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Distributed robust stabilization of networked multiagent systems with strict negative imaginary uncertainties

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Summary

This paper deals with the distributed robust stabilization problem for networked multiagent systems with strict negative imaginary (SNI) uncertainties. Communication among agents in the network is modelled by an undirected graph with at least one self-loop. A protocol based on relative state measurements of neighbouring agents and absolute state measurements of a subset of agents is considered. This paper shows how to design the protocol parameters such that the uncertain closed-loop networked multiagent system is robustly stable against any SNI uncertainty within a certain set for various different network topologies. Tools from negative imaginary (NI) theory are used as an aid to simplify the problem and synthesise the protocol parameters. We show that a state, input, and output transformation preserves the NI property of the network. Consequently, a necessary and sufficient condition for the transfer function matrix of the nominal closed-loop networked system to be NI and satisfy a DC gain condition is that multiple reduced-order equivalent systems be NI and satisfy a DC gain condition simultaneously. Based on the reduced-order systems, we derive sufficient conditions in an LMI framework which ensure the existence of a protocol satisfying the desired objectives. A numerical example is given to confirm the effectivenesses of the proposed results.

KEYWORDS

distributed control, multiagent systems, negative imaginary systems, robust control

1 | INTRODUCTION

The past two decades have witnessed great research advances in the field of distributed control of networked multiagent systems. Comprehensive surveys relating to these advances can be found in.[2-5] In general, the theory of networked multiagent systems is primarily concerned with the design of distributed control laws (also known as protocols) to guarantee a desired collective behaviour or a global control objective such as stability of a network, synchronization, consensus, etc. It is well known that the dynamics of agents and the interaction among them are two essential factors in the analysis and

Early preliminary parts of this work have been presented at ECC18.[1] The material here significantly expands and builds upon the earlier conference paper.

synthesis of control protocols for a desired behaviour or control objective. Moreover, synthesis of robust control protocols is inevitably required.

In recent years, a considerable amount of literature has been published on negative imaginary (NI) systems.[6-10] NI systems theory, which involved characterisation of NI systems and a robust stability result, was first introduced in the work of Lanzon and Petersen.[11] NI systems have a NI frequency response. For single-input single-output NI systems, the Nyquist plot of these systems lie on or below the real axis for all positive frequencies. Many physical systems can be modelled as NI systems. For example, flexible structures with colocated force actuators and position sensors exhibit the NI property.[11,12] Such systems are usually modelled as infinite dimensional distributed parameter systems.[13] However, for control design purpose, a finite-dimensional model is normally used which yields modelling errors due to these unmodelled dynamics. Such unmodelled dynamics usually belong to the strictly NI (SNI) class.[11] If the effect of these unmodelled dynamics is not taken into account in control design, these unmodelled dynamics may lead to instability and performance degradation of the controlled system.

In this paper, we aim to study the distributed robust stabilization problem for networked systems with SNI uncertainties.

1.1 | Motivation and Contribution

In the networked multiagent systems literature, dynamic uncertainties due to modelling error has not been much of a focus with only few researchers taking such uncertainties into account in the analysis and synthesis of distributed control protocols. Furthermore, the focus was mainly on uncertainties bounded in H_∞ norm. In this context, robust stability of multiagent dynamical systems was studied in the work of Hara and Tanaka[14] where three different types of multiplicative perturbations were considered. Robust synchronization of uncertain multiagent networks was addressed in the works of Trentelman et al[15] and Jongsma et al[16] with uncertainties in the form of additive perturbations in the work of Trentelman et al[15] and in the form of coprime factor perturbations in the work of Jongsma et al.[16] Robust consensus control for multiagent systems involving gap metric uncertainties was investigated in the work of Alvergue et al.[17] It is known[11] that the dynamic uncertainties of NI systems are mainly characterised by phase bounds, where the phase lies between $-\pi$ and 0. Using phase information in control design for lightly damped (ie, highly resonant) systems is much more effective than using gain information.[18,19] In other words, for such lightly damped systems gain stabilization, which is dependent on the small-gain theorem, leads to conservative design results.[11,18] On the other hand, phase stabilization, which ensures stability by restricting the phase of the open-loop system and by which the NI robust stability results were established, yields more powerful, less conservative, and robust control systems.[19,20] Thus, it is of interest to investigate how distributed control protocols can be designed to stabilize networked multiagent systems with SNI uncertainties via the NI systems theory. Although the work of Wang et al,[9] which studied robust cooperative control of multiple NI systems, took into account SNI uncertainties, the nominal plants were assumed to be NI. However, due to physical considerations, in some situations, what is only known about the system is that the uncertainty belongs to the SNI class. In such situations, the results in[8,18,20,21] showed that, if a controller is designed such that the closed-loop system is NI, then the robust feedback stability results in[6,11-13] can be applied to guarantee robustness to this class of uncertainties. However, the aforementioned papers only consider individual systems and do not consider networked multiagent systems. This motivates us to study the distributed robust stabilization problem of networked multiagent systems with SNI uncertainties.

The main contribution of this paper is thus to propose a solution to the distributed robust stabilization problem of networked multiagent systems with SNI uncertainties. We derive sufficient conditions for the existence of control protocol parameters such that the control protocol robustly stabilizes a networked multiagent system in presence of SNI uncertainties of certain DC size. Transformation of the large scale networked systems into equivalent reduced-order subsystems (also known as decomposition approach) is often used in multiagent systems literature to simplify the problem under consideration. Inspired mainly by works[15,16,22] and adopting such transformation techniques, we show that under certain assumptions on the network graph, a state, input, and output transformation preserves the NI property. Thus, for the transfer function matrix of the nominal closed-loop networked system to be NI and satisfy a DC gain condition, it is equivalent for multiple reduced-order equivalent systems to be NI and satisfy a DC gain condition simultaneously. Consequently, the aforementioned sufficient conditions are derived based on the reduced-order subsystems. The control protocol is also shown to ensure robust stability when variations in the network topology occur by only appropriately adjusting a positive scalar which is one of the control protocol parameters. An example is given to confirm the results of this paper.

2 | PRELIMINARIES

2.1 | Notation

Let $\mathbb{R}^{m\times n}$ and $\mathcal{R}^{m\times n}$ denote the set of $m\times n$ real matrices and real, rational, and proper transfer function matrices, respectively. Given a matrix A, A^T and A^* denote the transpose and the complex conjugate transpose of A, respectively. $\lambda_i(A)$ and $\lambda_{\max}(A)$ denote the ith and the largest eigenvalue (when the matrix has only real eigenvalues) of A, respectively. $\mathfrak{R}[\cdot]$ is the real part of a complex number. I_N denotes the identity matrix of dimension $N\times N$. 1_N is an $N\times 1$ vector with entries 1. $A\otimes B$ denotes the Kronecker product of matrices A and B. $\text{diag}(A_1, \dots, A_N)$ represents a block-diagonal matrix with matrices A_i for all $i\in\{1,\dots,N\}$ on the main diagonal. For a real symmetric matrix X, the notation $X>0$ ($X\geq 0$) means that matrix X is positive definite (positive semidefinite).

2.2 | Negative imaginary systems

In this paper, we restrict attention to NI systems without poles at the origin defined as follows.

Definition 1 (See the work of Xiong et al[12]). A square real, rational, and proper transfer function matrix $R(s)$ is termed NI if

1. $R(s)$ has no poles at the origin and in $\mathfrak{R}[s]>0$;
2. $j[R(j\omega)-R(j\omega)^*]\geq 0$ for all $\omega\in(0,\infty)$ except values of ω where $j\omega$ is a pole of $R(s)$;
3. if $j\omega_0$ with $\omega_0\in(0,\infty)$ is a pole of $R(s)$, then it is a simple pole and the residue matrix $K_0=\lim_{s\to j\omega_0}(s-j\omega_0)jR(s)$ is Hermitian and positive semidefinite.

Strictly NI systems are defined as follows.

Definition 2 (See the work of Lanzon and Petersen[11]). A square real, rational, and proper transfer function matrix $R(s)$ is termed SNI if

1. $R(s)$ has no poles in $\mathfrak{R}[s]\geq 0$;
2. $j[R(j\omega)-R(j\omega)^*]>0$ for $\omega\in(0,\infty)$.

The following lemma is used to check whether a system belongs to the class of NI or not.

Lemma 1 (See the work of Song et al[18]). *Let (A,B,C,D) be a state-space realization of $R(s)\in\mathcal{R}^{m\times m}$ where $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$, $C\in\mathbb{R}^{m\times n}$, $D\in\mathbb{R}^{m\times m}$ with $m\leq n$. If $\det(A)\neq 0$, $D=D^T$ and there exists a real matrix $Y=Y^T>0$ such that*

$$AY+YA^T\leq 0 \quad \text{and} \quad B+AYC^T=0, \tag{1}$$

then $R(s)$ is NI.

The following lemma characterises robust stability for NI systems. The result we use here is not the main theorem (see theorem 5 in the work of Lanzon and Petersen[11] or theorem 1 in the work of Xiong et al[12] for the main feedback stability theorem) as stated in the literature, but a corollary to the principal theorem stated in the same form as the small-gain theorem to suit our purpose. This was first proposed in the work of Lanzon and Petersen[11] for stable NI systems and later shown to be also valid for marginally stable NI systems in the work of Xiong et al.[12]

Lemma 2 (See the works of Lanzon and Petersen[11] and Xiong et al[12]). *Given $\gamma>0$ and a NI transfer function matrix $R(s)$, then the positive feedback interconnection $[\Delta(s),R(s)]$ is internally stable for all SNI transfer function matrices $\Delta(s)$ satisfying $\Delta(\infty)R(\infty)=0$, $\Delta(\infty)\geq 0$ and $\lambda_{\max}(\Delta(0))<(1/\gamma)$ (respectively, $\leq(1/\gamma)$) if and only if $\lambda_{\max}(R(0))\leq\gamma$ (respectively, $<\gamma$).*

Remark 1. It is worth mentioning that Definition 1 for NI systems without poles at the origin was extended in the work of Mabrok et al[13] to include poles at the origin and associated stability results were established in the works of Lanzon and Chen[6] and Mabrok et al.[13] Furthermore, the NI lemma given in the work of Lanzon and Petersen[11] for stable feedback systems, in the work of Xiong et al[12] for marginally stable systems, and in the work of Mabrok et al[21] for systems with possible poles at the origin contain necessary and sufficient conditions, but also imposes a minimality assumption on the state-space realisation. We restrict our work to the definitions stated above (1) to use Lemma 1 to facilitate control design since no minimality assumption is required and (2) since we are interested in robust stability of networked systems with SNI uncertainties of certain DC size via Lemma 2 (note that NI systems in Lemma 2 are restricted to have no poles at the origin).

2.3 | Graph theory

Graphs are used to model information exchange among agents in a network. The information flow is bidirectional in an undirected graph whereas directional in a directed graph. Detailed material on graph theory can be found in the works of Mesbahi and Egerstedt[23] and Ren et al.[24] We focus our attention here on undirected graphs. An undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ consists of a nonempty finite vertex set $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ and an edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ of unordered pairs of vertices, called edges. An edge in $\mathcal{G}$ is denoted by (v_i, v_j). If $(v_i, v_j) \in \mathcal{E}$, then vertices (ie, agents) v_i and v_j are adjacent (or neighbours) and can obtain information from each other. The set of neighbours of vertex v_i is defined as $\mathcal{N}_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$. An edge (v_i, v_i) is called a self-loop. A graph is said to be simple if it contains no self-loops and no repeated edges. A loop around vertex v_i means that agent v_i has access to its own absolute measurements. A path in a graph from v_i to v_j is a sequence of edges of the form $(v_i, v_{i+1}), (v_{i+1}, v_{i+2}), \dots, (v_{j-1}, v_j)$. An undirected graph is connected if there is an undirected path between every pair of distinct vertices. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined as $a_{ij} = a_{ji} = 1$ if $(v_i, v_j) \in \mathcal{E}$, $a_{ii} = 1$ if v_i has a self-loop, and 0 otherwise. Note that, for a simple graph, $a_{ii} = 0$ for all $i \in \{1, \dots, N\}$. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N\times N}$ of $\mathcal{G}$ is defined as $l_{ij} = -a_{ij}$, for $i \neq j$ and $l_{ii} = \sum_{k=1}^{N} a_{ik}$ for all $i \in \{1, \dots, N\}$. Based on the adjacency matrix, this definition can fit for both simple graphs and for graphs with self-loops.

The notation $\hat{\mathcal{L}}$ will hereafter be used to indicate the Laplacian matrix associated with a graph with self-loops.

Lemma 3 (See the work of Li et al[22]). *For a graph with at least one self-loop, the Laplacian matrix $\hat{\mathcal{L}}$ is positive definite, if the graph is connected.*

3 | PROBLEM FORMULATION

Consider a group of N linear uncertain agents. The dynamics of the ith agent are described by

$$\begin{aligned} \dot{x}_i(t) &= Ax_i(t) + B_1 w_i(t) + B_2 u_i(t), \\ z_i(t) &= C_1 x_i(t), \\ \hat{w}_i(s) &= \Delta_i(s)\hat{z}_i(s), \end{aligned} \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$, $w_i(t) \in \mathbb{R}^m$, $u_i(t) \in \mathbb{R}^p$, and $z_i(t) \in \mathbb{R}^m$ are the state, disturbance, control input, and controlled output of the ith agent, respectively, with $m \leq n$. The matrices $A \in \mathbb{R}^{n\times n}$, $B_1 \in \mathbb{R}^{n\times m}$, $B_2 \in \mathbb{R}^{n\times p}$, and $C_1 \in \mathbb{R}^{m\times n}$ are known constant matrices. The transfer function matrix $\Delta_i(s)$ represents the uncertainty in the dynamics of the ith agent, where $\hat{w}_i(s)$ and $\hat{z}_i(s)$ are the Laplace transform of $w_i(t)$ and $z_i(t)$, respectively.

Suppose that the uncertainty in the dynamics of each agent satisfies the following property and conditions.

Assumption 1. For all $i \in \{1, \dots, N\}$, the uncertainty $\Delta_i(s)$ is SNI and satisfies $\Delta_i(\infty) \geq 0$ and $\lambda_{\max}(\Delta_i(0)) \leq (1/\gamma)$, where $\gamma > 0$ is a prespecified number.

The uncertainty thus gives rise to the heterogeneity of the multiagent system.

Following the work of Li et al,[22] the control protocol for the ith agent is

$$u_i(t) = cK\left(\sum_{j=1}^{N} a_{ij}(x_i(t) - x_j(t)) + a_{ii}x_i(t)\right), \tag{3}$$

where $c > 0$ is the coupling strength to be selected, $K \in \mathbb{R}^{p\times n}$ is the control feedback gain matrix to be designed and a_{ij} are the elements of the adjacency matrix with $a_{ii} = 1\ \forall i \in \{1, \dots, q\}$, and $a_{ii} = 0\ \forall i \in \{q+1, \dots, N\}$. This protocol structure means that each agent receives the sum of relative state measurements with respect to its neighbours. In addition, a subset of agents receive their own absolute state measurements. Without loss of generality, it is assumed that the first q ($q \ll N$) agents have access to their own absolute state measurements. Consequently, the network graph that models the information exchange among the agents satisfies the following assumption.

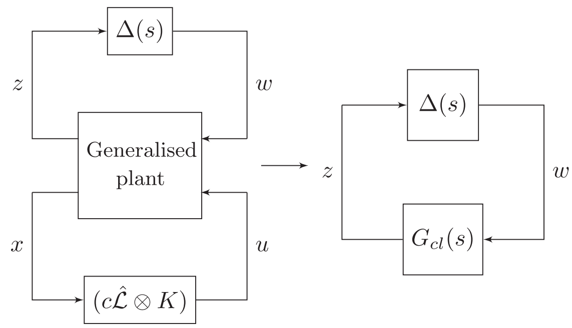


FIGURE 1 Networked multiagent system with strictly negative imaginary uncertainty

Assumption 2. The graph is connected, undirected and at least one vertex has a self-loop.

Dropping time dependency and Laplace variable dependency where it is clear from the context, it is clear that agent dynamics (2) can be rewritten as

$$\begin{aligned} \dot{x} &= (I_N \otimes A)x + (I_N \otimes B_1)w + (I_N \otimes B_2)u, \\ z &= (I_N \otimes C_1)x, \\ \hat{w} &= \Delta(s)\hat{z}, \end{aligned} \tag{4}$$

and control law (3) can be rewritten as

$$u = (c\hat{\mathcal{L}} \otimes K)x, \tag{5}$$

where $x = [x_1^T, \dots, x_N^T]^T \in \mathbb{R}^{nN}$, $w = [w_1^T, \dots, w_N^T]^T \in \mathbb{R}^{mN}$, $u = [u_1^T, \dots, u_N^T]^T \in \mathbb{R}^{pN}$, $z = [z_1^T, \dots, z_N^T]^T \in \mathbb{R}^{mN}$, $\Delta(s) = \mathrm{diag}(\Delta_1(s), \dots, \Delta_N(s))$, $\hat{w}$ is the Laplace of w, $\hat{z}$ is the Laplace of z, and $\hat{\mathcal{L}} \in \mathbb{R}^{N\times N}$ is the Laplacian matrix associated with $\mathcal{G}$. By applying protocol (5) (or equivalently (3) to each agent i in (2)) to the uncertain agents (4), the resulting uncertain closed-loop networked multiagent system becomes

$$\begin{aligned} \dot{x} &= \left((I_N \otimes A) + (c\hat{\mathcal{L}} \otimes B_2K)\right)x + (I_N \otimes B_1)w, \\ z &= (I_N \otimes C_1)x, \end{aligned} \tag{6}$$

and

$$\hat{w} = \Delta(s)\hat{z}. \tag{7}$$

Note that $\Delta(s)$ is SNI since each $\Delta_i(s), i \in \{1, \dots, N\}$ is SNI and satisfies $\Delta(\infty) \geq 0$ and $\lambda_{\max}(\Delta(0)) \leq 1/\gamma$ by noting that $\lambda_{\max}(\Delta(0)) = \max_{i=1,\dots,N}\lambda_{\max}(\Delta_i(0)) \leq 1/\gamma$. The transfer function matrix of the nominal closed-loop networked multiagent system from w to z is strictly proper and given by

$$G_{cl}(s) = C_{cl}(sI_{nN} - A_{cl})^{-1}B_{cl}, \tag{8}$$

where $A_{cl} = (I_N \otimes A) + (c\hat{\mathcal{L}} \otimes B_2K)$, $B_{cl} = (I_N \otimes B_1)$, $C_{cl} = (I_N \otimes C_1)$ and has an associated DC gain of

$$\lambda_{\max}(G_{cl}(0)) = \lambda_{\max}(C_{cl}(-A_{cl})^{-1}B_{cl}). \tag{9}$$

The uncertain networked multiagent system is depicted in Figure 1. According to Lemma 2, we can define the distributed robust stabilization problem as follows.

Definition 3. Given $\gamma > 0$, control protocol (3) is said to robustly stabilize the networked system with agent dynamics (2) against any SNI uncertainty satisfying Assumption 1 if it is designed such that the transfer function matrix (8) is NI and satisfies the DC gain condition $\lambda_{\max}(G_{cl}(0)) < \gamma$.

4 | PROBLEM REDUCTION AND ROBUST PROTOCOL SYNTHESIS

In order to address the distributed robust stabilization problem, the following technical lemmas are required.

Lemma 4. *Let $U \in \mathbb{R}^{N\times N}$ be any orthogonal matrix, $R(s) \in \mathcal{R}^{Nm\times Nm}$ and let $\tilde{R}(s) = (U^T \otimes I_m)R(s)(U \otimes I_m)$. Then, the following holds.*

1. *$\tilde{R}(s)$ is NI (respectively, SNI) if and only if $R(s)$ is NI (respectively, SNI).*
2. *$\lambda_{\max}(\tilde{R}(0)) = \lambda_{\max}(R(0))$.*

Proof. The proof is straightforward from the definition of NI (SNI) systems and properties of orthogonal matrices. □

Lemma 5 (See the work of Wang et al[10]). *$diag(R_1(s), \dots, R_N(s))$ is NI if and only if $R_i(s)$ are all NI for $i \in \{1, \dots, N\}$.*

The following lemma states that, under certain assumptions on the network graph, the NI property is preserved due to transformation.

Lemma 6. *Given $\gamma > 0$ and assume that the network topology $\mathcal{G}$ satisfies Assumption 2, let $\hat{\mathcal{L}}$ be the Laplacian matrix of $\mathcal{G}$ and let λ_i for all $i \in \{1, \dots, N\}$ be the eigenvalues of $\hat{\mathcal{L}}$. Then, the transfer function matrix (8) of the networked system (6) is NI and satisfies $\lambda_{\max}(G_{cl}(0)) < \gamma$ if and only if, for all $i \in \{1, \dots, N\}$, the transfer functions $\tilde{G}_i(s)$ of the following N isolated subsystems:*

$$
\begin{aligned}
\dot{\tilde{x}}_i &= (A + c\lambda_i B_2 K)\tilde{x}_i + B_1 \tilde{w}_i,\\
\tilde{z}_i &= C_1 \tilde{x}_i,
\end{aligned}
\tag{10}
$$

are all NI and satisfy $\lambda_{\max}(\tilde{G}_i(0)) < \gamma$ simultaneously, where $\tilde{G}_i(s) = C_1(sI - A - c\lambda_i B_2 K)^{-1}B_1$.

Proof. The idea of the proof is to transform the networked system (6) into a set of block-diagonal systems

$$
\begin{aligned}
\dot{\tilde{x}} &= ((I_N \otimes A) + (c\Lambda \otimes B_2 K))\tilde{x} + (I_N \otimes B_1)\tilde{w},\\
\tilde{z} &= (I_N \otimes C_1)\tilde{x}
\end{aligned}
\tag{11}
$$

in a similar manner to the decomposition approach used in[15,16,22,25] by letting $\tilde{x} = (U^T \otimes I_n)x$, $\tilde{w} = (U^T \otimes I_m)w$, $\tilde{z} = (U^T \otimes I_m)z$ and decomposing $\hat{\mathcal{L}}$ as $U^T\hat{\mathcal{L}}U = \Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_N)$, where $U \in \mathbb{R}^{N\times N}$ is an orthogonal matrix. Consequently, the transfer function matrix of the transformed system (11) from $\tilde{w}$ to $\tilde{z}$, which we denote $\tilde{G}_{cl}(s)$, can be expressed as

$$
\tilde{G}_{cl}(s) = \text{diag}\left(\tilde{G}_1(s), \dots, \tilde{G}_N(s)\right) = (U^T \otimes I_m)G_{cl}(s)(U \otimes I_m).
\tag{12}
$$

The desired conclusion then follows from Lemmas 4 and 5. □

Remark 2. According to Definition 3, the distributed robust stabilization problem is solved by designing a control protocol such that the transfer function matrix $G_{cl}(s)$ of the large scale nominal closed-loop networked system is NI and satisfies the DC gain condition. Lemma 6 states that a necessary and sufficient condition for $G_{cl}(s)$ to be NI and satisfy the DC gain condition is that N reduced-order subsystems, where each subsystem has the order of a single agent, satisfy the NI property and DC gain condition simultaneously. Consequently, the previous lemma plays a role in facilitating and simplifying the design procedure where the protocol parameters can be designed based on the reduced-order systems.

Remark 3. Reduction of the problem as stated in the previous remark is applicable since the uncertainty resulting from transformation remains SNI. If we denote the uncertainty of the transformed system as shown in Figure 2 by $\tilde{\Delta}(s)$, we have $\tilde{\Delta}(s) = (U^T \otimes I_m)\Delta(s)(U \otimes I_m)$. Since $\Delta(s)$ is SNI and satisfies $\Delta(\infty) \geq 0$, $\lambda_{\max}(\Delta(0)) \leq 1/\gamma$, then according to Lemma 4, so will $\tilde{\Delta}(s)$ be SNI and satisfy the corresponding conditions. As a result, under Assumption 2 of the network topology, internal stability of the system in Figure 1 is equivalent to the internal stability of the system in Figure 2.

Remark 4. We now give a justification for using graphs with at least one self-loop in this paper instead of simple graphs as used in for example.[15,16] Although the work of[15,16] assumed simple graphs in their approach to robust synchronization, the results therein are suitable only for the case were the dynamics of the nominal plants have no poles in the open-right half plane. That is, the work therein restricts the matrix A from containing eigenvalues with positive real parts. In this paper, we impose no such restrictions on the eigenvalues of matrix A. Thus, the graph which models the network topology cannot be simple but instead must contain at least one self-loop, as assumed in

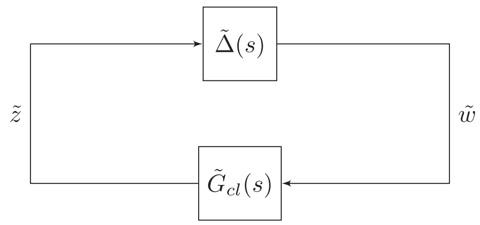


FIGURE 2 Transformed system

Assumption 2, because a simple connected graph will have one zero eigenvalue (see, eg, the work of Ren and Beard[26]) and, thus, the subsystem in (10) corresponding to $\lambda_1 = 0$ cannot be controlled to satisfy the NI property.

Lemma 6 reveals that, for networked dynamical system (6) to satisfy the NI property, it suffices to find a positive scalar c and a gain matrix K such that systems (10) satisfy the NI property simultaneously.

The following Theorem 1 gives sufficient conditions under which a $c > 0$ and a feedback gain matrix K exists such that the networked multiagent system is robustly stabilized by control protocol (3).

Theorem 1. *Given $\gamma > 0$, a network topology that satisfies Assumption 2 and an uncertain multiagent system (2) with $C_1B_2 = 0$, $m \le n$, and (A, B_2) controllable, if there exists a matrix $Y = Y^T > 0$ and a scalar $\tau > 0$ such that*

$$\begin{bmatrix} AY + YA^T - \tau B_2B_2^T & B_1 + AYC_1^T \\ B_1^T + C_1YA^T & 0 \end{bmatrix} \le 0, \tag{13}$$

$$C_1YC_1^T < \gamma I, \tag{14}$$

$$\det\left(AY - \frac{1}{2}\tau B_2B_2^T\right) \neq 0, \tag{15}$$

then there exists a feedback gain matrix K and a scalar $c \ge \frac{\tau}{\min_{i\in\{1,\ldots,N\}}\lambda_i}$ such that control protocol (3) robustly stabilizes the networked multiagent system in the presence of any SNI uncertainty satisfying Assumption 1. Moreover, a suitable feedback gain matrix K is given by $K = -0.5B_2^TY^{-1}$.

Proof. Since the LMI condition (13) holds for some matrix $Y > 0$ and some scalar $\tau > 0$ and since $c \ge \frac{\tau}{\lambda_i}$ for all $i \in \{1, \ldots, N\}$, it follows that, for all $i \in \{1, \ldots, N\}$,

$$\begin{bmatrix} AY + YA^T - c\lambda_iB_2B_2^T & B_1 + AYC_1^T \\ B_1^T + C_1YA^T & 0 \end{bmatrix} \le \begin{bmatrix} AY + YA^T - \tau B_2B_2^T & B_1 + AYC_1^T \\ B_1^T + C_1YA^T & 0 \end{bmatrix} \le 0 \tag{16}$$

as $\lambda_i > 0$ for all $i \in \{1, \ldots, N\}$. This implies that

$$AY + YA^T - c\lambda_iB_2B_2^T \le 0, \tag{17a}$$

$$B_1 + AYC_1^T = 0. \tag{17b}$$

Furthermore, since $C_1B_2 = 0$ by assumption, then (17b) can be written as

$$B_1 + AYC_1^T - 0.5c\lambda_iB_2B_2^TC_1^T = 0. \tag{18}$$

Now, let $K = -0.5B_2^TY^{-1}$. Via simple algebraic manipulation, (17a) and (18) become

$$(A + c\lambda_iB_2K)Y + Y(A + c\lambda_iB_2K)^T \le 0, \tag{19a}$$

$$B_1 + (A + c\lambda_iB_2K)YC_1^T = 0, \tag{19b}$$

for all $i \in \{1, \ldots, N\}$. Furthermore, (15) implies

$$\det\left(A - 0.5\tau B_2B_2^TY^{-1}\right) \neq 0,$$

which is equivalent to

$$\det(A + \tau B_2 K) \neq 0. \tag{20}$$

Now, since $c\lambda_i \geq \tau$ for all $i \in \{1, \dots, N\}$, it can be written as $c\lambda_i = \tau + \alpha_i$, where $\alpha_i \geq 0$. Then,

$$\begin{aligned}\det(A + c\lambda_i B_2 K) &= \det(A + \tau B_2 K + \alpha_i B_2 K) \\ &= \det(A + \tau B_2 K)\det(I + (A + \tau B_2 K)^{-1}\alpha_i B_2 K).\end{aligned} \tag{21}$$

We need to show that $\det(A + c\lambda_i B_2 K) \neq 0$ for all $i \in \{1, \dots, N\}$. Toward this end,

$$\det(A + c\lambda_i B_2 K) \neq 0 \Leftrightarrow \det\left(I + (A + \tau B_2 K)^{-1}\alpha_i B_2 K\right) \neq 0 \tag{22}$$

for all $i \in \{1, \dots, N\}$. It is easily seen that $\det(A + c\lambda_i B_2 K) \neq 0$ for $\alpha_i = 0, i \in \{1, \dots, N\}$. For $\alpha_i > 0, i \in \{1, \dots, N\}$, we have

$$\det\left(I + (A + \tau B_2 K)^{-1}\alpha_i B_2 K\right) = \alpha_i^n \det\left(\frac{1}{\alpha_i} I + (A + \tau B_2 K)^{-1} B_2 K\right) \tag{23}$$

and is nonzero if and only if $\frac{1}{\alpha_i} I + (A + \tau B_2 K)^{-1} B_2 K$ is nonsingular, which is satisfied when $\Re\{\lambda_j[(A + \tau B_2 K)^{-1} B_2 K]\} \geq 0\ \forall j$ since $1/\alpha_i$ for $i \in \{1, \dots, N\}$ is a positive scalar. Therefore, what is left is to show that $\Re\{\lambda_j[(A + \tau B_2 K)^{-1} B_2 K]\} \geq 0\ \forall j$

$$\begin{aligned}
AY + YA^T - \tau B_2 B_2^T \leq 0 &\Leftrightarrow \left(AY - \frac{1}{2}\tau B_2 B_2^T\right) + \left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^T \leq 0 \\
&\Leftrightarrow \left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^{-T} + \left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^{-1} \leq 0 \\
&\Leftrightarrow \left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^{-1} + \left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^{-T} \leq 0 \\
&\Rightarrow B_2^T\left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^{-1} B_2 + B_2^T\left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^{-T} B_2 \leq 0 \\
&\Leftrightarrow \left[B_2^T\left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^{-1} B_2\right] I + I\left[B_2^T\left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^{-T} B_2\right] \leq 0 \\
&\Rightarrow \Re\left\{\lambda_j\left[B_2^T\left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^{-1} B_2\right]\right\} \leq 0 \quad \forall j \\
&\Leftrightarrow \Re\left\{\lambda_j\left[\left(AY - \frac{1}{2}\tau B_2 B_2^T\right)^{-1} B_2 B_2^T\right]\right\} \leq 0 \quad \forall j \\
&\Leftrightarrow \Re\left\{\lambda_j\left[Y^{-1}\left(A - \frac{1}{2}\tau B_2 B_2^T Y^{-1}\right)^{-1} B_2 B_2^T\right]\right\} \leq 0 \quad \forall j \\
&\Leftrightarrow \Re\left\{\lambda_j\left[\left(A - \frac{1}{2}\tau B_2 B_2^T Y^{-1}\right)^{-1} B_2 B_2^T Y^{-1}\right]\right\} \leq 0 \quad \forall j \\
&\Leftrightarrow \Re\left\{\lambda_j\left[(A + \tau B_2 K)^{-1} B_2 K\right]\right\} \geq 0 \quad \forall j.
\end{aligned} \tag{24}$$

It follows that

$$\det(A + c\lambda_i B_2 K) \neq 0 \quad \text{for all } i \in \{1, \dots, N\}. \tag{25}$$

Consequently, $\tilde{G}_i(s)$ is NI for all $i \in \{1, \dots, N\}$ by Lemma 1.

It remains to show that the DC gain of each subsystem is less than γ. Since the LMI condition (14) holds, via (25) and (19b), it follows that

$$\begin{aligned}\gamma I > C_1 Y C_1^T &= C_1(A + c\lambda_i B_2 K)^{-1}(A + c\lambda_i B_2 K) Y C_1^T \\ &= C_1(-A - c\lambda_i B_2 K)^{-1} B_1 = \tilde{G}_i(0)\end{aligned} \tag{26}$$

for all $i \in \{1, \dots, N\}$. Consequently, $\lambda_{\max}(\tilde{G}_i(0)) < \gamma$ for all $i \in \{1, \dots, N\}$.

From Lemma 6, we conclude that $G_{cl}(s)$ is NI and $\lambda_{\max}(G_{cl}(0)) < \gamma$.

Now, since $G_{cl}(s)$ is strictly proper, we have $\Delta(\infty)G_{cl}(\infty) = 0$, and since the uncertainty satisfies Assumption 1, it follows from Lemma 2 that control protocol (3) robustly stabilizes the networked multiagent system. □

Remark 5. $C_1B_2 = 0$ means that the transfer function from u_i to z_i $\forall i \in \{1, \dots, N\}$ has a relative degree strictly greater than unity. This is hence often fulfilled in practice due to strictly proper actuator dynamics and strictly proper plant dynamics.

Remark 6. By imposing the assumption $C_1B_2 = 0$ in Theorem 1, we get simpler solvable conditions (13) to (15), which do not involve the network topology.

Remark 7. The determinant condition appears because the NI property excludes poles at the origin. This nonconvex condition is not troublesome as a feasible solution for Y and τ can always be obtained first by solving the LMI conditions and then checking whether the computed values satisfy the determinant condition or not. If they do not, then a small increase in τ often resolves the problem.

Thus, the steps required to design the protocol can be summarized in the following algorithm.

1. Solve the LMI conditions (13) and (14) for $Y > 0$ and $\tau > 0$. Then, check whether the determinant condition (15) is satisfied or not. If not, perturb τ and/or Y to satisfy all of (13) to (15).
2. Let the feedback gain matrix be $K = -0.5B_2^TY^{-1}$.
3. Select the coupling strength c to satisfy $c \geq \frac{\tau}{\min\limits_{i\in\{1,\dots,N\}} \lambda_i}$, where λ_i $\forall i \in \{1, \dots, N\}$ are the eigenvalues of $\hat{\mathcal{L}}$ (note that the minimum value of c that can be selected is when the equal sign holds).

Remark 8. The benefit of the aforementioned design procedure is that the feedback gain matrix K is first designed without any knowledge of the network graph. Then, the coupling strength c is adjusted to handle the effect of the network topology. Thus, once a feedback gain matrix K is designed, robust stability against SNI uncertainties with certain DC size is achieved via control protocol (3) for various different network graphs that satisfy the condition $\lambda_i \geq \tau/c$ for all $i \in \{1, \dots, N\}$. Clearly, this inequality is satisfied for a rich class of Laplacian matrices and associated network topologies. Consequently, by selecting a large enough value for the coupling strength c, both robust stability to agents dynamics and robustness to variations in the network topology can be guaranteed.

Remark 9. Although we build on the work of Li et al,[22] it is important to observe that the results here are not a specialisation of the results in the work of the aforementioned authors[22] because we consider a distinct problem from the aforementioned work.[22] It is assumed in the work of Li et al[22] that the agents are subject to external disturbances in $\mathfrak{L}_2[0, \infty)$ and the problem consider therein is to evaluate the performance of a networked multiagent system subject to these external disturbances. In this paper, we consider the situation where agents are subject to dynamical uncertainties (modelling errors), which belong to the SNI class and the problem here is to maintain stability of the network in the presence of SNI uncertainties with a certain DC size. Li et al[22] studied a suboptimal H_∞ control problem, where distributed controllers need to be found such that the H_∞ norm of a transfer function is less than a desired tolerance. Thus, it is essential that the gain be small over all frequency ranges. On the contrary, the distributed robust stabilization problem we consider here requires to find distributed controllers such that a transfer function matrix satisfies the NI property and only the DC gain value be restricted on one side, which is less conservative. While the work of Li et al[22] derive conditions for the existence of controllers to have unbounded H_∞ performance region to ensure a level of robustness with respect to the communication topology, the results we present in this paper derive conditions for the existence of controllers that robustly stabilize networked systems in the presence of dynamical uncertainties that belong to the SNI class and achieving robustness to variations in the network topology.

Remark 10. It is worth mentioning that the robust stabilization problem we address in this paper is somehow different and not comparable to the consensus problem addressed for example in the works of Ren et al.[24,26] Modelling errors in the agents dynamics were not considered in the works of Ren et al.[24,26] Furthermore, the consensus problem addressed therein requires convergence of the states to an unspecified common value depending on the initial state information. This can only be satisfied with simple network graphs where zero is a simple eigenvalue of the Laplacian matrix $\mathcal{L}$ and hence span$\{1_N\}$ is contained in the null space of $\mathcal{L}$; consequently, consensus is guaranteed.[24] The robust stabilization problem we address here is mainly concerned with guaranteeing robust stability of the networked system in presence of modelling errors, which belong to the SNI class. As stated in Remark 4, it is essential the network graph contains at least one self-loop as for simple graphs where zero is a simple eigenvalue of $\mathcal{L}$ the subsystem in (10) corresponding to this zero eigenvalue of $\mathcal{L}$ cannot be controlled to satisfy the NI property, consequently robust stabilization cannot be guaranteed. Nevertheless, it may be of interest to investigate consensus to a desired reference/trajectory as a next step provided robust stability against SNI uncertainties is satisfied first for the networked system. However, this may be challenging and not straightforward and is beyond the scope of this paper.

5 | NUMERICAL EXAMPLE

The example in the work of Song et al[18] is modified to design distributed controllers for systems with heterogeneous SNI uncertainties. Consider a group of $N = 6$ uncertain systems connected over a network topology. The block diagram of the ith uncertain systems is depicted in Figure 3 and the network topology that models the communication among the systems and the associated Laplacian matrix are shown in Figure 4. Each of the six systems contains an uncertain flexible structure with colocated force actuation and position sensing and, thus, the transfer function of the ith flexible structure $M_i(s)\ \forall i \in \{1, \ldots, 6\}$ is SNI. For control design purpose, $M_i(s)$ has been replaced by unity gain and the resulting modelling error $\Delta_i(s) = M_i(s) - 1$ is an SNI uncertainty as shown in Figure 3. It is assumed that $\Delta_i(s)$ satisfies Assumption 1 with $\gamma = 1$. Via results of this paper, parameters K and c of distributed control protocol (3) can be designed to ensure robust stability of the closed-loop networked system against SNI uncertainties and also ensure robustness to variations in the network topology.

To this end, the dynamics of the ith system can be obtained from Figure 3 in the form of (2) with $x_i = [x_{i1}^T, x_{i2}^T, x_{i3}^T]^T$ and matrices

$$A = \begin{bmatrix} -2 & 1 & 0 \\ 1 & -5 & 1 \\ 0 & 0 & -4 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

It is easy to see that $C_1B_2 = 0$, $m \leq n$, and (A, B_2) are controllable. It can also be seen from Figure 3 that the transfer function from u_i to z_i has a relative degree strictly greater than unity, which emphasises the statement in Remark 5.

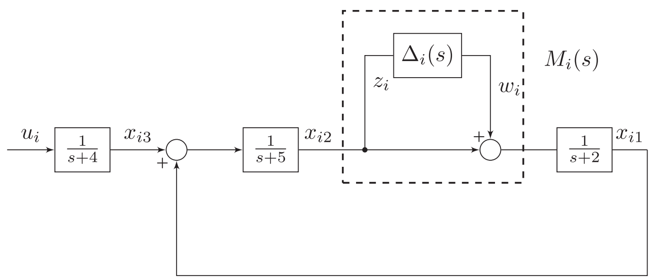
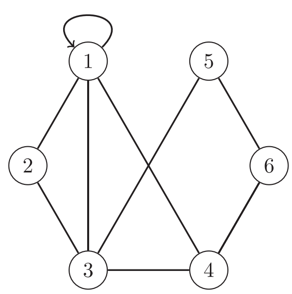


FIGURE 3 Block diagram of the ith uncertain system to be controlled



$$\hat{\mathcal{L}} = \begin{bmatrix} 4 & -1 & -1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ -1 & -1 & 4 & -1 & -1 & 0 \\ -1 & 0 & -1 & 3 & 0 & -1 \\ 0 & 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & 0 & -1 & -1 & 2 \end{bmatrix}$$

FIGURE 4 Network topology and associated Laplacian matrix

Using the YALMIP[27] and SeDuMi[28] toolboxes to solve the LMI conditions as according to step 1 in the algorithm, we obtain the feasible solutions

$$Y = \begin{bmatrix} 3.7674 & 0.4547 & -0.4949 \\ 0.4545 & 0.0909 & 0 \\ -0.4946 & 0 & 0.4946 \end{bmatrix} > 0,$$

and $\tau = 2.7377$. We check that $\det(AY - \frac{1}{2}\tau B_2 B_2^T) = -3.3474 \neq 0$. No perturbations to Y and τ are necessary. From step 2 in the algorithm, the feedback gain matrix is given by

$$K = -\frac{1}{2}B_2^T Y^{-1} = [\,-0.5 \;\; 2.5 \;\; -1.5\,].$$

The minimum eigenvalue of $\hat{\mathcal{L}}$ in Figure 4 is 0.1266. We hence select the coupling strength c according to step 3 in the algorithm to be $c = 43$ (twice the minimum value). Thus, Theorem 1 states that the control protocol with the values of K and c as computed above robustly stabilizes the networked system against any SNI uncertainty having a DC gain less than or equal to unity.

To illustrate this and to avoid construction of the 18th-order ($N = 6$ and $n = 3$) overall plant dynamics in (6), we simply demonstrate that each of the N subsystems $\tilde{G}_i(s)$ given by (10) within the transformed overall plant dynamics (11) are all individually NI and satisfy the DC gain conditions. Figure 5 gives the Nyquist plots of the six subsystems; that is, $\tilde{G}_i(s)$, $\forall i \in \{1, \dots, 6\}$. It is clear from the plots that the systems have a NI frequency response. Furthermore, Table 1 gives the explicit transfer functions for $\tilde{G}_i(s)$ from which it is easy to verify that each $\tilde{G}_i(s)$ is stable. Table 1 also shows that the nonsingular determinant condition (25) is satisfied for each of the six subsystems and that the DC gains of $\tilde{G}_i(s)$, $i \in \{1, \dots, 6\}$ are all equal to 0.1 since they have been all set to be equal to $C_1 Y C_1^T$, which are less than unity.

Moreover, control protocol (3) with the same values of K and c as designed above also guarantees a level of robustness to variations in the network topology. To see this, consider the four different network topologies in Figure 6 constructed by adding or/and removing links from the original network topology in Figure 4. Control protocol (3) with the same values

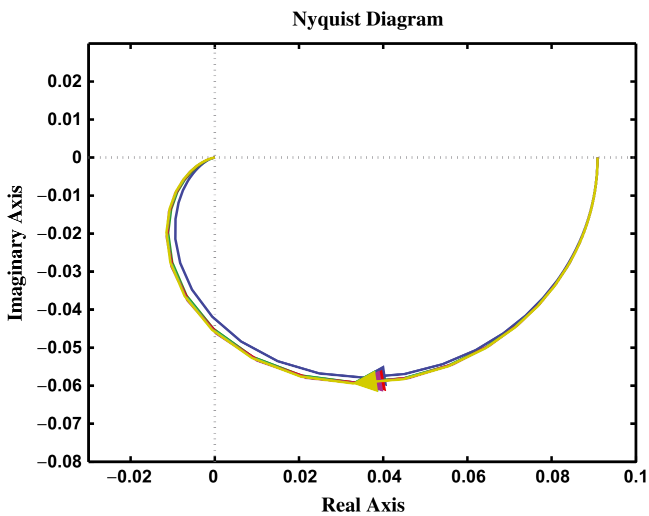


FIGURE 5 Nyquist plots of $\tilde{G}_i(s), \forall i \in \{1, \dots, 6\}$, $c = 43$ [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Verifying the negative imaginary property for $\tilde{G}_i(s)$, $i \in \{1, \dots, 6\}$

i	λ_i	$\tilde{G}_i(s)$	$\tilde{G}_i(0) = C_1 Y C_1^T$	$\det(A + c\lambda_i B_2 K)$
1	0.1266	$\frac{s+9.504}{s^3+19.23s^2+82.98s+104.5}$	0.1	−104.5
2	1.2205	$\frac{s+57.05}{s^3+90.29s^2+462.9s+627.6}$	0.1	−627.6
3	2.3293	$\frac{s+105.2}{s^3+162.3s^2+847.9s+1158}$	0.1	−1157.7
4	3.0647	$\frac{s+137.2}{s^3+210.1s^2+1103s+1509}$	0.1	−1509.4
5	4.9643	$\frac{s+219.8}{s^3+333.5s^2+1763s+2418}$	0.1	−2417.6
6	5.2945	$\frac{s+234.1}{s^3+355s^2+1875s+2575}$	0.1	−2575.5

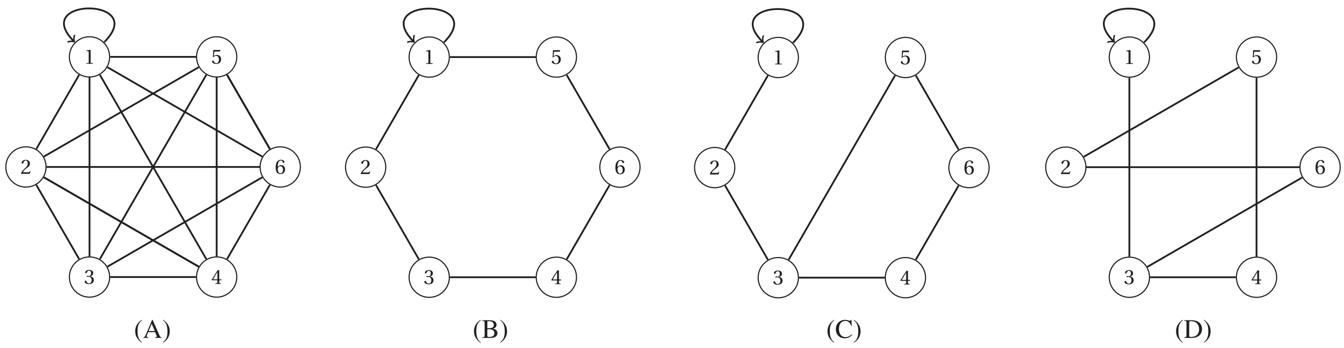


FIGURE 6 Four different network topologies

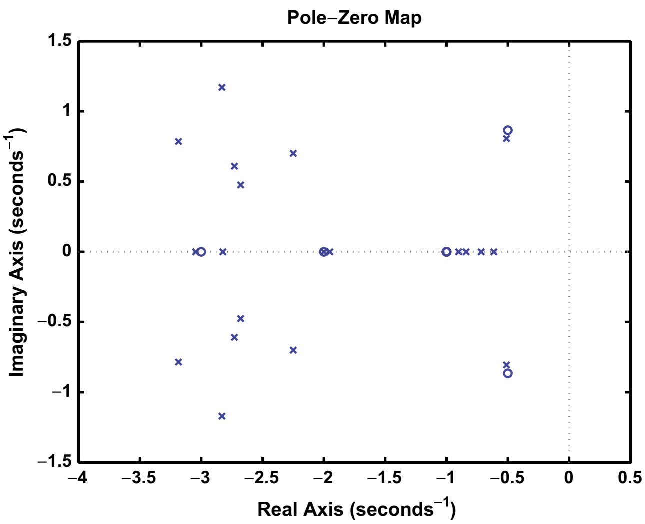


FIGURE 7 Poles and zeros of $G_{cl}(I - \Delta G_{cl})^{-1}$ corresponding to original network topology; poles are marked by x, and zeros are marked by o [Colour figure can be viewed at wileyonlinelibrary.com]

of K and c as designed above is guaranteed to achieve robust stability for all four various different networked systems (ie, agents may be connected over any of the four network topologies) in the presence of SNI uncertainties with DC gains less than or equal to unity since $c = 43$ is greater than the minimum value of c corresponding to each of these network graphs. The minimum values of c, which correspond to the network graphs of Figure 6A to 6D, are 18.764, 25.1627, 40.8283, and 34.3070, respectively.

We can also easily demonstrate that, for some specific uncertainties, the conclusion holds. For instance, choose $\Delta_1(s) = 0.5/(s+1)$, $\Delta_2(s) = (1-s)/(1+s)$, $\Delta_3(s) = 1/(s+3)$, $\Delta_4(s) = 1/(s^2+3s+2)$, $\Delta_5(s) = 1/(s+1)^2$, and $\Delta_6(s) = (0.5s+1)/(s^2+s+1)$, which are SNI. $\Delta(s)$ in Figure 1 has $\lambda_{\max}(\Delta(0)) = 1 \leq 1/\gamma$. A pole-zero map of $G_{cl}(I - \Delta G_{cl})^{-1}$ is shown in Figure 7 for the original network topology in Figure 4. Since all closed-loop poles are in the left half plane, we conclude that the heterogeneous perturbed closed-loop system of Figure 1 is internally stable.

6 | CONCLUSIONS

This paper has studied the distributed robust stabilization problem for networked multiagent systems with SNI uncertainties. It was shown that a state, input, and output transformation preserves the NI property of the network when the network topology is modelled by an undirected graph with self-loops. This result was shown to be useful in control protocol design as the problem simplified to finding parameters which ensured that each of the multiple reduced-order systems satisfy the NI property. The synthesis procedure involved the design of two separate parameters, ie, one which is a scalar that handles the effect of the network topology and the other one was a state feedback gain matrix. The advantage of this design procedure lies in the ability of the control protocol to maintain robust stability in the face of SNI uncertainties for different network topologies by simply appropriately adjusting this coupling scalar while leaving the state feedback gain matrix unchanged. A numerical example was given to show the usefulness of the results.

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