Addendum to "Feedback stability of negative imaginary systems"

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***Abstract*—The statement of [1, Lemma 6] is perfectly correct. The proof is also perfectly fine when the kernel of A is non-trivial. However, the proof in [1, Lemma 6] is silent in the situation when the matrix A only has a trivial kernel. The purpose of this brief addendum is to fill the gap in the original published proof via very minor changes to the originally published proof.**

***Index Terms*—Negative imaginary systems, robust control, feedback stability.**

I. Addendum

The following lemma statement is identical to [1, Lemma 6]. The following proof is nearly identical to the proof of [1, Lemma 6] with only very small changes so that the originally missed situation (i.e. when A only has a trivial kernel) is now *also* captured.

The notation is identical to [1].

Lemma 1: [1, Lemma 6]: Let $M(s)$ be a negative imaginary system and $N(s)$ be a strictly negative imaginary system. Assume $s = j\omega_0$ with $\omega_0 \in (0, \infty)$ is a simple pole of $M(s)$. Then $I - M(s)N(s)$ has no transmission zero at $s = j\omega_0$.

Proof: Since $s = j\omega_0$ with $\omega_0 \in (0, \infty)$ is a simple pole of $M(s)$, $M(s)$ can be factored as $M(s) = M_1(s) + \frac{-jA}{s-j\omega_0}$ where $A = A^* \geq 0$ and $M_1(s)$ is analytic in the neighborhood of $s = j\omega_0$.

Choose a sufficiently small $\delta > 0$.

Since $M(s)$ is a negative imaginary system,

$j[M(j\omega) - M(j\omega)^*] \geq 0 \quad \forall\omega \in \{0 < |\omega - \omega_0| < \delta\}$

$\Leftrightarrow$ $[jM(j\omega)] + [jM(j\omega)]^* \geq 0 \quad \forall\omega \in \{0 < |\omega - \omega_0| < \delta\}$

$\Leftrightarrow$ $[jM_1(j\omega) + \frac{A}{j(\omega-\omega_0)}] + [jM_1(j\omega) + \frac{A}{j(\omega-\omega_0)}]^* \geq 0 \quad \forall\omega \in \{0 < |\omega - \omega_0| < \delta\}$

$\Leftrightarrow$ $[jM_1(j\omega)] + [jM_1(j\omega)]^* \geq 0 \quad \forall\omega \in \{0 < |\omega - \omega_0| < \delta\}$

$\Rightarrow$ $[jM_1(j\omega_0)] + [jM_1(j\omega_0)]^* \geq 0$ (by continuity of real rational functions and because $M_1(s)$ is analytic near $s = j\omega_0$).

Also, since $N(s)$ is a strictly negative imaginary system,

$j[N(j\omega) - N(j\omega)^*] > 0 \quad \forall\omega \in (0, \infty)$

$\Leftrightarrow$ $[jN(j\omega)] + [jN(j\omega)]^* > 0 \quad \forall\omega \in (0, \infty)$

$\Leftrightarrow$ $[jN(j\omega)]^{-1} + [jN(j\omega)]^{-*} > 0 \quad \forall\omega \in (0, \infty)$ (since $N(j\omega)$ has to be nonsingular $\forall\omega \in (0, \infty)$ to satisfy $[jN(j\omega)] + [jN(j\omega)]^* > 0 \; \forall\omega \in (0, \infty)$)

$\Rightarrow$ $[jN(j\omega_0)]^{-1} + [jN(j\omega_0)]^{-*} > 0$.

From the above two conditions, we now have

$$[[jN(j\omega_0)]^{-1} + [jM_1(j\omega_0)]] + [[jN(j\omega_0)]^{-1} + [jM_1(j\omega_0)]]^* > 0. \tag{1}$$

However, we need to show that $I - M(s)N(s)$ has no transmission zero at $s = j\omega_0$, which is equivalent to $[jN(s)]^{-1} + [jM(s)]$ has no transmission zero at $s = j\omega_0$.

We show this via contradiction. Suppose $s = j\omega_0$ with $\omega_0 > 0$ is a transmission zero of $[jN(s)]^{-1} + [jM(s)]$. Then $\exists y \in \mathbb{C}^m$ with $y \neq 0$ such that $[[jN(s)]^{-1} + [jM(s)]]y = 0$ at $s = j\omega_0$. Expanding the above equation, we have that the given y must satisfy $[jN(s)]^{-1}y + [jM_1(s)]y = \frac{-Ay}{s-j\omega_0}$. But in the limit as $s \to j\omega_0$, the left-hand side is finite and the right-hand side is infinite when $y \notin \ker(A)$. Hence $y \in \ker(A)$ (which could be either a trivial kernel or a non-trivial kernel). Thus, $[[jN(s)]^{-1} + [jM(s)]]y = 0$ at $s = j\omega_0$ implies $[jN(j\omega_0)]^{-1}y + [jM_1(j\omega_0)]y = 0$ which in turn implies that $y = 0$ via (1). Hence, by contradiction, $I - M(s)N(s)$ has no transmission zero at $s = j\omega_0$. ∎

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References

[1] A. Lanzon and H. J. Chen, "Feedback stability of negative imaginary systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5620–5633, Nov. 2017.