An Innovative Tri-rotor Drone and Associated Distributed Aerial Drone Swarm Control

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Abstract

This paper presents a novel unmanned aerial vehicle platform based on a three rotor configuration, which can achieve the highest level of manoeuvrability in all 6 dimensions (i.e. 3D position and 3D attitude). The three propellers can be tilted independently to obtain full force and torque vectoring authority, such that this new aerial robotic platform can overcome the limitations of a classic quadrotor UAV that can not change its attitude while hovering at a stationary position. A robust feedback linearization controller is developed to deal with this highly coupled and nonlinear dynamics of the proposed tri-rotor UAV, which linearises the dynamics globally using geometric transformations to produce a linear model that matches the Jacobi linearization of the nonlinear dynamics at the operating point of interest. A distributed formation control tracking protocol is then proposed to control a swarm of tri-rotor UAVs. The 3D position and 3D attitude of each vehicle can be controlled independently to follow a desired time-varying formation. The effectiveness of the designed control strategy is illustrated in a realistic virtual reality simulation environment based on real hardware parameters from a physical construction.

Keywords:
Aerial robotics, distributed systems, formation control, optimal control, multi-agent, unmanned aerial vehicles.

1. Introduction

In recent years, cooperative control of multi-rotor Unmanned Aerial Vehicles (UAVs) have received significant attention from both the practical engineering and academic communities due to their broad prospect in applications [1]. When working together, they are able to perform complex tasks with excellent efficiency and reliability, such as search and rescue [2], crop and weed management in agriculture [3], oil pipeline surveillance [4], etc. Aiming at more efficient configurations in terms of size, autonomy, flight range, payload capacity and other factors, some innovative vehicle platforms are developed by researchers [5]. One of such aerial robotic platforms that holds new and significant properties is the tri-rotor UAV, which is cost effective with more flexibility and agility [6],[7].

The proposed tri-rotor UAV has three rotors arranged in an equilateral triangular configuration and each rotor is attached to a servo motor that can independently change the rotating direction of the propeller. Thus, complete 3D thrust and 3D torque vectoring authority is achieved, which means that the vehicle does not have a nominally upright flying orientation: it can fly in any orientation chosen by the user. Any time-dependent 3D position trajectory can be tracked at the same time as tracking any time-dependent 3D attitude trajectory. This configuration guarantees the UAV a high level of flexibility and maneuverability for attitude control and position movement. Compared to the quadrotor, this innovative configuration also requires less hover power and hence provides longer flight time [8], which makes it ideal for deployment in various missions.

To the best of the authors’ knowledge, no prior literature has studied a tri-rotor UAV configuration with completely independent tilted-rotor capability on all three rotors. The tri-rotor UAV introduced in [9] only has one servo motor that is installed on the arm, which can not hold different attitudes while hovering. A triangular quadrotor is proposed in [8], which contains a single large rotor fixed on the main body. This configuration requires more power to hover and causes uncompensated gyroscopic drift.

In contrast to a quadrotor UAV, which has zero angular momentum in hover, a tri-rotor UAV has persistent angular moment, and hence also gyroscopic dynamics due to the asymmetric configuration of the system which poses significant control systems complexities. Furthermore, independent attitude and trajectory tracking can and should be considered simultaneously. However, the control algorithm in [10] only considers attitude stabilization (as opposed to simultaneous independent attitude and trajectory tracking) and the control design proposed in [11] only focus on the static hovering. In this paper, both these two objectives (i.e. simultaneous independent 3D attitude and 3D trajectory tracking) are considered for the tri-rotor UAV in order to overcome the limitation of quadrotors and thus create more possibilities when performing special tasks through aerial robotic platforms.

Furthermore, swarm robotics is a field of multi-robotics where a group of robots are controlled in a distributed way to perform complex tasks in a more efficient way than using a single robot [12]. As a key control technique in swarm
2. Preliminaries

In this section, notation, definitions and basic concepts on graph theory are introduced.

2.1. Notation and Definitions

Let $I_n \in \mathbb{R}^{n \times n}$ denote the identity matrix of dimension $n$ and $1_n \in \mathbb{R}^n$ be the vector with all entries equal to one. $\text{diag}(a_i)$ represents a diagonal matrix with diagonal entries $a_i$. The Kronecker product is denoted by $\otimes$. We use the superscript $T$ and $^*$ to denote the transpose and complex conjugate transpose of a matrix respectively. For $\lambda \in \mathbb{C}$, $\text{Re}(\lambda)$ is the real part of $\lambda$.

2.2. Graph Theory

Consider a weighted and directed graph $G = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ with a nonempty set of $N$ nodes $\mathcal{V} = \{1, 2, \ldots, N\}$, a set of edges $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, and associated adjacency matrix $\mathcal{A} = \{a_{ij}\} \in \mathbb{R}^{N \times N}$. An edge rooted at node $i$ and ended at node $j$ is denoted by $(i, j)$, which means information can flow from node $i$ to node $j$. $a_{ij}$ is the weight of edge $(i, j)$ and $a_{ij} > 0$ if $(i, j) \in \mathcal{E}$. Assume that there are no repeated edges and no self loops. Node $j$ is called a neighbour of node $i$ if $(i, j) \in \mathcal{E}$. Define the in-degree matrix as $D = \text{diag}(d_i) \in \mathbb{R}^{N \times N}$ with $d_i = \sum_{j=1}^{N} a_{ij}$. The Laplacian matrix $L \in \mathbb{R}^{N \times N}$ of $G$ is defined as $L = D - \mathcal{A}$. A directed graph has or contains a directed spanning tree if there exists a node, called the root, such that there exists a directed path from this node to every other nodes.

**Lemma 1** ([29]). If $G$ contains a spanning tree, then zero is a simple eigenvalue of $L$ with associated right eigenvector $1_N$, and all the other $N - 1$ eigenvalues have nonnegative real parts.

The following assumption of graph topology holds throughout this paper.

**Assumption 1.** The directed graph $G$ contains a spanning tree and the root node $i$ can obtain information from the leader node.

3. Mathematical Modeling

In this section, we dynamically modeling the proposed tri-rotor UAV.

3.1. System Description

The configuration of the tri-rotor UAV is illustrated in Fig. 1, which was first proposed in our earlier work [6]. The UAV has a triangular structure with three arms and a force generating unit plus a revolute joint at the end of each arm. All three arms have identical length $l$. Each force generating unit includes a fixed pitch propeller driven by a brushless DC motor to provide thrust. The motors can be powered by a single battery pack located at the centre of mass or by three separate battery packs located at an equal distance from the centre of mass and each other. The propeller-motor assembly is attached to the body arm via a servo motor that can rotate in a vertical plane to tilt the propeller-motor assembly with an angle $\alpha_{hi}$ (the subscript...
's' denotes servo) in order to produce a horizontal component of the generated force, which is shown in Fig. 2.

All three propellers can be tilted independently to give full thrust vectoring authority. Then the UAV becomes a full six-degrees-of-freedom (6-DOF) vehicle in which all motions can be achieved independently by changing speed of the propellers and tilting angles of the servo motor directly. This configuration enables vehicle attitude (i.e. 3D orientation) and vehicle translation (i.e. 3D movement) to be independently controlled.

In order to develop the dynamic model of the proposed tri-rotor UAV, the following right hand coordinate systems shown in Fig. 3 are considered: \((X_e, Y_e, Z_e)\) represents the earth coordinate system, which is assumed to be inertial (i.e. fixed). \((X_b, Y_b, Z_b)\) denotes the body coordinate system, where the origin \(O_b\) is fixed to the center of mass of the vehicle. This coordinate system moves with the vehicle. \((X_l, Y_l, Z_l)\) with \(i \in \{1, 2, 3\}\) is the local coordinate system of each propeller-motor assembly. The location of the origin of each local coordinate system coincides with the intersection of the UAV arm and the propeller-motor assembly, where \(X_l\) is extended outside the \(i^{th}\) arm of UAV along the same line as the arm and \(Z_l\) is along the direction of the motor shaft axis when the servo angle is zero.

In this section, the superscript \(b, e\) and \(i\) are used to denote the corresponding coordinate system in which vectors are expressed. The subscript \(i\) refers to the \(i^{th}\) propeller, servo motor or brushless DC motor with \(i \in \{1, 2, 3\}\). The nominal mathematical model is based on the following assumptions:

1) Fast actuators are assumed, so the dynamics of actuators are neglected.
2) Propellers are considered to be rigid, thus blade flapping is not considered in the model.
3) The body structure is rigid and the mass is fixed.

It should be noted that although we do not consider these factors in model design, they can still be included as perturbations and uncertainties when carrying out simulation or experiment to test the robustness of the proposed control system in the next section.

In order to obtain the dynamical equations of motion of the vehicle, both forces and torques acting on the UAV need to be analyzed.

### 3.2. Forces Analysis

There are two main forces acting on the tri-rotor, which are the propulsive force and the gravitational force respectively.

The total propulsive force \(F_p\) is equal to the algebraic sum of the three individual propulsive forces produced by each propeller. The individual propulsive forces \(F_{pi}\) at the local coordinate systems are given by:

\[
F_{pi} = \begin{bmatrix}
0 \\
k_f \omega_{p_{mi}}^2 \sin (\alpha_s) \\
k_f \omega_{p_{mi}}^2 \cos (\alpha_s)
\end{bmatrix}, \quad i \in \{1, 2, 3\},
\]

where \(k_f\) is the thrust coefficient of the propeller that can be easily determined from static thrust tests \([30]\), \(\omega_{p_{mi}}\) is the rotational speed of the \(i^{th}\) brushless DC motor and \(\alpha_s\) is the tilting angle of the \(i^{th}\) servo motor.

To obtain the propulsive forces in the body coordinate system, consider the following rotation matrices from propeller
The total propulsive force is then given by

\[ F_p = R_{b_{13}}^b F_{p_{1}}^b + R_{b_{23}}^b F_{p_{2}}^b + R_{b_{3}}^b F_{p_{3}}^b = k_f H_f \rho \]  

(3)

where

\[ H_f = \begin{bmatrix} 0 & -\sqrt{3} & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \]

and \( \rho = \begin{bmatrix} \alpha^2 \sin(\alpha_i) \\ \alpha^2 \sin(\alpha_i) \\ \alpha^2 \sin(\alpha_i) \\ \alpha^2 \cos(\alpha_i) \\ \alpha^2 \cos(\alpha_i) \\ \alpha^2 \cos(\alpha_i) \end{bmatrix} \).

The total drag (or reaction) torque in the body coordinate system can be written as

\[ \tau_d = F_{g}^b \times F_{p}^b = k_f H_f \rho + mg \Theta. \]  

(9)

3.3. Torques Analysis

There are two main torques acting on the tri-rotor, which are the propulsive torque and drag torque.

The propulsive torque is the torque caused by the thrust generated from the propellers near the centre of mass of the UAV. Since there are three identical arms with length \( l \), the propulsive torque for each actuator is given by

\[ \tau_i = l \times F_{p}^b, \quad i \in \{1, 2, 3\}, \]  

(10)

where

\[ t_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad t_2 = \begin{bmatrix} -\sqrt{3} \\ -1 \\ 0 \end{bmatrix}, \quad t_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}. \]

and \( F_{p}^b = R_{b_{p_i}}^b F_{p_i} \).

Then the total propulsive torque with respect to the body coordinate system can be written as:

\[ \tau_p^b = \tau_{p_{1i}}^b + \tau_{p_{2i}}^b + \tau_{p_{3i}}^b = k_f H_f \rho. \]  

(11)

where

\[ H_i = l \begin{bmatrix} 0 & 0 & 0 & \sqrt{3} & -\sqrt{3} \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}. \]

The drag torque is expressed as the torque caused by the aerodynamic drag forces, which is in the opposite direction to the rotation direction of propeller. Thus, the resulting drag torque on the \( i \)-th propeller is given by \( k_d \omega_m^2 \), where \( k_d \) is the drag torque to speed coefficient resulting from the rotation of the propeller.

The drag torque on the propellers causes an equal reaction torque on the vehicle which can be expressed in the local coordinate systems as

\[ \tau_d^i = \begin{bmatrix} 0 \\ -k_d \omega_m^2 \sin(\alpha_i) \\ -k_d \omega_m^2 \cos(\alpha_i) \end{bmatrix}, \quad i \in \{1, 2, 3\}. \]  

(12)

The total drag (or reaction) torque in the body coordinate system can be written as

\[ \tau_d = \tau_{d_{1i}} + \tau_{d_{2i}} + \tau_{d_{3i}} = R_{b_{13}}^b \tau_d^1 + R_{b_{23}}^b \tau_d^2 + R_{b_{3}}^b \tau_d^3. \]  

(13)
Finally, the total torque acting on the tri-rotor UAV in the body coordinate system can be written as
\[ \tau^b = I^b_p \omega^b + (k_f H_l - k_d H_f)p. \]  

### 3.4. Dynamic Model

Under the assumption stated earlier that the tri-rotor UAV is a rigid body of fixed mass, the vehicle’s translational and rotational dynamics can be calculated by the Newton-Euler’s second law of motion [31] in the body coordinate frame as
\[ F^b = m(\dot{v}_b + S(\omega_b)v_b), \]  
\[ \tau^b = I^b_p \omega^b + S(\omega_b)I^b_p \omega^b, \]
where \( v_b \) is the vehicle’s translational velocity measured in the body coordinate frame and \( \omega^b \) is the vehicle’s angular velocity. The vehicle’s inertia matrix expressed in the body coordinate frame and the skew matrix constructed from the vector \( \omega^b = [p\ q\ r]^T \) are given by
\[ I^b_p = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & -I_{yz} \\ 0 & I_{yz} & I_{zz} \end{bmatrix}, \]
and
\[ S(\omega^b) = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix}. \]

Now, substituting \( F^b \) and \( \tau^b \) from (9) and (14) gives
\[ k_f H_l p + mg\Theta = m(\dot{v}_b + S(\omega_b)v_b), \]
\[ (k_f H_l - k_d H_f)p = I^b_p \omega^b + S(\omega_b)I^b_p \omega^b. \]

Let \( \eta = [\phi \ \theta \ \psi]^T \) and \( \xi = [x\ y\ z]^T \) denote respectively the attitude vector and the position vector with respect to the earth coordinate system. Then
\[ \eta^e = \Psi \omega^b, \]
\[ \xi^e = R^e_p v^b, \]
describe the relations between velocities and positions [32], where \( \Psi \) relates the instantaneous angular velocities around the \( X_b \)-axis, \( Y_b \)-axis and \( Z_b \)-axis to the rate of change of the roll, pitch and yaw angles. It is given in [33] as
\[ \Psi = \begin{bmatrix} 1 & \sin(\phi)\tan(\theta) & \cos(\phi)\tan(\theta) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \sin(\phi)\sec(\theta) & \cos(\phi)\sec(\theta) \end{bmatrix}, -\frac{\pi}{2} < \theta < \frac{\pi}{2}. \]

Therefore, the dynamic model of the tri-rotor can be described in a compact form as
\[ \dot{v}_b = g\Theta - S(\omega_b)v_b + \frac{k_f}{m} H_l p, \]
\[ \dot{\omega}_b = -(I^b_p)^{-1} S(\omega_b)I^b_p \omega^b + (I^b_p)^{-1} (k_f H_l - k_d H_f)p. \]

Choosing the state vector as
\[ x = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6 \ x_7 \ x_8 \ x_9 \ x_{10} \ x_{11} \ x_{12}]^T \]
\[ = [u_0 \ v_b \ w_b \ p \ r \ \phi \ \theta \ \psi \ x \ y \ z]^T, \]
and the input vector as
\[ u = \rho = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}, \]
then the set of (24)-(27) can be written in the state-space form as
\[ \dot{x}_1 = x_2 x_6 - x_3 x_5 + g \sin(x_8) - \frac{\sqrt{3}k_f}{2m} u_2 + \frac{\sqrt{3}k_f}{2m} u_3, \]
\[ \dot{x}_2 = x_3 x_4 - x_1 x_6 - g \sin(x_7) \cos(x_8), \]
\[ \dot{x}_3 = x_1 x_5 - x_2 x_4 - g \cos(x_7) \sin(x_8), \]
\[ \dot{x}_4 = \frac{x_5 x_6 (I_{xx} - I_{zz}) + I_{yz} (x_3^2 - x_2^2)}{2I_{xx}} + \frac{\sqrt{3}k_f}{2I_{xx}} u_2, \]
\[ \dot{x}_5 = \frac{x_4 x_6 (I_{yy} - I_{zz}) - I_{xx} I_{zz} - I_{yy} I_{xx}}{I_{yy} I_{zz} - I_{xx} I_{yy}} + \frac{\sqrt{3}k_f}{2I_{xx}} u_3, \]
\[ \dot{x}_6 = \frac{x_4 x_6 (I_{yy} - I_{zz}) + I_{xx} I_{zz} - I_{yy} I_{xx}}{I_{yy} I_{zz} - I_{xx} I_{yy}} + \frac{\sqrt{3}k_f}{2I_{xx}} u_2, \]
\[ \dot{x}_7 = \frac{2I_{xx} k_f l + I_{zz} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} + \frac{2I_{yy} k_f l + I_{xx} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} u_3, \]
\[ \dot{x}_8 = \frac{I_{xx} k_f l + I_{zz} k_d}{I_{yy} I_{zz} - I_{xx} I_{yy}} + \frac{2I_{yy} k_f l + I_{xx} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} u_3, \]
\[ \dot{x}_9 = \frac{I_{xx} k_f l - 2I_{zz} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} + \frac{I_{yy} k_f l - 2I_{xx} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} u_3, \]
\[ \dot{x}_{10} = \frac{I_{xx} k_f l - 2I_{zz} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} + \frac{I_{yy} k_f l - 2I_{xx} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} u_2, \]
\[ \dot{x}_{11} = \frac{2I_{yy} k_f l - I_{xx} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} + \frac{2I_{xx} k_f l - I_{zz} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} u_2, \]
\[ \dot{x}_{12} = \frac{2I_{yy} k_f l - I_{xx} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} + \frac{2I_{xx} k_f l - I_{zz} k_d}{2(I_{yy} I_{zz} - I_{xx} I_{yy})} u_2. \]
ing problem for tri-rotor robotic swarms. The output vector is chosen as

\[
\begin{bmatrix}
\phi \\
\theta \\
\psi \\
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10} \\
x_{11} \\
x_{12}
\end{bmatrix}
= \begin{bmatrix}
x_7 \\
x_8 \\
x_9 \\
x_{10} \\
x_{11} \\
x_{12}
\end{bmatrix},
\]

(42)

4. Control System Design

The objective of this section is to design a robust distributed formation control protocol for swarms of the proposed tri-rotor UAV. Since the dynamical model of a single tri-rotor UAV is highly coupled and nonlinear, a robust feedback linearization technique is first applied to each tri-rotor to obtain simpler closed-loop dynamics. Then the swarm of identical tri-rotor UAVs is controlled through an optimal distributed formation control protocol which solves the time-varying formation tracking problem for tri-rotor robotic swarms.

Remark 1. Note that the real inputs \(\omega_m\) and \(\alpha_i\) are mapped into the control inputs \(u_i\) via the nonlinear mapping (29). It can be shown that this nonlinear mapping is invertible thus giving actuator signals \(\omega_m\) and \(\alpha_i\) for use in real application. The physical actuator inputs \(\omega_m\) and \(\alpha_i\) can be calculated back from the control inputs \(u_i\) via

\[
\alpha_i = \arctan \left( \frac{u_i}{u_{i+3}} \right) \quad \text{and} \quad \omega_m = \sqrt{u_i^2 + u_{i+3}^2}, \quad \forall i \in \{1, 2, 3\}.
\]

4.1. Robust Feedback Linearization

Consider a single nonlinear system with \(n\) states, \(m\) inputs, and \(m\) outputs described by

\[
\dot{x} = F(x) + G(x)u = F(x) + \sum_{i=1}^{m} G_i(x)u_i,
\]

(43)

\[
y = [H_1(x), \ldots, H_m(x)]^T,
\]

(44)

where \(x(t) \in \mathbb{R}^n\) denotes the state vector, \(u(t) \in \mathbb{R}^m\) is the control input, \(y(t) \in \mathbb{R}^m\) is the output vector, and \(F(x), G_1(x), \ldots, G_m(x), y\) are smooth vector fields defined on an open subset of \(\mathbb{R}^n\).

Suppose that this system satisfies the well-known conditions for feedback linearization [34]: The relative degree of \(H_i\) is equal to \(r_i\) for \(i \in \{1, \ldots, m\}\) such that \(r_1 + \cdots + r_m = n\), and the decoupling matrix

\[
M(x) = \begin{bmatrix}
L_{G_1}L_{F_1}^{-1}H_1(x) & \cdots & L_{G_m}L_{F_m}^{-1}H_1(x) \\
\vdots & \ddots & \vdots \\
L_{G_1}L_{F_m}^{-1}H_m(x) & \cdots & L_{G_m}L_{F_m}^{-1}H_m(x)
\end{bmatrix},
\]

(45)

is invertible, where \(L_i(.)\) denotes the Lie derivative operator [34]. It is then possible to find a feedback linearizing control law of the form

\[
u(x, w) = \alpha_c(x) + \beta_c(x)w,
\]

(46)

where \(w(t)\) is a new control input, and \(\alpha_c(x) = -M^{-1}(x)\left[L_{G_1}L_{F_1}^{-1}H_1(x) \cdots L_{G_m}L_{F_m}^{-1}H_m(x)\right]^T, \beta_c(x) = M^{-1}(x)\), such that on application of the control law in (46), the nonlinear state-equation (43) reduces into the linear state-equation

\[
\dot{x}_c = A_c x_c + B_c w,
\]

(47)

where \(A_c\) and \(B_c\) are matrices of the Brunovsky canonical form [34], and a change of coordinates \(\phi_c(x) = \phi_{c1}(x) \cdots \phi_{cm}(x)\) and \(\phi_{c1}(x) = [H_1(x) L_{F_1}(x) \cdots L_{F_m}(x)]^T\).

The robust feedback linearization technique [26], on the other hand, exactly transforms the nonlinear state-equation into a linear state-equation that is equal to the Jacobi linear approximation of the original nonlinear state-equation around the origin. This can then be controlled using linear techniques [27]. In the robust feedback linearization case, the linearized state-equation becomes

\[
\dot{x}_r = A_r x_r + B_r v,
\]

(48)

where \(A_r = \partial_x F(0)\) and \(B_r = G(0)\). The nonlinear state-equation (43) is geometrically transformed into the linear state-equation of any operating point, not only in a small neighborhood of the origin point. [26] argues that classical feedback linearization may be non robust in the presence of uncertainties as any system is transformed into a chain of integrators (i.e. Brunovsky form) whereas robust feedback linearization preserves some system information.
The robust feedback linearization control law is

$$ u(x,v) = \alpha(x) + \beta(x)v, \quad \text{(49)} $$

where

$$ \alpha(x) = \alpha_r(x) + \beta_r(x)LU^{-1}\phi_r(x), \quad \text{(50)} $$

$$ \beta(x) = \beta_r(x)R^{-1}, \quad \text{(51)} $$

$$ \phi_r(x) = U^{-1}\phi_r(0), \quad \text{(52)} $$

$$ L = -M(0)\partial_x\alpha_r(0), \quad \text{(53)} $$

$$ R = M^{-1}(0), \quad \text{(54)} $$

$$ U = \partial_x\phi_r(0). \quad \text{(55)} $$

Now we apply the robust feedback linearization to the dynamics of tri-rotor UAV system. The relative degrees are $r_1 = 2$, $r_2 = 2$, $r_3 = 2$, $r_4 = 3$, and $r_5 = 2$, resulting in a vector relative degree $r = 12$, which is equal to the number of states. The decoupling matrix $M(x)$ can also be written in a compact form as given in [6]:

$$ M(x) = \left[ \Psi(I_p^0)^{-1}(k_H - k_dH_f) \right]. \quad \text{(57)} $$

It can be verified that $\text{det}(M(x)) \neq 0$ as the pitch angle is assumed to be in the range of $-\pi/2 < \theta < \pi/2$, such that $M(x)$ is always invertible in this case. As a result, the conditions for feedback linearization are satisfied.

After calculating the classic Brunowksi form linearizing input (46) and applying the formulas for the robust feedback linearization (49)-(56), the system can then be robust feedback linearized into

$$ \dot{x}_r = A_r x_r + B_r v, \quad \text{(58)} $$

$$ y = C_r x_r. \quad \text{(59)} $$

The state space matrix $A_r$, $B_r$, and $C_r$ are shown at the bottom of the page.

Furthermore, $L$, $R$ and $U$ are calculated by

$$ L = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad \text{(63)} $$

$$ R = \begin{bmatrix} 0 & -\frac{\lambda_1}{m} & \frac{\lambda_1}{m} & 0 & \frac{m}{m} & \frac{m}{m} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\lambda_2}{m} & \frac{\lambda_2}{m} & -\frac{\lambda_3}{m} & \frac{\lambda_3}{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\lambda_2}{m} & \frac{\lambda_2}{m} & -\frac{\lambda_3}{m} & \frac{\lambda_3}{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\lambda_4}{m} & \frac{\lambda_4}{m} & -\frac{\lambda_5}{m} & \frac{\lambda_5}{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{\lambda_4}{m} & \frac{\lambda_4}{m} & -\frac{\lambda_5}{m} & \frac{\lambda_5}{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -\frac{\lambda_4}{m} & -\frac{\lambda_5}{m} & -\frac{\lambda_5}{m} & -\frac{\lambda_6}{m} & -\frac{\lambda_6}{m} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \text{(64)} $$

$$ U = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}. \quad \text{(65)} $$

and $\phi_r$ and $\phi_r$, are given by

$$ \phi_r = [x_7 \ x_9 \ x_{10} \ x_{11} \ x_{12} \ x_7 \ x_8 \ x_9 \ x_{10} \ x_{11} \ x_{12}]^T, \quad \text{(66)} $$

$$ \phi_r = [x_{10} \ x_{11} \ x_7 \ x_8 \ x_9 \ x_7 \ x_8 \ x_9 \ x_{10} \ x_{11} \ x_{12}]^T. \quad \text{(67)} $$

From (56) we know $x_r = \phi_r(x)$. Finally, $\alpha(x)$ and $\beta(x)$ can then be computed from (50) and (51) directly.

### 4.2. Distributed Optimal Formation Protocol Design

In practical applications, some states do not need to be measured by sensors for controller design. For example, the vehicle’s translational velocity in the body coordinate frame $v_i^b$ is not used in the robust feedback linearization controller. It can hence be obtained by using an observer on input and output information of the feedback linearized system. In this section, we propose a distributed optimal formation protocol which uses the nearhood state estimation information for controller design and the local output estimation error information for the observer design. The scheme for controlling the dynamics of attitude and position of each tri-rotor UAV, based on robust feedback linearization and distributed optimal output feedback formation protocol, is illustrated in Fig. 4.

Consider a set of $N$ tri-rotor UAVs. Suppose that each tri-rotor UAV has the identical linearized dynamics described by

$$ \dot{x}_i = A_r x_i + B_r v_i, \quad \text{(68)} $$

$$ y_i = C_r x_i. \quad \text{(69)} $$

It can be easily verified that $(A_r, B_r, C_r)$ is stabilizable and detectable.

The dynamics of the leader node, labeled 0, is given by

$$ x_0 = A_r x_0, \quad \text{(70)} $$

$$ y_0 = C_r x_0. \quad \text{(71)} $$

where $x_0 \in \mathbb{R}^n$ is the state, $y_0 \in \mathbb{R}^p$ is the output. It can be considered as a command generator, which generates the desired target trajectory. The leader can be observed from a subset of agents in a graph. If node $i$ observes the leader, an edge $(0, i)$ is said to exist with weighting gain $g_i > 0$ as a pinned node. We denote the pinning matrix as $G = \text{diag}(g_i) \in \mathbb{R}^{N \times N}$.

The desired formation is specified by the vector $h = [h_1^T, h_2^T, \ldots, h_N^T]^T$ with $h_i \in \mathbb{R}^p$ being a preset vector known by the corresponding $i$th agent. It should be noted that the formation problem reduces to a consensus problem when $h_i = 0 \ \forall \ i \in \{1, \ldots, N\}$.
Denote the estimate of the state \( x_i \) by \( \hat{x}_i \in \mathbb{R}^n \) and let the state estimation error be \( \hat{x}_i = x_i - \hat{x}_i \). Then the consequent estimate of the output \( y_i \) is given by \( \hat{y}_i = C_{ij} \hat{x}_j \) and the output estimation error for node \( i \) is given by \( \hat{y}_i = y_i - \hat{y}_i \). Consider the following distributed optimal formation protocol

\[
v_i = cK \sum_{j \in \mathcal{N}_i} a_{ij} \left( \hat{x}_j - h_i \right) - \left( \hat{x}_i - h_i \right) + cK_{ij} \left( x_0 - (\hat{x}_i - h_i) \right) + \gamma_i, \tag{72}
\]

\[
\dot{x}_i = A_{ij} \hat{x}_i + B_{ij} v_i - cF \gamma_i, \tag{73}
\]

where \( c > 0 \) is the scalar coupling gain, \( K \in \mathbb{R}^{m \times m} \) is the feedback control gain matrix, \( F \in \mathbb{R}^{m \times m} \) is the observer gain, and \( \gamma_i \in \mathbb{R}^m \) represents the formation compensation signal to be designed.

\[
A_r = \begin{bmatrix}
0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0
0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots
0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0
0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0
0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0
0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0
0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0
\end{bmatrix}
\tag{60}
\]

\[
B_r = \begin{bmatrix}
0 & \gamma_1 & \ldots & \gamma_n & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0
0 & 0 & \ldots & 0 & 0 & \gamma_1 & 0 & \ldots & 0 & 0 & 0 & 0
0 & 0 & \ldots & 0 & 0 & 0 & \gamma_1 & \ldots & 0 & 0 & 0 & 0
\ \vdots & \ \vdots & \ \ddots & \ \vdots & \ \vdots & \ \vdots & \ \vdots & \ \ddots & \ \vdots & \ \vdots & \ \vdots & \ \vdots
0 & 0 & \ldots & 0 & 0 & \gamma_1 & \ldots & \gamma_n & 0 & 0 & 0 & 0
0 & 0 & \ldots & 0 & 0 & 0 & \gamma_1 & \ldots & \gamma_n & 0 & 0 & 0
0 & 0 & \ldots & 0 & 0 & 0 & 0 & \ldots & \gamma_1 & \ldots & \gamma_n & 0
0 & 0 & \ldots & 0 & 0 & 0 & \ldots & \gamma_1 & \ldots & \gamma_n & 0 & 0
\end{bmatrix}
\tag{61}
\]

\[
C_r = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\ \vdots & \ \vdots & \ \vdots & \ \vdots & \ \vdots & \ \vdots & \ \vdots & \ \vdots & \ \vdots & \ \vdots & \ \vdots & \ \vdots
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\tag{62}
\]

Let \( x = [x_{i1}^T, x_{i2}^T, \ldots, x_{iN}^T]^T \) \( \dot{x} = [\dot{x}_{i1}^T, \dot{x}_{i2}^T, \ldots, \dot{x}_{iN}^T]^T \) \( \ddot{x} = [\ddot{x}_{i1}^T, \ddot{x}_{i2}^T, \ldots, \ddot{x}_{iN}^T]^T \). Under a control protocol with directed topology, the tri-rotor UAV swarm can be written in a compact form as

\[
\dot{x} = (I_N \otimes A_r)x - c[(L + G) \otimes B_r \hat{K}](\hat{x} - \hat{x}_0) + c[(L + G) \otimes B_r \hat{K}]h + (I_N \otimes B_r) \gamma, \tag{74}
\]

\[
\dot{\hat{x}} = (I_N \otimes A_r)\hat{x} - (I_N \otimes cFC_r)(x - \hat{x}) - c[(L + G) \otimes B_r \hat{K}](\hat{x} - \hat{x}_0) + c[(L + G) \otimes B_r \hat{K}]h + (I_N \otimes B_r) \gamma. \tag{75}
\]

It follows the fact that matrix \( B_r \) given in (61) is of full rank, there always exists a nonsingular matrix \( \tilde{B}_r^T \tilde{B}_r \) with \( \tilde{B} \in \]
Let $\lambda_i(i \in \{1, \ldots, N\})$ be the eigenvalues of $(L + G)$. Then the tri-rotor UAV swarm with directed interaction topology asymptotically converges to the formation specified by $(x_0 + h_i) \in \mathbb{R}^n$ for all $i \in \{1, \ldots, N\}$ if the following conditions hold for all $i \in \{1, \ldots, N\}$.

$$\begin{align*}
BA_ih_i - Bh_i &= 0, \\
A_i - c\lambda_iB_iK \text{ and } A_i + cFC_i \text{ are Hurwitz,}
\end{align*}$$

and $\gamma_i = B\Phi_i - BA_ih_i$ for all $i \in \{1, \ldots, N\}$. \hfill (81)

Proof. Let formation tracking error for each UAV be $\Phi_i = x_{ui} - h_i - x_0$ and $\Phi = [\Phi_1', \Phi_2', \ldots, \Phi_N']'$. Then the global formation error dynamics with directed interaction topology can be written as

$$\begin{align*}
\dot{\Phi} &= [I_N \otimes A_i - c(L + G) \otimes B_iK] \Phi \\
&\quad + c(L + G) \otimes B_iK \check{x} \\
&\quad + (I_N \otimes \dot{A}_i)h - (I_N \otimes I_N)\dot{h} \\
&\quad + (I_N \otimes B_i)\gamma.
\end{align*}$$

The global observer error dynamics is

$$\dot{x} = I_N \otimes (A_i + cFC_i)\check{x}. \hfill (80)$$

In view of Assumption 1, all the eigenvalues of matrix $(L + G)$ have positive real parts [36]. It is well known that there exists a nonsingular $T$ such that $T^{-1}(L + G)T$ is in the Jordan canonical form $J$. Let $\theta = (T^{-1} \otimes I_N)[\Phi] = [\theta_1', \theta_2', \ldots, \theta_N']'$. Then multi-agent system can be represented in terms of $\theta$ as

$$\begin{align*}
\dot{\theta} &= (I_N \otimes A_i - cJ \otimes B_iK)\theta \\
&\quad + c[T^{-1}(L + G) \otimes B_iK] \check{x} \\
&\quad + (T^{-1} \otimes A_i)h - (T^{-1} \otimes I_N)\dot{h} \\
&\quad + (T^{-1} \otimes B_i)\gamma.
\end{align*}$$

If condition (76) holds, then for all $i \in \{1, \ldots, N\}$

$$\begin{align*}
BA_i h_i - Bh_i &= 0, \\
A_i - c\lambda_iB_iK \text{ and } A_i + cFC_i \text{ are Hurwitz,}
\end{align*}$$

By letting $\gamma_i = B\Phi_i - BA_ih_i$, it follows that

$$\begin{align*}
\dot{BA}_i h_i - Bh_i + BB_i \gamma_i &= 0. \\
\end{align*}$$

From (82) and (83) and the fact that $[\check{B}^T, \check{B}^T]^T$ is nonsingular, one gets

$$A_i h_i - \dot{h}_i + B_i \gamma_i = 0. \hfill (84)$$

which means that

$$(IN \otimes A_i)h - (I_N \otimes I_N)\dot{h} + (I_N \otimes B_i)\gamma = 0. \hfill (85)$$

Pre-multiplying the both sides of (85) by $T^{-1} \otimes I_N$ yields

$$(T^{-1} \otimes A_i)h - (T^{-1} \otimes I_N)\dot{h} + (T^{-1} \otimes B_i)\gamma = 0. \hfill (86)$$

Then (81) reduces to the following dynamics

$$\dot{\theta} = (I_N \otimes A_i - cJ \otimes B_iK)\theta + c(T^{-1}(L + G) \otimes B_iK)\check{x}. \hfill (87)$$

From (87) and (80), it can be obtained that

$$\begin{align*}
\begin{bmatrix}
\dot{\theta} \\
\dot{x}
\end{bmatrix} &= \begin{bmatrix}
A_i & B_e \\
0 & I_N \otimes (A_i + cFC_i)
\end{bmatrix} \begin{bmatrix}
\theta \\
x
\end{bmatrix},
\end{align*}$$

where

$$A_e = I_N \otimes A_i - cJ \otimes B_iK,$$

$$B_e = cT^{-1}(L + G) \otimes B_iK.$$

Therefore, the global error system in (88) is asymptotically stable if and only if both $A_e$ and $I_N \otimes (A_i + cFC_i)$ are Hurwitz, and the latter can be satisfied due to the detectability of $(C, A_i)$. Note the the state matrix $A_e$ is either block diagonal or block upper-triangular. Hence the stability of (88) is equivalent to the stability of the $N$ subsystems defined with the diagonal blocks. Therefore, $A_e - c\lambda_iB_iK$ is Hurwitz $\forall i \in \{1, \ldots, N\}$ if and only if $I_N \otimes A_i - cJ \otimes B_iK$ is Hurwitz. Therefore, $\dot{\theta}$ converges asymptotically to the origin which is equivalent to stating that $x_{ui}$ converges asymptotically to $x_0 + h_i$ for all $i \in \{1, \ldots, N\}$. \hfill \square

Next we will show how to select state variable feedback control gain $K$ to guarantee stability on arbitrary directed graphs containing a spanning tree by using LQR based optimal design and proper choice of the coupling gain $c$. The following theorem is an extension of a result in [28], which only considers the consensus problem. In the case where $h = 0$, the optimal formation tracking protocol (72) becomes the optimal consensus tracking protocol of [28], so it can be viewed as a special case of the result in the current paper.

Theorem 2. Let $Q = Q^T \in \mathbb{R}^{n \times n}$ and $R = R^T \in \mathbb{R}^{m \times m}$ be positive definite matrices. Let $P$ be the unique positive definite solution of the algebraic Riccati equation

$$A_e^T P + PA_e + Q - PB_e R^{-1} B_e^T P = 0. \hfill (89)$$

Then, under Assumption 1 and condition (76), the distributed formation tracking control protocol (72) with

$$K = R^{-1} B_e^T P \hfill (90)$$

and $\gamma_i$ set as in (81) ensures that the tri-rotor UAV swarm with directed interaction topology asymptotically converges to the formation specified by $(x_0 + h_i) \in \mathbb{R}^n$ for all $i \in \{1, \ldots, N\}$ if the coupling gain

$$c \geq \frac{1}{2\lambda_i} \hfill (91)$$

with $\lambda_i = \min_{\{1, \ldots, n\}} \text{Re}(\lambda_i)$, where $\lambda_i$ are the eigenvalues of $(L + G)$.
Proof. Consider the stability of the following subsystem
\[ \dot{\delta}_i = (A_r - cA_r B_r K) \delta_i, \]  
where \( \delta \) denotes the formation tracking closed-loop error. Construct the following Lyapunov candidate function
\[ V_i = \delta_i^T P \delta_i. \]
Taking the derivative of \( V_i \) along the trajectory of subsystem gives
\[ \dot{V}_i = \delta_i^T (PA_r + A_r^T P - cA_r^T (B_r B_r^T)P - cA_r P B_r K) \delta_i. \]
Substituting \( K = R^{-1} B_r^T P \) and \( A_r^T P + PA_r = -Q + PB_r R^{-1} B_r^T P \) into (92) one has
\[ \dot{V}_i = [1 - 2c \text{Re}(\lambda_i)] \delta_i^T (PB_r R^{-1} B_r^T P) \delta_i - \delta_i^T Q \delta_i. \]
It can be seen that if condition (91) holds, then \( \dot{V}_i < 0 \). Therefore, \( A_r - cA_r B_r K \) is Hurwitz for all \( i \in \{1, \ldots, N\} \) by Lyapunov theory [37]. This completes the proof.

The ARE in (89) is extracted by minimizing the following performance index for each tri-rotor UAV
\[ J_i = \frac{1}{2} \int_0^\infty \left( \delta_i^T Q \delta_i + v_i^T R v_i \right) dt. \]
The design Riccati matrices \( Q \) and \( R \) can be selected to adjust the relative cost of formation tracking error and control effort. This allows the cooperative control system to be tuned to trade-off between the speed of formation tracking and the speed of DC motors to achieve it.

Remark 2. In order to enhance the robustness of our tri-rotor UAV, suppose external white noises \( \epsilon_1 \) and \( \epsilon_2 \) are added to (68) and (69) respectively, which satisfy that
\[ E[\epsilon_1 \epsilon_1^T] = \tilde{Q}, E[\epsilon_2 \epsilon_2^T] = \tilde{R}, E[\epsilon_1 \epsilon_2^T] = 0, \]
where \( E \) donates the expected value, and \( \tilde{Q} \) and \( \tilde{R} \) are positive definite matrices. Then a local optimal observer gain \( F \) can be calculated by a similar approach (see [38] for further details) as
\[ F = PC_r^T R^{-1}, \]
where \( P \) is the unique positive definite solution of ARE
\[ A_r P + PA_r^T - PC_r^T R^{-1} C_r P + \tilde{Q} = 0. \]
This optimal observer is also known as Kalman-Bucy filter [39], which has been widely used in system state estimation. It has been demonstrated [40] to have many advantages, including optimality of state estimation in the presence of white noise and external disturbance [41].

With the above analysis, the procedure to construct the control law \( u_i \) is given in Algorithm 1.

Validation of internal stability using closed-loop data from experiments can be performed using technique described in [42]. This is useful as one would also expect unmodelled dynamics.

Algorithm 1 Procedure for construction of the control law of a tri-rotor UAV robotic swarm

1. initialize state variables for a tri-rotor UAV robotic swarm;
2. for each vehicle \( i \in \{1, \ldots, N\} \) do
3. select the desired formation reference \( h_i \in \mathbb{R}^n \);
4. if formation feasibility condition (76) is satisfied then
5. compute distributed feedback gain \( K \) using (90) and (89);
6. select coupling gain \( c \) according to condition (91);
7. compute local optimal observer gain \( F \) using (98) and (99);
8. set \( \gamma_i \in \mathbb{R}^n \) according to (78);
9. set the distributed optimal formation control protocol \( v_i \) as in (72) and (73);
10. set the robust feedback control law \( u_i \) as in (49);
11. else
12. back to Step 3;
13. end if
14. end for

5. Simulation Results

The numerical simulation carried out in this section is based on real hardware as shown in Fig. 5. The electric propulsion unit of the tri-rotor UAV includes energy storage units (battery packs), electronic speed control units (ESC), electric motors (brushless DC motors), and propellers. Also, an embedded system is installed on the main body, which includes an on-board microcontroller (OBM), a data acquisition module (DAQ) and a sensor module (IMU). The measured model parameters of the tri-rotor UAV are given in Table 1.

The simulation environment has been designed and implemented in Simscape Multibody™ and Simulink® for more realistic results as this provides a 3D graphical display of physical devices. Simscape Multibody™ is used to develop the dynamic model of the tri-rotor UAV based on physical components such as joints, constraints, force elements, and sensors. The designed control system is implemented in Simulink®. Furthermore, a time delay of 0.01s in servo motor responses and a maximum speed saturation constraint of 12000 RPM on the electric motors are considered in the simulation model to mimic real physical considerations.
valid regardless of the target is static or time-varying. In this case, $h_j$ is selected as

$$h_j = \begin{bmatrix} 10 \cos \left( t + \frac{i \pi}{20} \right) \\ -10 \sin \left( t + \frac{i \pi}{20} \right) \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (\text{for } i \in \{1, 2, \ldots, 6\}),$$

where the desired offsets of 3D attitude and 3D position with respect to the reference signal for each agent are represented by last six rows, and the first six rows are the derivatives of them according to the change of coordinates given in (67).

It can be verified that the formation tracking feasibility condition (76) in Theorem 1 is satisfied. Then the optimal state-feedback gain $K$ and coupling gain $c$ can be obtained using the approach in Theorem 2. The local optimal observer gain $F$ for each UAV can also be selected easily by solving the corresponding ARE based on the estimation of noise. These design gains are hence given by

$$c = 5,$$

$$K = \begin{bmatrix} -2.1 & 0.3 & -12 & -0.1 & -2.3 & 3.2 & -0.9 & -9.4 & 18 & -2.2 & 0.5 & 2.6 \\ 2.1 & -2.9 & 12 & 2.6 & 12 & 3.1 & 11 & 7.6 & 17 & 2.8 & -3.5 & 2.6 \\ 2.6 & 2.5 & 12 & -2.4 & 13.3 & 3.1 & -11.9 & 8.9 & 17 & 3.7 & 2.9 & 2.6 \\ -20 & 0 & 85 & -0.3 & 17 & 0.1 & -81 & -0.5 & -25 & 0 & 18 \\ 10 & -18 & 85 & 17 & -8.4 & -0.8 & 77 & 40 & -3.6 & 12 & -22 & 18 \\ 10 & 18 & 85 & -17 & 8.4 & -0.8 & 77 & 40 & -3.6 & 12 & 22 & 18 \end{bmatrix},$$

$$F = \begin{bmatrix} -0.12 & -0.01 & 10.08 & -0.01 & 9.83 & 0 & 0 \\ -10.14 & -0.12 & 0 & 9.83 & 0 & 0 & 0 \\ -0 & -0.18 & 0 & 0 & 1.25 & 0 & 0 \\ 160.26 & 0 & -0.12 & 5.57 & 0 & 0 & 0 \\ 0 & 208.61 & 50.72 & -6.01 & -0.12 & 0 & 0 \\ 0 & 50.72 & 916.32 & -0.13 & 0 & 0 & 0 \\ 17.9 & 0 & 0 & -0.03 & -0.20 & 0 & 0 \\ 0 & 1.60 & 42.77 & -0.01 & 0 & 0 & 0 \\ 0 & -0.01 & 1.60 & 4.43 & 0 & 0 & 0 \\ -0.20 & -0.01 & 0 & 0 & 4.43 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.5864 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

On using robust control law (49) with optimal distributed formation control protocol (72) and (73), the trajectory of each tri-rotor UAV is given by Fig. 7. The 3D visualisation of distributed formation of the tri-rotor UAV swarm are illustrated in Fig. 8. The attitude tracking performance with respect to roll,
pitch and yaw angles is shown in Fig. 9. The position tracking of hovering is shown in Fig. 10. From all these figures, it can be seen that the tri-rotor swarm forms a regular hexagonal formation with circular time-variation in the x-y plane after 25s and the surveilled target lies in the middle of the circular rotation at ground level. The attitude of each tri-rotor UAV varies with time along the circular trajectory so that each tri-rotor points (e.g. its onboard camera) to the target located at the centre of the circle. It is concluded that the desired formation and attitude tracking of the UAV swarm is achieved independently, and the designed control system preserves good robustness properties when subjected to simulated aerodynamic disturbances and model uncertainties.

6. Conclusion

In this paper, we have proposed a new tri-rotor unmanned aerial vehicle which is more efficient and flexible than a quadrotor UAV. A formation tracking problem of a networked tri-rotor UAV swarm has also been solved using a distributed formation control protocol.

To achieve this, the dynamical model was first derived based on force and torque kinematic analysis and subsequent translational and rotational dynamic modelling. A robust feedback linearization controller was then developed to deal with this highly coupled and nonlinear tri-rotor UAV to achieve a feedback linearized system through geometric transformation that is valid at any operating point but matches the Jacobi linearization of the system at the operating point of interest. The technique preserves robustness as it does not invert all nonlinear dynamics, unlike classic feedback linearization. An distributed optimal formation tracking control protocol was then developed for the tri-rotor robotic swarm, which guarantees that the target time-varying position and time-varying attitude of each UAV can be achieved independently. Finally, simulation results were given in a realistic environment based on 3D graphical display and physical visualisations. It has been shown that the proposed tri-rotor UAV swarm is able to track a desired time-varying formation whilst independently tracking different time-varying attitudes. A target surveillance task was performed effectively by these tri-rotor UAVs, which lays the foundation for some more complex collaborative tasks to be explored.

Future work will take obstacle avoidance and power management as shown in [43] into consideration, the proposed distributed controller will be applied to real hardware, and robust methods such as [44], [45] will be exploited in the design of the distributed control protocol.

Acknowledgment

This work was supported by the Engineering and Physical Sciences Research Council (EPSRC) [grant number EP/R008876/1]. All research data supporting this publication are directly available within this publication.

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