is this ratio which is directly related to the control activity [1].

However, if one applies two different control laws (one for the non-contact phase and one for the contact phase) and switch between them when the system is entering or leaving contact, then the closed-loop system may enter a limit cycle with the manipulator oscillating to and fro between the two control laws. To eliminate this effect we define the signal $\mathcal{L}_\mu^{\xi,\omega_n}(x_{r_i}, \dot{x}_{r_i})$ to be the low-pass filtered version of the signal $\mathcal{L}_\mu(x_{r_i}, \dot{x}_{r_i})$ through the second-order filter[2] $\frac{\omega_n^2}{\ell^2 + 2\xi\omega_n\ell + \omega_n^2}$, and $\mathcal{L}_\mu(x_{r_i}, \dot{x}_{r_i})$ is defined to be 1 when $\dot{x}_{r_i} + \mu x_{r_i} \geq 0$ and 0 otherwise. The

[2]Hereafter, ℓ will be the Laplace variable; so as not to confuse it with the sliding surfaces s.

reasons for choosing such a continuous gating signal will not be discussed here due to space constraints, however the reader is referred to [6] for a detailed discussion. Consequently, we modify equation (16) by multiplying its right-hand side with $\mathcal{L}_{\mu}^{\xi,\omega_n}(x_{r_i}, \dot{x}_{r_i})$ as follows:

$$\ddot{\alpha}_i + 2\lambda\dot{\alpha}_i + \lambda^2\alpha_i = \left[\frac{\lambda^2 F_d}{k} + (\ddot{x}_{r_i} + 2\lambda\dot{x}_{r_i} + \lambda^2 x_{r_i}) - \Psi_i \,\mathrm{sat}\,\frac{s_i}{\Phi_i} - \psi_i \,\mathrm{sat}\,\frac{(-kp_i)}{(k\phi_i)}\right]\mathcal{L}_{\mu}^{\xi,\omega_n}(x_{r_i}, \dot{x}_{r_i}). \quad (23)$$

Summing up, the resulting multivariable controller for the dynamical system described by equation (4), assuming known k, is given by equations (7), (8), (12), (13), (15), (20), (21) and (23).

3.2 Unknown Environment Stiffness

The assumption that the environment stiffness k is known will now be removed by deriving an adaptive algorithm that estimates it. Adaptation only makes sense when the system is in contact with the environment and is in force control. During this time $\mathcal{L}_{\mu}^{\xi,\omega_n}(x_{r_i}, \dot{x}_{r_i}) = 1$ [see equation (23)]. The procedure used here to derive an adaptive observer uses some of the ideas described in [3], although the techniques used in this study are different from those presented there.

In practice k is unknown — only an estimate $\hat{k}$ is available. Consequently, since $\alpha_i(t)$ depends on k [see equation (23)], then only an estimate of $\alpha_i(t)$ is available. Denote this estimate by $\hat{\alpha}_i(t)$. If this estimate is used in the control law, then equation (8) becomes:

$$u_i = (\ddot{x}_{r_i} - \ddot{\hat{\alpha}}_i) - 2\lambda[\dot{x}_i - (\dot{x}_{r_i} - \dot{\hat{\alpha}}_i)] - \lambda^2[x_i - (x_{r_i} - \hat{\alpha}_i)] - \Psi_i \,\mathrm{sat}\,\frac{s_i}{\Phi_i}. \quad (24)$$

Define $\theta = 1/k$, $\hat{\theta} = 1/\hat{k}$ and $\Omega = \lambda^2 F_d$ and consider the following adaptive observer structure:

$$\ddot{\hat{\alpha}}_i + 2\lambda\dot{\hat{\alpha}}_i + \lambda^2\hat{\alpha}_i = \left[(\hat{\theta}\Omega + \dot{\hat{\theta}}\bar{\Omega}_i) + (\ddot{x}_{r_i} + 2\lambda\dot{x}_{r_i} + \lambda^2 x_{r_i}) - \Psi_i \,\mathrm{sat}\,\frac{s_i}{\Phi_i} - (\hat{k}\phi_i) \,\mathrm{sat}\,\frac{(-\hat{k}p_i)}{(\hat{k}\phi_i)}\right]\mathcal{L}_{\mu}^{\xi,\omega_n}(x_{r_i}, \dot{x}_{r_i}) \quad (25)$$

with

$$\dot{\bar{\Omega}}_i + \rho\bar{\Omega}_i = \Omega, \qquad 0 < \rho < 2\lambda, \quad (26)$$

$$\dot{\hat{\theta}} = -\gamma\bar{\Omega}_i\tilde{F}_i, \qquad 0 < \gamma. \quad (27)$$

Note that the adaptation gain γ is to be chosen small enough so that adaptation is slow when compared to the closed-loop system dynamics produced by the sliding-mode controller. This is required to ensure two different time-scales in design [2]. Note also that we have only one adaptive law, as the environment stiffness k is assumed to be the same in all directions i. Substituting equation (25) with $\mathcal{L}_{\mu}^{\xi,\omega_n}(x_{r_i}, \dot{x}_{r_i}) = 1$ into equation (24), then the resulting control law into the system dynamics (6) and making use of the contact relations $F_i = -kx_i$ stated earlier, we get:

$$\ddot{\tilde{F}}_i + 2\lambda\dot{\tilde{F}}_i + \lambda^2\tilde{F}_i = k\Big\{(\hat{\theta}\Omega + \dot{\hat{\theta}}\bar{\Omega}_i - \theta\Omega) - g(X)_{ii}^{-1}\big[f(X)_i + c_i(X,u,F) + m(X)_{ii}F_i\big] - (1 - g(X)_{ii}^{-1})\ddot{x}_i - (\hat{k}\phi_i)\,\mathrm{sat}\,\frac{(-\hat{k}p_i)}{(\hat{k}\phi_i)}\Big\}. \quad (28)$$

Introducing $\tilde{\theta} = \hat{\theta} - \theta$ and noting that $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$ (as θ is constant), we observe that $\hat{\theta}\Omega + \dot{\hat{\theta}}\bar{\Omega}_i - \theta\Omega = (\frac{d}{dt} + \rho)\{\tilde{\theta}\bar{\Omega}_i\}$, by equation (26). Now, taking the Laplace Transform $\mathscr{L}$ of the above equation gives:

$$\tilde{F}_i(\ell) = k\frac{(\ell + \rho)}{(\ell^2 + 2\lambda\ell + \lambda^2)}\mathscr{L}\left\{\tilde{\theta}\bar{\Omega}_i\right\} - \frac{k}{(\ell^2 + 2\lambda\ell + \lambda^2)} \times \mathscr{L}\Big\{g(X)_{ii}^{-1}\big[f(X)_i + c_i(X,u,F) + m(X)_{ii}F_i\big] + (1 - g(X)_{ii}^{-1})\ddot{x}_i + (\hat{k}\phi_i)\,\mathrm{sat}\,\frac{(-\hat{k}p_i)}{(\hat{k}\phi_i)}\Big\}.$$

The second input will be approximately equal to zero since $(-\hat{k}p_i)$ is varying so as to cancel out the affects of the other terms[3]. It will in fact be exactly zero in the absence of uncertainty and will otherwise be very small. Define $W(\ell) = k\frac{(\ell+\rho)}{(\ell^2+2\lambda\ell+\lambda^2)}$. Then the equation above reduces to $\tilde{F}_i(\ell) \approx W(\ell)\mathscr{L}\{\tilde{\theta}\bar{\Omega}_i\}$, which can be rewritten in state-space form as:

$$\dot{\sigma} = A\sigma + b\{\tilde{\theta}\bar{\Omega}_i\} \quad (29)$$

$$\tilde{F}_i = c^T\sigma \quad (30)$$

where σ is the state vector and A is Hurwitz. Furthermore, since $\rho < 2\lambda$ [by equation (26)], then it can be easily shown that $W(\ell)$ is strictly positive real and consequently by the Kalman-Yakubovich-Popov lemma [7]:

$$\exists\, P = P^T > 0,\ Q = Q^T > 0\ :\ A^TP + PA = -Q \quad (31)$$

$$b^TP = c^T. \quad (32)$$

Thus, choosing $V = \sigma^TP\sigma + \tilde{\theta}^2/\gamma$ as a Lyapunov candidate function, differentiating it and using equations (29) to (32), gives $\dot{V} = -\sigma^TQ\sigma + 2\tilde{\theta}[\bar{\Omega}_i\tilde{F}_i + \dot{\tilde{\theta}}/\gamma]$. This reduces to $\dot{V} = -\sigma^TQ\sigma < 0\ \forall\sigma \neq 0$ by using equation (27), which in turn implies that $\tilde{F}_i \to 0$ and $\tilde{\theta} \to 0$ as $t \to \infty$ by equations (29) and (30).

Summing up, the resulting adaptive sliding-mode controller for the dynamical system described by equation (4) is obtained by combining the controller derived in the previous section with the adaptive observer given in this section. Hence recalling that here $x_{d_i} = x_{r_i} - \hat{\alpha}_i$, $\hat{k} = 1/\hat{\theta}$ and $\Omega = \lambda^2F_d$; then the final controller is given by equations (7), (12), (13), (15) and (21) with k replaced by $\hat{k}$, (24), (25), (26), and (27).

[3] This can be seen, after some algebra, by eliminating u_i from equations (6) and (22).

4 Physical Implementation

The controller derived above was tested on the planar two-degree of freedom robot arm shown in Figure 1. This figure also shows the environment (a perspex plate) and the force sensor (a strain gauge bridge) which was mounted behind the environment, rather than on the end-effector, due to ease of construction. The chosen reference signal was a repeating circular trajectory of radius 7cm and period 4 seconds, as illustrated in Figure 2. This figure also shows the location of the environment. The resulting semi-circular path traced by the end-effector and the reaction force exerted by the manipulator on the environment are depicted in Figure 3. The effects of adaptation can clearly be seen in the reaction force data. Finally, Figure 4 confirms that trajectory control and force regulation were achieved without excessive control activity.

5 Conclusions

A *single* controller which achieves bounded trajectory-tracking and force-regulation errors in the presence of bounded uncertainty and disturbances and unknown environment stiffness was derived. The control action was also designed to be smooth (i.e. differentiable) at the contact boundary, thus ensuring that the controller 'gracefully' handles changes in constraints. The price paid for using only one control law is increased controller complexity. The experimental results presented in this paper demonstrate the applicability of the controller and show how the controller handles changes in constraints.

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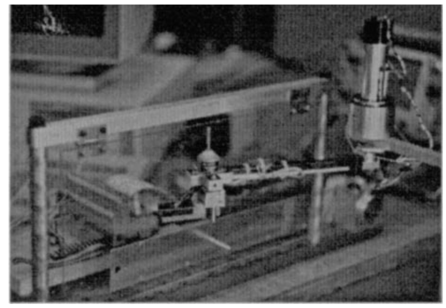

Figure 1: End-effector and environment

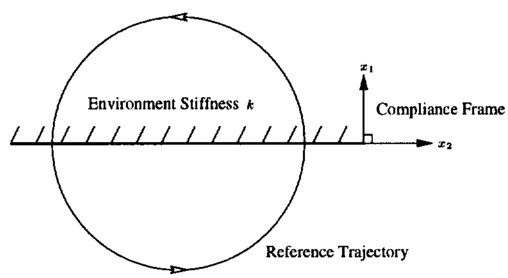

Figure 2: Reference trajectory

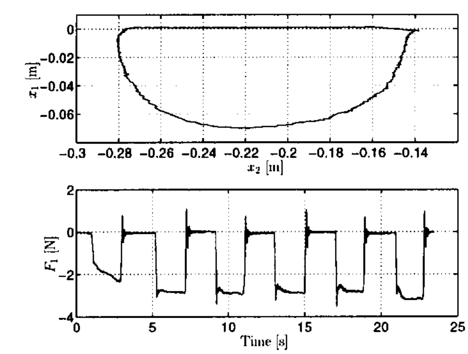

Figure 3: End-effector motion and reaction force

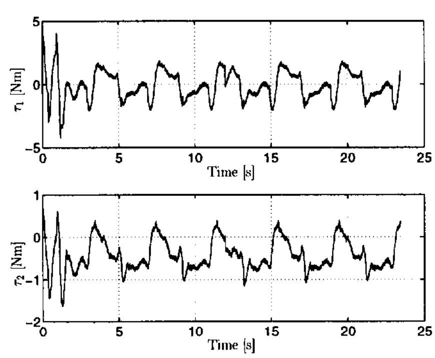

Figure 4: Joint torques