Generalised Control-Oriented Modelling Framework for Multi-Energy Systems

Technical Report

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Abstract

A requirement for new sources of flexibility in meeting energy demand and the growing interest in the interaction between energy sectors have drawn attention to Multi-Energy Systems (MES), which can offer numerous benefits, e.g. providing flexibility to counteract the intermittency of renewable generation and increase energy efficiency. In order to maximise these benefits, advanced control methods are required, such as Model Predictive Control (MPC). MPC is increasingly recognised as a suitable control methodology for energy applications due to its ability to incorporate economic and operational objectives whilst respecting various technical, regulatory and environmental constraints. However, as its name suggests, MPC is critically reliant on the availability of a sufficiently accurate system model that allows it to forecast and, therefore, optimise future system behaviour. This report presents a novel generalised modelling framework for MES, which is particularly suited to design of optimisation based control schemes such as MPC. The proposed modelling framework is capable of representing energy converter topologies of arbitrary complexity containing multiple energy vectors, as well as multi-directional energy flow, multi-generation and multi-mode devices, a wide range of controllable producers/consumers, energy storage and flexible loads. To demonstrate the effectiveness of the developed modelling framework a representative case study based on three buildings at the University of Manchester is used. The model of the case study’s MES is firstly obtained using the proposed approach and then it is incorporated into a certainty equivalent MPC scheme. The numerical evaluation demonstrates the ability of the controller to minimise the cost of purchasing different energy vectors whilst satisfying operational constraints, including the requirements of various types of controllable flexible demand.
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Chapter 1

Introduction

To support progression toward a low-carbon future and away from dependency on non-renewable energy resources, electrification of the heat and transport sectors as well as increased penetration of low-carbon technology is expected to continue at ever increasing pace. This also means an increase in so-called prosumers, as distributed generation technologies provide the means for energy users to produce energy locally along with consuming energy provided by the wider infrastructure. While this shift presents great challenges for distribution network operators, it is also an opportunity for novel energy management methodologies to be developed and applied. This is due to the increase in the number of flexibility sources made available through the integration of low-carbon technologies, which in many cases represent controllable system components. Electrification of heat and transport entails new loads, such as heat pumps and electric vehicles, which are associated with inherent storage capabilities (e.g. thermal storage, thermal load inertia, plug-in electric vehicle batteries) and can be harnessed to further increase system flexibility. In particular, by employing programs designed to influence desirable changes in consumer’s energy use, i.e. demand-side management (DSM) [1]. By considering the inherently multi-variable nature of energy systems and exploiting the interaction between different energy vectors, a variety of benefits in power system operation can be realised such as increased flexibility in load balancing, frequency response and reserve [2]. These implications have influenced significant interest in the concept of multi-energy systems (MES). Correctly managing the newly available flexibility sources has the potential to significantly reduce the requirement for electricity network re-enforcement that would otherwise be required as they can assist an economically and/or environmentally optimal dispatch of generation/load, thus accommodating more renewable energy sources (RES) and maximising capacity utilisation of the existing infrastructure.

A promising control methodology to accommodate these requirements is model predictive control (MPC). MPC is suited to this type of application since it incorporates multi-step optimisation which accounts for system changes over time. In addition, limiting/inequality constraints are explicitly handled allowing the inclusion of both hard (e.g. system’s physical and regulatory limits) and soft (e.g. user’s comfort thresholds) constraints [3].
As its name suggests, MPC is critically reliant on the availability of a sufficiently accurate system model that allows it to forecast and, therefore, optimise future system behaviour. In particular, the model should suitably balance the trade-off between accuracy/detail and complexity/tractability. With aim to meet this requirement a novel, generalised, approach to the modelling of MES is presented in this report. The approach is particularly suited, but not limited, to optimisation-based control schemes such as MPC.

1.1 Related Literature

The energy hub has been a prevalent concept for the modelling of MES over the last decade. Energy hubs are described as units with inputs and outputs in which the storage and conversion of different energy carriers take place [4]. In the original formulation, a hub is comprised of converter and storage elements. Converter elements represent how efficiently energy carriers are converted from one form to another, whilst storage elements represent energy storage devices. An excellent resource, containing software tools for energy hub modelling can be found in [5]. The energy hub modelling approach has been applied to a variety of operational challenges associated with MES such as expansion planning [6, 7], optimal design [8, 9], resource optimisation [10, 11] and reliability assessment [12]. The energy hub is capable of representing MES flexibility provided by energy storage and the increased degrees of freedom in the energy resource dispatch. However, as suggested in [13], existing energy hub approaches do not allow the modelling of arbitrarily complex converter topologies (e.g. including bi-directional power flow and splitting of energy flow at any point in the converter arrangement).

Alternative approaches to MES modelling are included in [14–17]. However, these approaches are either unsuitable for control-oriented applications due their design [14, 17] and/or stakeholder benefit [15] aspects, or are restricted by focussing on specific technologies [16].

The modelling approaches within the aforementioned studies do not capture the full range of flexibility benefits made available when considering MES and particularly those achievable through integrated demand side management (IDSM) which extends the domain of DSM to MES, where instead of just shifting or modulating their energy use (as with DSM), energy users are able to switch the primary source of the consumed energy [18]. Contributions focussing on IDSM include [19–21] but are not fully comprehensive approaches as they either focus on demand response (DR) [19, 20], which is a subset of DSM and relates to the DSM programs implemented to alter energy use in response to particular events such as periods of poor system reliability or high wholesale energy cost, or they focus on specific energy vectors as is the case in [21].

The optimisation of DSM for the electrical power system is a widely researched topic, with models relating to specific energy resources being recently proposed in [22–24]. However, as these approaches all relate solely to electrical energy systems they do not include
multi-energy technologies that unlock additional flexibility and enable IDSM, such as co-generation. It is worth mentioning that the DSM models in [23] cover a range of flexible demand characteristics captured by the model presented in this report but only focus on a single primary energy vector.

IDSM and the energy hub modelling approach have been combined in [25–31]. The IDSM models included in these references focus on a variety of specific aspects but none offer a fully comprehensive approach to modelling the flexibility possible in MES. For example, the DR model in [29] allows modulation of the demand profile and does not consider the case where the demand profile (or at least a portion of it) is fixed and can only be shifted in time.

MES with IDSM is also considered more specifically for microgrid operation with DR in [32–34]. However, these contributions only consider curtailable and/or shiftable demands.

Most of the references reviewed thus far do not address the control problem of automatically operating MES in the presence of uncertain disturbances. The exceptions are [33] and [34], both of which include operational considerations. Also, MPC has recently been applied to MES [35–39]. However, with these approaches IDSM is either not included (which is the case in [35]) or the proposed models do not include all the possible types of demand flexibility, i.e. [36–39].

1.2 Summary of Contribution

To the best of the authors’ knowledge, no formalised approach to MES modelling for control applications has yet been proposed that incorporates the full range of flexibility sources available in MES. In order to address this, an innovative generalised approach to modelling MES is presented in this report. The approach includes a wide range of controllable elements which are found in MES and its modularity removes need for spending excessive time developing bespoke mathematical models for particular systems/scenarios since the models of pre-defined components can be readily incorporated into the overall MES system description. In an addition to the proposed representation of arbitrarily complex energy conversion topologies, which has already been preliminarily introduced in [13], devices with distinct operating configurations can be readily modelled, allowing the possibility of capturing highly non-linear conversion efficiencies and/or multi-mode devices. Also, a set of fundamental flexible demand models is introduced allowing individual models to be joined in any possible combination and to form compound models representing a full range of IDSM flexibility. The flexible demand models include novel extensions of existing approaches to accommodate fully adjustable demands, readily linked to energy user comfort. In accordance with the control centred focus of the modelling framework, the updating of inter-temporal constraints relating to physical dynamic relationships (e.g. storage, thermal loads), as well as sequential dispatch schedules, are explicitly modelled making the approach suitable for on-line applications as opposed to static ‘look-ahead’ optimisation.
1.3 Report Outline

The remainder of this report is organised as follows. In Chapter 2 an overview of the control oriented modelling framework is provided and the key components of the proposed MES model are introduced. The first of these components, namely the energy conversion model, is described in detail in Chapter 3. In Chapter 4 the storage model component is described. A prosumer model component is described in Chapter 5 along with a detailed description of the flexible demand modelling approach and its extension to energy producers. In Chapter 6, a particular case-study is introduced that illustrates capabilities of the proposed modelling approach when integrated within MPC control framework. The simulated results relating to the case study are presented in Chapter 6.3 before the report is concluded and discussion of future work is provided in Chapter 7.
Chapter 2

Control Oriented Modelling Framework for Multi-Energy Systems

The MES modelling approach proposed in this report assumes that for certain sections of MES it is reasonable to represent the transportation networks of the various energy carriers within the section as non-dynamic 'lossy' connections. Such sections are termed energy exchange zones (EEZ) and extend the well known energy hub concept [40]. As with the energy hub, the spatial boundaries of a particular EEZ depend upon a given application (e.g. a building, district, city etc.) and are essentially defined by the validity of assuming non-dynamic and linear relationships in the transportation networks. An EEZ contains three types of components, which are the energy conversion model (ECM), the storage model (SM) and prosumer model (PM). The flow, conversion and splitting of the energy carriers are modelled by means of the ECM. Description of various energy storage devices that may exist within an EEZ is provided using the SM. Finally, different types of demand and/or production that may exist within an EEZ are modelled using PM. In a given EEZ, only one ECM component may exist. This is in contrast to SM and PM components, which are not limited by number of occurrences within a single EEZ.

A schematic of a single generic EEZ that exists within a larger multi-zone MES is depicted in Figure 2.1. The ECM component provides an interface between SM and PM sub-components in a given EEZ and a wider multi-energy network. Separation of energy conversion, storage dynamics and various prosumer characteristics, as shown in Figure 1, facilitates modular approach to the modelling of MES. Also, the proposed approach for the ECM allows configurable bi-directional converter topologies to be described in a compact manner whilst maintaining a reasonable representation of the underlying physical system. In the following sections, modelling of these three types of EEZ components will be described in greater detail. Whilst the remainder of the report considers a single EEZ, the proposed modelling framework can be readily applied to general MES composed of multiple EEZs interconnected with transportation networks as indicated in Figure 2.1.
Figure 2.1: General Model of Multi-EEZ MES
Chapter 3

Energy Conversion Model (ECM)

The ECM, preliminarily introduced in [13], represents a novel extension of the original energy hub concept used as a paradigm for representing energy conversion within MES. In particular, the ability of ECM modelling approach to represent energy conversion topologies of arbitrary complexity represents one of the key contribution of this report. No restrictions are imposed on the direction of power flow through a particular arrangement of converters nor the number and/or position of energy carrier splits within. This is achieved whilst still maintaining a reasonable representation of physical conversion devices, with particular reference to multi-output converters such as combined heat and power (CHP) devices. Furthermore, the proposed methodology is well suited to software implementation thus expediting wider model development.

ECM representation is based on graph theory and is fully characterised by the set of nodes it contains and the associated arcs that provide mutual interconnections between the nodes. Each node of the graph has an energy carrier associated with it. The arcs of the graph represent directional paths through the ECM along which the power can flow and are denoted as \((n_i \rightarrow n_j)\) where \(n_i\) and \(n_j\) correspond to an arc’s source node and sink node, respectively. Each arc has an associated conversion factor, denoted as \(\eta(n_i \rightarrow n_j)\). The power that flows to \(n_j\) node is then given by \(P(n_i \rightarrow n_j)(k)\eta(n_i \rightarrow n_j)\), where \(P(n_i \rightarrow n_j)(k)\) is the power flowing from node \(n_i\) during sampling period \(k\). Also, two index sets are associated with each node. These are \(I_{\cdots n_i}\), which denotes the index set of source nodes that connect to \(n_i\), and \(L_{n_i \rightarrow \cdot}\), which denotes the index set of sink nodes that \(n_i\) connects to.

Each node in the ECM belongs to one of the following four categories: terminal nodes, sum nodes, switch nodes and transmitter nodes. Terminal nodes represent connections of the ECM to externalities that include prosumers, storage elements and wider energy network. Terminal nodes can be further sub-divided into input, output and input/output nodes depending on whether power flows in or out of ECM. Input nodes have a single outgoing arc and facilitate import of energy to ECM. Output nodes, conversely, have a single incoming arc and facilitate export of energy out of ECM. Lastly, input/output nodes have both a single incoming and single outgoing arc connected to the same adjacent node.
Input/output nodes allow interface of ECM with other entities within MES system that both import and export energy, e.g. prosumers and storages. The power flowing into or out of the ECM’s terminal node $n_t$, denoted as $P_{n_t}$, is related to the incoming and/or outgoing arc powers according to the following equation:

$$P_{n_t}(k) = \sum_{i \in \mathcal{I}_{n_t}} P_{(n_t \rightarrow n_i)}(k)\eta_{(n_t \rightarrow n_i)} - \sum_{j \in \mathcal{I}_{n_t}} P_{(n_i \rightarrow n_t)}(k)$$  

(3.1)

Furthermore, non-negativity constraints are imposed on all ECM arc flow variables which ensures that (3.1) respects a sign convention such that $P_{n_t} \geq 0$ for input nodes and $P_{n_t} \leq 0$ for output nodes.

Sum nodes describe the combining and splitting of a particular energy carrier within ECM. For each sum node the algebraic sum of incoming and outgoing power flows is equal to zero:

$$\sum_{i \in \mathcal{I}_{n_t}} P_{(n_t \rightarrow n_i)}(k)\eta_{(n_t \rightarrow n_i)} - \sum_{j \in \mathcal{I}_{n_t}} P_{(n_i \rightarrow n_t)}(k) = 0$$  

(3.2)

Sum nodes represent junctions of energy flow and can be used to describe particular parts of a multi-energy system, such as electrical busbar or pipework manifold.

Switch nodes, similarly to sum nodes, obey the relationship between their incoming and outgoing arcs given in (3.2). However, additional binary decision variables, denoted as $\delta$, are associated with each outgoing arc of a switch node to obey the condition (3.3a) and the constraint (3.3b) which ensure that only a single outgoing arc is active at any instant in time.

$$P_{(n_i \rightarrow n_j)}(k) > 0 \iff \delta_{(n_i \rightarrow n_j)}(k) = 1 \quad \forall j \in \mathcal{I}_{n_t}$$  

(3.3a)

$$\sum_{j \in \mathcal{I}_{n_t}} \delta_{(n_i \rightarrow n_j)}(k) \leq 1$$  

(3.3b)

Therefore, switch nodes facilitate modelling of multi-mode devices, such as a heat pump that can operate in both heating and cooling modes.

Transmitter nodes are used to model the conversion of energy through a co-generation plant, such as CHP, by requiring that for each of its outgoing arc its flow value is equal to the sum of incoming arc flows’ values, as described in (3.4). For each transmitter node there are as many equations as there are outgoing arcs.

$$P_{(n_i \rightarrow n_j)}(k) = \sum_{i \in \mathcal{I}_{n_t}} P_{(n_t \rightarrow n_i)}(k)\eta_{(n_t \rightarrow n_i)} \quad \forall j \in \mathcal{I}_{n_t}$$  

(3.4)

To prevent the unrealistic case of simultaneous power flow in two parallel but directionally opposite arcs between two nodes, binary decision variables are associated with the particular arcs that obey an equivalent condition to (3.3a) and the following mutual exclusivity constraint:

$$\delta_{(n_i \rightarrow n_j)}(k) + \delta_{(n_j \rightarrow n_i)}(k) \leq 1$$  

(3.5)
3.1 Example 1 - ECM

Consider the energy converter arrangement depicted in Figure 3.1 where energy carriers available for import from the wider energy infrastructure are electricity and gas. It is also possible for electrical power to be exported if the prosumer is generating. Imported gas is converted into both heat and electricity through a CHP device. Further, a dual-mode heat pump provides a means by which electricity is used to transfer useful heat or cooling, depending on its operating mode. In Figure 3.1, bi-directional flow is possible along paths that do not end with an arrow head. The graph representation of the example ECM is depicted in Figure 3.2. Each node of the graph has an energy carrier associated with it. In this case, the set of energy vectors present in the system are electricity, gas, heating and cooling which are denoted as $e$, $g$, $h$ and $c$ respectively. As more than one node is associated with the same energy carrier, each node is also assigned an index to differentiate itself from others with the same associated energy carrier. For example, three nodes are associated with electricity and they are denoted $e_1$, $e_2$ and $e_3$ respectively. The power flow paths and efficiencies of the energy conversion model can be represented by a connection matrix, $M \in \mathbb{R}^{N_{nd} \times N_{nd}}$, where $N_{nd}$ is the number of nodes in the ECM. The row indices of $M$ correspond to source nodes and the column indices correspond to sink nodes. If an element of $M$ is equal to 0 then there exists no conversion path between the two nodes in the direction defined by the order of node pairs which denote the arc. No node can be connected with itself and so all diagonal elements of $M$ are 0. Also, any element of $M$ equal to 1 indicates a lossless connection between two nodes. The matrix $M$ relating to the example in Figure 3.2 is given in Equation (3.6). The matrix representation is useful as it allows the ECM
In the example ECM depicted in Figure 3.2 nodes $e_1$, $e_3$, $g_1$, $h_2$ and $c_1$ are terminal nodes, drawn as squares, with $g_1$ an input node, $h_2$ and $c_1$ output nodes, and $e_1$ and $e_3$ input/output nodes.

The switch node, $^s e_4$, can also be identified, notationally, by the leading $s$ superscript. As discussed, switch nodes share the same relationship between their incoming and outgoing arcs as the sum nodes (i.e. Equation (3.2)). However, additional binary decision variables are associated with each outgoing arc of a switch node, i.e. the outgoing arcs of node $^s e_4$ are associated with binary decision variables $\delta(^{s e_4 \rightarrow h_1})$ and $\delta(^{s e_4 \rightarrow h_2})$.

The transmitter node is denoted with a leading $t$ superscript i.e. $^t n_t$. The outgoing arc flows are converted, with corresponding efficiency, to the energy carrier types of the adjacent

\[
M = \begin{pmatrix}
0 & \eta(e_1 \rightarrow e_2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\eta(e_2 \rightarrow e_1) & 0 & \eta(e_2 \rightarrow e_3) & \eta(e_2 \rightarrow e_4) & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \eta(e_3 \rightarrow e_2) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \eta(s e_4 \rightarrow h_1) & 0 & \eta(s e_4 \rightarrow c_1) & 0 \\
0 & 0 & 0 & 0 & 0 & \eta(g_1 \rightarrow g_2) & 0 & 0 & 0 & 0 \\
0 & \eta(g_2 \rightarrow e_2) & 0 & 0 & 0 & 0 & \eta(t g_2 \rightarrow h_1) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \eta(h_1 \rightarrow h_2) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
\]
nodes. This approach fixes the ratio of power flow in the output arcs for a given total input power flow.

If terminal nodes represent power flows that can be controlled (e.g. inputs, storage flows, controllable/flexible loads etc.) then their corresponding variables represent degrees of freedom in the system model. In addition to the controllable terminal nodes, degrees of freedom exist in the model by way of parallel paths between a given pair of nodes. For example, if energy can flow between two terminal nodes via two different paths then there exists an additional degree of freedom in the system.

The proposed representation of the converter model allows a system of equations to be written down in a straightforward manner. This can be done manually through inspection of the graph, or by computer via a search of $M$. In the most general case, where each terminal node represents a degree of freedom (i.e. all ECM inputs and outputs are control system manipulating variables), the vector of continuous system variables contains each arc power flow variable as well as the value of each terminal node. The system of equations which describe the example ECM are shown in (3.7). One equation is required for each terminal node where (3.7a), (3.7b), (3.7e), (3.7i) and (3.7j) relate to $e_1, e_3, g_1, h_2$ and $c_1$ respectively. One equation is also required for each sum and switch node where (3.7c), (3.7d) and (3.7h) relate to $e_2, e_4$ and $h_1$ respectively. Since the transmitter node has two outgoing arcs it requires two equations i.e. (3.7f) and (3.7g).

$$
\begin{align*}
P_{e_1} + P_{(e_2 \rightarrow e_1)} \eta_{(e_2 \rightarrow e_1)} &= P_{(e_1 \rightarrow e_2)} \tag{3.7a} \\
P_{e_3} + P_{(e_2 \rightarrow e_3)} \eta_{(e_2 \rightarrow e_3)} &= P_{(e_3 \rightarrow e_2)} \tag{3.7b} \\
P_{(e_1 \rightarrow e_2)} \eta_{(e_1 \rightarrow e_2)} + P_{(e_3 \rightarrow e_2)} \eta_{(e_3 \rightarrow e_2)} + P_{(g_2 \rightarrow e_2)} \eta_{(g_2 \rightarrow e_2)} &= P_{(e_2 \rightarrow e_1)} + P_{(e_2 \rightarrow e_3)} + P_{(e_2 \rightarrow e_4)} \tag{3.7c} \\
\end{align*}
$$

$$
\begin{align*}
P_{(e_2 \rightarrow e_4)} \eta_{(e_2 \rightarrow e_4)} &= P_{(e_4 \rightarrow c_1)} + P_{(e_4 \rightarrow h_1)} \tag{3.7d} \\
P_{g_1} &= P_{(g_1 \rightarrow g_2)} \tag{3.7e} \\
P_{(g_2 \rightarrow e_2)} &= P_{(g_1 \rightarrow g_2)} \eta_{(g_1 \rightarrow g_2)} \tag{3.7f} \\
P_{(g_2 \rightarrow h_1)} &= P_{(g_1 \rightarrow g_2)} \eta_{(g_1 \rightarrow g_2)} \tag{3.7g} \\
P_{(e_4 \rightarrow h_1)} \eta_{(e_2 \rightarrow h_1)} + P_{(g_2 \rightarrow h_1)} \eta_{(g_2 \rightarrow h_1)} &= P_{(h_1 \rightarrow h_2)} \tag{3.7h} \\
P_{h_2} &= -P_{(h_1 \rightarrow h_2)} \eta_{(h_1 \rightarrow h_2)} \tag{3.7i} \\
P_{c_1} &= -P_{(e_4 \rightarrow c_1)} \eta_{(e_4 \rightarrow c_1)} \tag{3.7j} \\
\end{align*}
$$

The outgoing arc energy flow represents the input flow and the incoming energy flow to the incident sink node represents the output flow. In the case of the example shown in Figure 3.2, the heat pump is modelled using nodes $e_4, h_1$ and $c_1$, arc energy flows $P_{(e_4 \rightarrow h_1)}$ and $P_{(e_4 \rightarrow c_1)}$ along with the weightings $\eta_{(e_4 \rightarrow h_1)}$ and $\eta_{(e_4 \rightarrow c_1)}$ which represent the heat pump’s coefficient of performance and energy efficiency ratio respectively. In general, physical single-mode-single-input-single-output converters are represented by graph arcs whose source nodes
are of terminal or sum type and whose weighting does not equal 1. Multiple-mode-single-
input-single output devices are modelled using arcs sharing a common switch type source
node. Single-input-multiple-output converters, such as CHP devices, are represented by
transmitter nodes combined with their outgoing arcs. The sum of the incoming arcs to the
source transmitter node represent the input flow, and the incoming arc flows to each adjacent
sink node represent the respective output flow of the conversion associated with the arc. In
the example, the CHP plant is modelled using nodes \( g2, e2 \) and \( h1 \) as well as arc power
flows \( P_{(g2 \rightarrow e2)} \) and \( P_{(h2 \rightarrow h1)} \) along with \( \eta_{(g2 \rightarrow e2)} \) and \( \eta_{(h2 \rightarrow h1)} \) which represent the respective
gas-to-electricity and gas-to-heat conversion efficiencies of the device.

As shown, bi-directional power flow through the ECM is represented by parallel but
directionally opposite arcs between adjacent nodes. Such a representation allows for dif-
ferent conversion efficiencies to apply in each direction. However, it is not a physically
realisable scenario to have power flowing in both directions simultaneously. Therefore, arcs
that represent bi-directional flows are associated with binary decision variables to determine
the configuration of the conversion model. Referring again to the example system and its
graph representation in Figure 3.2, restriction on power flow in both directions simultane-
ously between nodes \( e1 \) and \( e2 \) is ensured by implementing the logical conditions (3.8)-(3.9)
and enforcing the non-negativity constraints (3.10)-(3.11) and mutual exclusivity constraint
(3.12).

\[
\begin{align*}
P_{(e1 \rightarrow e2)} &> 0 \iff \delta_{(e1 \rightarrow e2)} = 1 \quad (3.8) \\
P_{(e2 \rightarrow e1)} &> 0 \iff \delta_{(e2 \rightarrow e1)} = 1 \quad (3.9) \\
P_{(e1 \rightarrow e2)} &\geq 0 \quad (3.10) \\
P_{(e2 \rightarrow e1)} &\geq 0 \quad (3.11) \\
\delta_{(e1 \rightarrow e2)} + \delta_{(e2 \rightarrow e1)} &\leq 1 \quad (3.12)
\end{align*}
\]

In the example, equivalent binary decision variables and constraints also exist for the two
arcs between nodes \( e2 \) and \( e3 \). In general, wherever bi-directional flow is possible between a
pair of nodes, it is required that two binary decision variables be added to the system model.

In addition to the constraints on binary decision variables, non-negativity constraints ap-
ply to arc power flow decision variables, along with upper bounds to represent the maximum
possible power flow along any given arc in the converter model. Input terminal nodes are
bounded by non-negativity constraints as well as an upper bound which represents capacity
constraint imposed on input energy. Output terminal nodes are bounded by non-positivity
constraints as well as lower bounds which represent capacity constraints imposed on the out-
put energy. Input/output terminal nodes have both a negative lower bound and a positive
upper bound in order to impose capacity limits on the output and input energy, respectively. Inequality constraints also provide the means by which the binary decision variables for bi-directional power flow are implemented in the model, following the approach described in [41]. The set of inequality constraints for the continuous system variables in the example
is thus described by (3.14):

\[
X_{ECM} \geq \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \Delta_{ECM} \end{bmatrix} X_{ECM} \tag{3.13}
\]

\[
X_{ECM} \leq \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & \Delta_{ECM} \end{bmatrix} \bar{X}_{ECM} \tag{3.14}
\]

where \( X_{ECM} \) is the vector of decision variables, \( \bar{X}_{ECM} \) the vector of associated upper bounds and \( \bar{X}_{ECM} \) the vector of associated lower bounds. Also, \( \Delta_{ECM} \) is a diagonal matrix containing the ECM binary decision variables for each associated outgoing arc. The vectors, \( X_{ECM}, \bar{X}_{ECM} \) and \( \bar{X}_{ECM} \) are defined in (3.15):

\[
X_{ECM} = \begin{bmatrix} P_{ECM-T} \\ P_{ECM} \\ P_{ECM-\delta} \end{bmatrix} \tag{3.15}
\]

\[
\bar{X}_{ECM} = \begin{bmatrix} P_{ECM-T} \\ 0 \\ \varepsilon \end{bmatrix} \tag{3.16}
\]

\[
\bar{X}_{ECM} = \begin{bmatrix} P_{ECM-T} \\ P_{ECM} \\ P_{ECM-\delta} \end{bmatrix} \tag{3.17}
\]

where, in the example case, \( P_{ECM-T} = [P_{e1}, P_{e3}, P_{g1}, P_{h2}, P_{c1}]^T \) is the vector of terminal node variables, \( P_{ECM} = [P_{(g1\rightarrow g2)}, P_{(g1\rightarrow h1)}, P_{(g2\rightarrow h1)}, P_{(h1\rightarrow h2)}] \) is the vector of arc power flow variables that are not associated with binary decision variables and \( P_{ECM-\delta} = [P_{(e1\rightarrow e2)}, P_{(e2\rightarrow e1)}, P_{(e2\rightarrow e3)}, P_{(e3\rightarrow e2)}] \) is the vector of arc power flow variables associated with binary decision variables. The corresponding variables in the \( \bar{X}_{ECM} \) and \( \bar{X}_{ECM} \) vectors represent the respective upper bounds and lower bounds. In order to prevent feasible solutions where \( P_{(n_i\rightarrow n_j)} = 0 \) and \( \delta_{(n_i\rightarrow n_j)} = 1 \), the lower bound of each arc power flow variable associated with a binary decision variable is the binary variable itself, multiplied by a small number, \( \varepsilon \) (typically the machine precision).

For notational simplicity, the variables and constraints presented in this section have all been related at a single point in time. This is allowable as the ECM system model contains no inter-temporal constraints.
Chapter 4

Storage Model

The modular nature of the proposed approach allows for the storage dynamics to be represented by a general n-th order discrete-time state space model, i.e. $x(k+1) = Ax(k) + Bu(k)$; $y(k) = Cx(k) + Du(k)$. Note that the generality of state-space models allows multi-input, multi-output SM to be considered. Inputs to a SM generally consist of charging/discharging power, denoted as $v(k)$, as well as $N_w$ other disturbance inputs, denoted as $w(k) \in \mathbb{R}^{N_w}$, that affect storage, i.e. $u(k) = [v(k) \ w(k)^T]^T$. The output of SM, denoted as $y(k)$, represents the stored energy at sampling instant $k$. Storage models are interfaced with the rest of the MES via ECM’s terminal input/output nodes. Equation (4.1) indicates how the interface between a SM and ECM is accomplished by setting SM input $v(k)$ equal to the negated node power of the associated terminal node of the ECM, in order to maintain the adopted sign convention. Thus, the power associated with the terminal input/output node of ECM to which a SM is connected is positive during discharging and negative during charging. Further, as SM are connected to terminal nodes, (3.5) ensures that simultaneous charging and discharging is prohibited.

$$v(k) = -P_{n_i}(k) \tag{4.1}$$

4.1 Example 2 - Adding Storage Model to ECM

To connect an energy storage device to the ECM within this framework, it is necessary to add an input/output node to the ECM. In this section, the example ECM introduced in Section 3.1 will be extended in order to accommodate an energy storage device. In particular, an additional input/output terminal node, $h3$, will be added to the ECM, connected to the sum node $h1$ to provide the point of connection for the storage device. The carrier type corresponding to the node to which the storage device is connected indicates the particular form of energy stored in the device; in this example the energy stored is heat (carrier type $h$). The updated configuration is depicted in Figure 4.1. The inclusion of $h3$ requires two additional continuous decision variables be included in the ECM, namely $P_{(h1 \rightarrow h3)}$ and
\( P_{(h_3 \rightarrow h_1)} \). Also, as described in Section 3, two additional binary decision variables, \( \delta_{(h_1 \rightarrow h_3)} \) and \( \delta_{(h_3 \rightarrow h_1)} \) are required to prevent simultaneous power flow in each direction. Thus, the arc flow \( P_{(h_1 \rightarrow h_3)} \) represents the charging power flow to the storage device and the arc weighting \( \eta_{(h_1 \rightarrow h_3)} \) is equal to the charging efficiency \( \eta_{h_3}^+ \) of the storage device connected to node \( h_3 \). Similarly, the arc weighting \( P_{(h_3 \rightarrow h_1)} \) represents the discharging power flow of the device and its arc weighting \( \eta_{(h_3 \rightarrow h_1)} \) is equal to the discharging efficiency \( \eta_{h_3}^- \) of the device which is equal to \( \eta^- = 1/\eta^+ \). Consequently, the absolute value taken by the terminal node \( h_3 \) is the amount of charging or discharging power flow dependant on the state of \( \delta_{(h_1 \rightarrow h_3)} \) and \( \delta_{(h_3 \rightarrow h_1)} \).

![Figure 4.1: Example Energy Conversion Model Graph with Additional Node, \( h_3 \), to Accommodate Storage Device](image-url)
Chapter 5

Prosumer Model (PM)

In the wider context of energy systems and the ‘smart grid’, the prosumer is defined as an energy user that both produces and consumes energy, i.e. producer-consumer. As such, within the context of the proposed modelling framework, the prosumer model (PM) represents a terminus of the MES where energy can be consumed and/or produced. Crucially, the model incorporates additional degrees of freedom allowing it to model dispatchable generation and/or IDSM.

The PM is represented by a set of components. Each component models a particular type of energy demand or production (e.g. local generator or RES). A particular PM is denoted, \( p_{j,n_i} \), where \( j \) is an index corresponding to the PM when more than one is connected to a terminal ECM node \( n_i \). The net energy demand of a particular prosumer at sampling instant \( k \), denoted as \( P_{p_{j,n_i}}(k) \) is given by (5.1).

\[
P_{p_{j,n_i}}(k) = \sum_{m=1}^{N_{G,p_{j,n_i}}} G_m(k) - \sum_{n=1}^{N_{L,p_{j,n_i}}} L_n(k)
\]

Where \( G_m(k) \geq 0 \) and \( L_n(k) \geq 0 \) represent prosumer’s \( m^{th} \) generating component and \( n^{th} \) load component, respectively. Also, \( N_{G,p_{j,n_i}} \) is the total number of energy generators within the PM and \( N_{L,p_{j,n_i}} \) is the total number of energy consumers. The sign of \( P_{p_{j,n_i}}(k) \) indicates whether it is producing or consuming, i.e. positive \( P_{p_{j,n_i}}(k) \) represents production and negative \( P_{p_{j,n_i}}(k) \) represents consumption. As such, end users that are either sole consumers or producers can be represented by (5.1) with \( G_m(k) = 0 \) or \( L_n(k) = 0 \) respectively.

As discussed in the following sub-sections prosumer components can be one of two types: fixed or flexible. In the remainder of this section only consumer type components of the PM will be considered since the potential flexibility of generation type components can be described by using the same approach.
5.1 Fixed Demand

Fixed demand components represent energy consumption that cannot be modified either in the rate or time of usage and, therefore, must be met completely and instantaneously. Examples include electrical demand for lighting and television as well as ‘on-demand’ heating to produce hot water drawn from a tap. Within the proposed framework decision variables are included to represent the consumption with their values fixed at each sampling instant by equality constraints.

5.2 Flexible Demand

Each flexible demand component (FDC) is characterised by a particular schedule of its associated decision variables over a future scheduling horizon with length equal to $N_H \in \mathbb{I}$ and divided into individual scheduling periods, denoted as $h \in \{0, 1, \ldots, N_H - 1\}$. Also, it is assumed that the values of the decision variables are constant during each scheduling period. FDC are classified using four different flexibility characteristics:

- **Shiftable**: Commencement of the demand is not fixed and can be moved within the scheduling horizon but no interruptions are allowed.

- **Adjustable**: Baseline energy demand can be exceeded or curtailed at particular sampling instants in the scheduling horizon.

- **Pliable**: The energy demand profile can be manipulated through the scheduling horizon, provided the total energy meets its requirement.

- **Interruptible**: Energy demand, once commenced at some fixed scheduling period, can be paused between particular sampling instants in the scheduling horizon.

Figure 5.1 shows how the four flexibility characteristics can be combined to represent different types of demand. In this report the model representing the combination of shiftable adjustable pliable interruptible (SAPI) demand and shown in the centre of Figure 5.1 will be described in detail. The other combinations in Figure 5.1 are special cases of the SAPI demand that result in simplified versions of the model. The model is based on [42], with novel extensions and developments including the adoption of slack decision variables and soft constraints to quantify mismatches to the baseline energy requirements for loads. In particular, the slack variables allow for an explicit link to comfort based adjustable loads that require additional dynamic models for their inclusion. Due to space limitations, this feature of the modelling framework is excluded from this report. However, an example can be found in the associated technical report [43].

In the proposed modelling framework the time-profile of the energy consumption for a particular demand $n$ is split into $N_n$ segments that, once started, cannot be interrupted.
Energy consumption of the \(i^{th}\) segment of demand \(n\) during time period \(h\) is denoted as \(l_{n,i}(h)\) and is given by:

\[
l_{n,i}(h) = \tilde{l}_{n,i}(h) + \Delta l_{n,i}^+(h) - \Delta l_{n,i}^-(h) \quad \forall n, i, h
\]

where \(\tilde{l}_{n,i}(h)\) is the energy consumption related to the baseline energy requirement for the \(i^{th}\) segment of demand \(n\) during time period \(h\), and \(\Delta l_{n,i}^+(h)\) and \(\Delta l_{n,i}^-(h)\) quantify the amount of energy for the demand \(n\) that has been curtailed or exceeded, respectively, for each segment \(i\) during each time period \(h\). The baseline energy requirement represents a preferred amount of energy to be consumed for each segment if adjustable flexibility is not required. The cumulative sum over the scheduling horizon of the baseline energy consumption for the \(i^{th}\) segment of demand \(n\) must remain within the baseline lower and upper bounds of the segment’s total energy requirement, denoted \(E_{n,i}\) and \(\bar{E}_{n,i}\) respectively:

\[
E_{n,i} \leq \sum_{h=0}^{N_H-1} \tilde{l}_{n,i}(h) \leq \bar{E}_{n,i} \quad \forall n, i
\]

In the cases of shiftable and/or interruptible and/or adjustable demand type the baseline segment time-profile is fixed and is not a degree of freedom. For these cases \(\tilde{E}_{n,i} = \bar{E}_{n,i} = E_{n,i}\) and (5.3) collapses to an equality constraint. Conversely, in the case of any demand that includes pliable type flexibility, the baseline time-profile is not pre-determined and represents an additional degree of freedom that is determined whilst respecting the constraint imposed on the baseline total energy requirement specified in (5.3). Furthermore, since the total energy requirement is fixed in the cases of shiftable and/or interruptible and/or pliable demand types, its curtailment/increase is not allowed and hence \(\Delta l_{n,i}^+(h) = 0 \quad \forall i, h\) and
\[ \Delta l_{n,i}(h) = 0 \quad \forall i, h. \] However, in the case of a flexible demand that combines adjustable flexibility with any of the other three types, \( \Delta l_{n,i}^+ \) and \( \Delta l_{n,i}^- \) represent degrees of freedom. Hence, in order to be able to represent any combination of the four demand types, degrees of freedom include the baseline energy consumption time-profile, given by \( l_{n,i}(h) \), as well as both curtailment and increase in energy demand, given by \( \Delta l_{n,i}^-(h) \) and \( \Delta l_{n,i}^+(h) \) respectively. In the case of adjustable flexibility characteristic both demand curtailment (\( \Delta l_{n,i}^-(h) \)) and increase (\( \Delta l_{n,i}^+(h) \)) are manipulated through appropriate cost-function selection, i.e. equally large penalties must be imposed on each variable under 'normal' operation, when neither demand curtailment or increase is required, and only one the responses (curtailment or increase) be incentivised at any given \( h \).

The resulting energy consumption of a particular demand \( n \) at a given time \( h \) is represented by \( L_n(h) \) and is related to the actual energy consumption of its energy segments in the following way:

\[
L_n(h) = \sum_{i=1}^{N_n} l_{n,i}(h) \quad \forall n
\]  

(5.4)

An additional three sets of binary decision variables are utilised to commence or to halt individual demand segments. The first set, \( \delta^p_{n,i}(h) \), contains segment processing variables which determine whether or not an energy segment \( i \) is to be processed during each time step \( h \). In particular, \( \delta^p_{n,i}(h) = 1 \) if and only if energy segment \( i \) is in process at time \( h \). The second set, \( \delta^c_{n,i}(h) \), is comprised of segment complete variables which indicate whether or not a particular energy segment has been completed. More specifically, \( \delta^c_{n,i}(h) = 1 \) if and only if segment \( i \) has been completed before time step \( h \). The third set, \( \delta^w_{n,i}(h) \), consists of segment waiting variables which indicate whether or not a demand has been halted between completed and yet-to-be-completed segments i.e. \( \delta^w_{n,i}(h) = 1 \) if and only if energy segment \( i - 1 \) has been completed and segment \( i \) is waiting to begin.

The eight sets of decision variables associated with the general SAPI FDC are listed in (5.5).
\begin{align}
\mathbf{l}_n &= \left[ l_{n,1}(0), \ldots, l_{n,1}(N_H - 1) \right]^T, \ldots, \left[ l_{n,N_n}(0), \ldots, U_{n,N_n}(N_H - 1) \right]^T \in \mathbb{R}_{\geq 0}^{N_n \times N_H} \quad (5.5a) \\
\mathbf{\delta}_n^p &= \left[ \delta_{n,1}^p(0), \ldots, \delta_{n,1}^p(N_H - 1) \right]^T, \ldots, \left[ \delta_{n,N_n}^p(0), \ldots, \delta_{n,N_n}^p(N_H - 1) \right]^T \in \mathbb{R}_{\geq 0}^{N_n \times N_H} \quad (5.5b) \\
\mathbf{\delta}_n^c &= \left[ \delta_{n,1}^c(0), \ldots, \delta_{n,1}^c(N_H - 1) \right]^T, \ldots, \left[ \delta_{n,N_n}^c(0), \ldots, \delta_{n,N_n}^c(N_H - 1) \right]^T \in \mathbb{R}_{\geq 0}^{N_n \times N_H} \quad (5.5c) \\
\mathbf{\delta}_n^w &= \left[ \delta_{n,2}^w(0), \ldots, \delta_{n,2}^w(N_H - 1) \right]^T, \ldots, \left[ \delta_{n,N_n}^w(0), \ldots, \delta_{n,N_n}^w(N_H - 1) \right]^T \in \mathbb{R}_{\geq 0}^{N_n \times N_H} \quad (5.5d) \\
\mathbf{\bar{l}}_n &= \left[ \bar{l}_{n,1}(0), \ldots, \bar{l}_{n,1}(N_H - 1) \right]^T, \ldots, \left[ \bar{l}_{n,N_n}(0), \ldots, \bar{l}_{n,N_n}(N_H - 1) \right]^T \in \mathbb{R}_{\geq 0}^{N_n \times N_H} \quad (5.5e) \\
\mathbf{\Delta l}_n^+ &= \left[ \Delta l_{n,1}^+(0), \ldots, \Delta l_{n,1}^+(N_H - 1) \right]^T, \ldots, \left[ \Delta l_{n,N_n}^+(0), \ldots, \Delta l_{n,N_n}^+(N_H - 1) \right]^T \in \mathbb{R}_{\geq 0}^{N_n \times N_H} \quad (5.5f) \\
\mathbf{\Delta l}_n^- &= \left[ \Delta l_{n,1}^-(0), \ldots, \Delta l_{n,1}^-(N_H - 1) \right]^T, \ldots, \left[ \Delta l_{n,N_n}^-(0), \ldots, \Delta l_{n,N_n}^-(N_H - 1) \right]^T \in \mathbb{R}_{\geq 0}^{N_n \times N_H} \quad (5.5g) \\
\mathbf{L}_n &= \left[ L_n(0), \ldots, L_n(N_H - 1) \right]^T \in \mathbb{R}^{N_H} \quad (5.5h)
\end{align}

The inequalities relating to the slack variables are provided in (5.6) where (5.6a) and (5.6b) determine the (hard) limits on the size of the energy requirement deviation variables \( \Delta l_{n,i}^+ \) and \( \Delta l_{n,i}^- \), with \( \Delta I_{n,i}^+ \) and \( \Delta I_{n,i}^- \) representing the corresponding upper bounds. Similarly, (5.6c) determine the minimum \( \underline{l}_{n,u} \) and maximum \( \bar{l}_{n,u} \) amount of energy that can be scheduled for any particular demand segment at a particular instant. In (5.6a)-(5.6c), the multiplication of the segment processing binary decision variables by the lower and/or upper bounds ensures that the corresponding continuous decision variable cannot be non-zero unless the particular segment \( i \) is currently under way. The length of a particular demand segment, given in terms of a number of scheduling periods, is constrained from below and above by \( \mathcal{N}_{n,i}^p \) and \( \mathcal{N}_{n,i}^p \) respectively.

\begin{align}
0 &\leq \Delta l_{n,i}^-(h) \leq \Delta I_{n,i}^-(h) \delta_{n,i}^u(h) \quad \forall n, i, h \quad (5.6a) \\
0 &\leq \Delta l_{n,i}^+(h) \leq \Delta I_{n,i}^+(h) \delta_{n,i}^u(h) \quad \forall n, i, h \quad (5.6b) \\
\underline{l}_{n,i}(h) \delta_{n,i}^u(h) &\leq l_{n,i}(h) \leq \bar{l}_{n,i}(h) \delta_{n,i}^u(h) \quad \forall n, i, h \quad (5.6c) \\
\mathcal{N}_{n,i}^p &\leq \sum_{h=0}^{N_H-1} \delta_{n,i}^p(h) \leq \mathcal{N}_{n,i}^p \quad \forall n, i \quad (5.6d)
\end{align}

The following inequalities imposed on segment processing and segment complete binary decision variables ensure that once an energy segment has commenced, it is not interrupted.
and must run to completion:

\[ \delta_{n,i}(h) + \delta_{n,i}^c(h) \leq 1 \]
\[ \forall n, i, h \]  
\[ \delta_{n,i}(h-1) - \delta_{n,i}^p(h) \leq \delta_{n,i}^c(h) \]
\[ \forall n, i \, \forall h \in \{1 : N_H - 1\} \]  
\[ \delta_{n,i}(h-1) \leq \delta_{n,i}^c(h) \]
\[ \forall n, i \, \forall h \in \{1 : N_H - 1\} \]  

(5.7a)

(5.7b)

(5.7c)

The correct sequencing of individual segments is ensured by the following inequality constraint imposed on segment processing and segment complete binary decision variables:

\[ \delta_{n,i}^p(h) \leq \delta_{n,i}^c(h) \]
\[ \forall n, h \, \forall i \in \{2 : N_n\} \]  

(5.8)

The segment waiting decision variables allow halting of the corresponding energy segment after its predecessor has been completed:

\[ \delta_{n,i}^w(h) = \delta_{n,i-1}^c(h) - (\delta_{n,i}^p(h) + \delta_{n,i}^c(h)) \]
\[ \forall n, h \, \forall i \in \{2 : N_n\} \]  

(5.9)

The halting period is bounded from below and above by \( N_n^w \) and \( \overline{N}_n^w \):

\[ N_n^w \leq \sum_{h=0}^{N_H-1} \delta_{n,i}^w(h) \leq \overline{N}_n^w \]
\[ \forall n, h \, \forall i \in \{2 : N_n\} \]  

(5.10)

Finally, (5.11) specifies the equality constraint which ensures that no segment of demand \( n \) can run at particular scheduling instants listed in vector \( T_n^p \), which correspond to the user time preferences for the demand:

\[ \delta_{n,i}^p(h) = 0 \]
\[ \forall n, i \, \forall h \in T_n^p \]  

(5.11)

As stated, the other categories of flexible demand shown in Figure 5.1 are special cases of the SAPI. In particular, the constraints described above take a particular form and not all the decision variable vectors in (5.5) are required, depending on the classification. For example, S, P and I demands do not require the slack variables \( \Delta l^+ \) and \( \Delta l^- \). Due to space limitations, the mathematical description of the fundamental demand types S,A,P and I are not covered in this report. However, the interested reader is referred to the technical report, which includes more detailed descriptions and illustrations associated with each of the different fundamental demand types [43].
5.3 Flexible Demand Fundamental Types

As stated in Section 5.2, the other categories of flexible demand shown in Figure 5.1 are special cases of the SAPI. In particular, the constraints described in Section ?? take a particular form depending on the classification. The four fundamental sets shown in Figure 5.1 (S,A,P and I) will be described in the following sub-sections. To assist their description, a vector $D_n \in \mathbb{R}_{\geq 0}$ is introduced whose elements correspond to a nominal energy consumption profile for a particular demand $n$, i.e. each element of $D_n$ corresponds to a desired amount of energy that is consumed by the demand during one scheduling period. Therefore, the number of elements in $D_n$, denoted as $\text{dim}(D_n)$, corresponds to the number of scheduling periods the demand will span if run continuously, which can also be thought of as the uninterrupted duration of the demand.

5.3.1 Shiftable Demand

Shiftable demand (SD) is energy consumption with a fixed un-interruptible profile where a degree of freedom exits as to when its commencement takes place. The model characteristics are shown in (5.12).

\begin{align*}
N^i_n &= \text{dim}(D_n) \tag{5.12a} \\
N^w_{n,i} &= 0 \quad \forall h, i \tag{5.12b} \\
N^p_{n,i} &= \frac{N^p_{n,i}}{N^p_{n,i}} = 1 \quad \forall i \tag{5.12c} \\
\Delta l^-(h) &= 0, \quad \Delta l^+(h) = 0 \quad \forall h, i \tag{5.12d} \\
E_{n,i} &= D(i) \quad \forall i \tag{5.12e}
\end{align*}

As in (5.12a), the number of energy segments for a SD correspond to the length of the demand profile. Therefore, the duration of each segment is equal to a single scheduling period as shown in (5.12c). Once the segment has begun processing, the energy demand of each segment is equal to the corresponding element of $D_n$ as is shown in (5.12e). The equality constraint (5.12d) means that the decision variable sets $l_n, \Delta l^+_n$ and $\Delta l^-_n$ are not required. Further, (5.12b) implies that (5.9) becomes $\delta_c^{n,i} = (\delta^p_n(h) + \delta^c_n(k)) \quad \forall h, \forall i \in \{2 : N^i_n\}$ and decision variable set $\delta_n^w$ is also redundant.

5.3.2 Adjustable Demand

The energy delivered to an adjustable demand (AD) during each scheduling period can be adjusted against some nominal energy requirement. However, no degree of freedom exists as to when commencement occurs, nor can the demand be interrupted. Therefore, if $h_c$ denotes the scheduling period when the adjustable demand comes on-line, the constraints of (5.13)
apply.

\[ N^i_n = \dim(D_n) \]  
\[ \delta^p_{n,1}(h_c) = 1 \]  
\[ \overline{N}_{n,i} = 0 \quad \forall h \quad \forall i \in \{2 : N^i_n\} \]  
\[ \underline{N}_{n,i}^p = \overline{N}_{n,i}^p = 1 \quad \forall i \]  
\[ E_{n,i} = D(i) \quad \forall i \]

Similar to shiftable demand, the number of energy segments for an AD corresponds to the number of elements in \( D_n \) (see (5.13a)), no delay between segments is permitted (see (5.13c)), segment energy requirements correspond to demand profile (see (5.13e)) and the duration of each segment is equal to a single sampling period (as in (5.13d)). However, the scheduling period during which the demand must commence is fixed by (5.13b) and the slack variables \( \Delta l^-_{n,i}(h) \) and \( \Delta l^+_{n,i}(h) \) can become positive. Due to constraint (5.13c), the decision variable set \( \delta^w_n \) is not required. For a particular demand, positive slack variables may represent a violation of an energy user’s comfort parameters. An example of this is provided in Section 5.4.

### 5.3.3 Pliable Demand

A pliable demand (PD) has a fixed overall energy demand and, as with adjustable demand, no degree of freedom exists as to when commencement occurs. However, there is no restriction on its particular profile through the scheduling horizon. As such, the constraints of (5.14) apply.

\[ N^i_n = 1 \]  
\[ \delta^p_{n,1}(h_c) = 1 \]  
\[ \overline{N}_{n,i} = 0 \quad \forall h \quad \forall i \in \{2 : N^i_n\} \]  
\[ \Delta l^-_{n,i}(h) = 0, \quad \Delta l^+_{n,i}(h) = 0 \quad \forall h, i \]

An PD needs only one energy segment as shown in (5.14a) and so no inter-segment delays are possible. The period of its commencement is fixed as in (5.14b) and, according to (5.14d) no mismatch with the nominal energy requirement is permitted. This means that decision variable sets \( \tilde{l}_n \), \( \Delta l^+_n \) and \( \Delta l^-_n \) are not required.

### 5.3.4 Interruptible Demand

Interruptible demand (ID) has a fixed energy consumption profile. Its commencement cannot be shifted in the scheduling horizon. It has a fixed energy demand but can be paused for
certain delays through the scheduling horizon. The relevant constraints are shown in (5.15).

\[ N_n^i = \text{dim}(D_n) \]  
\[ N_{n,i}^p = \overline{N}_{n,i}^p = 1 \quad \forall i \]  
\[ \delta_{n,1}^p(h_u) = 1 \]  
\[ \Delta l_{n,i}^-(h) = 0, \quad \Delta l_{n,i}^+(h) = 0 \quad \forall h, u \]  
\[ E_{n,i} = D(i) \quad \forall i \]  

As with the previous two categories, the energy segments correspond to the demand profile (i.e. (5.15a), (5.15b) and (5.15e)). The energy requirements of each segment must be met as required by (5.15d) but inter-segment delays can be positive. The ID model is simplified with comparison to the SAPI as the decision variable sets \( \tilde{l}_n \), \( \Delta l_{n,i}^+ \) and \( \Delta l_{n,i}^- \) are not required.

It is proposed, that by further adapting the constraints of the flexible demand model, the other sub-categories seen in Figure 5.1 can be mathematically defined.

### 5.3.5 Receding Horizon Constraints Update

If the constraints introduced in the previous section, which model the allowable behaviour of a controllable demand, are part of an optimisation problem solved repeatedly by a predictive controller, it is then necessary to account for the effect of the control actions determined by the solution of the previous optimisation (at \( k - 1 \)) on the constraints of the current optimisation (\( k \)). This section describes the conditions for when particular constraints are updated. In order to facilitate this discussion the notation \( (h|k) \) is adopted which corresponds to the scheduling period \( h \) in the schedule devised at control step \( k \).

If, after solving the optimisation at a particular control step \( k \), the controller assigns energy to any of the demand segments at \( (0|k) \) then the overall energy requirement of the segment will be diminished at the next sampling instant, and so the energy requirement of each segment becomes a function of the sampling instant i.e.

\[ E_{n,i}(k) = E_{n,i}(k - 1) - l_{n,i}(0|k - 1) \quad \forall n, i, k \]  

where \( E_{n,i}(k) \) denotes the energy requirements of segment \( i \) of demand \( n \) at control step \( k \) and \( l_{n,i}(0|k - 1) \) denotes the energy assigned during the first scheduling period \( (h = 0) \) after the schedule was devised at the previous sampling instant \( k - 1 \). Also, note constraint (5.6d). If, for example, an energy segment may not be active longer than \( \overline{N}_{n,i}^p \) periods and is processed at \( (0|k - 1) \) then at \( k \), the segment must not be active longer than \( \overline{N}_{n,i}^p - 1 \) periods. Similarly, if the energy segment must last at least \( \overline{N}_{n,i}^p \) periods and is processed at \( (0|k - 1) \) then the minimum requirement reduces to \( \overline{N}_{n,i}^p - 1 \) at \( k \). Therefore, as in (5.16),
the timing limits on particular energy segments become a function of $k$:

$$N_{n,i}^p(k) = \begin{cases} N_{n,i}^p(k-1) - 1, & \text{if } \delta_{n,i}^p(0|k-1) = 1 \\ 1, & \text{if } N_{n,i}^p(k-1) = 1 \\ N_{n,i}^p(k-1), & \text{otherwise} \end{cases} \tag{5.17a}$$

$$N_{n,i}^w(k) = \begin{cases} N_{n,i}^w(k-1) - 1, & \text{if } \delta_{n,i}^w(0|k-1) = 1 \\ 1, & \text{if } N_{n,i}^w(k-1) = 1 \\ N_{n,i}^w(k-1), & \text{otherwise} \end{cases} \tag{5.17b}$$

Note that the second case in (5.17a) and (5.17b) ensures that $N_{n,i}^p$ can never be less than 1, which would result in an infeasibility.

The upper and lower bounds of the inequalities in (??) also become a function of $k$ as, if a segment is waiting during (0|k − 1), the allowable time delays for the segment are reduced. This is shown in (5.18):

$$N_{n,i}^w(k) = \begin{cases} N_{n,i}^w(k-1) - 1, & \text{if } \delta_{n,i}^w(0|k-1) = 1 \\ 0, & \text{if } \delta_{n,i}^w(0|k-1) = 0 \\ N_{n,i}^w(k-1), & \text{otherwise} \end{cases} \tag{5.18a}$$

$$N_{n,i}^w(k) = \begin{cases} N_{n,i}^w(k-1) - 1, & \text{if } \delta_{n,i}^w(0|k-1) = 1 \\ 0, & \text{if } \delta_{n,i}^w(0|k-1) = 0 \\ N_{n,i}^w(k-1), & \text{otherwise} \end{cases} \tag{5.18b}$$

If a segment processing during (0|k − 1) has satisfied its energy requirements, then to ensure the segment is not restarted within the scheduling horizon of $k$ (if it were being processed during (0|k − 1) it would not have been marked complete) it is necessary to set $\delta_{n,i}^c$ according to:

$$\delta_{n,i}^c(0|k) = \begin{cases} 1, & \text{if } \delta_{n,i}^p(0|k-1) = 1 \text{ and } \hat{E}_{n,i}(k) = 0 \\ 0, & \text{otherwise} \end{cases} \tag{5.19a}$$

Finally, the time preference vector $T_{n}(k)$ becomes a function of time and the values of its elements, denoted $t_i$, are decremented as $k$ increases until they reach 0 at which point they are removed from the vector altogether:

$$T_{n}^p(k+1) = \begin{cases} t_i = t_i - 1, & \text{if } t_i - 1 \geq 0 \\ T_{n}^p(k) \setminus \{t_i\}, & \text{otherwise} \end{cases}, \quad \forall t_i \in T_{n}^p(k) \tag{5.20}$$

The scheduling of a particular controllable demand is only incorporated into the system model, when it is brought on-line. This could be due to the interaction of an energy user (e.g. a device is switched on) or the demand may be repetitive and will automatically come back on-line once its energy requirement has been satisfied and the maximum duration it may take to complete has elapsed (e.g. an energy consuming process that repeats daily).
5.4 Comfort Based Flexible Demands

As was indicated in Section 5.2, certain flexible demand models within the framework can correspond to energy usage that is directly linked to exogenous variables related to end-user comfort. For example, temperature, ventilation air flow or concentration of CO\(_2\) in a building. Consideration of such comfort variables requires the employment of explicit models that describe their relationship to energy use which must then be linked to the modelling framework. In this section, an example considering temperature as the exogenous variable and a simple dynamic model will be used to describe its relationship to heat. The evolution of temperature within a particular room is described by the following two state continuous time state-space model:

\[
\begin{bmatrix}
    \frac{d}{dt} T_w(t) \\
    \frac{d}{dt} T_r(t)
\end{bmatrix} = \begin{bmatrix}
    \frac{1}{(R_w C_w)} & \frac{1}{(R_w C_r)} \\
    \frac{1}{(R_r C_w)} & \frac{1}{(R_r C_r)}
\end{bmatrix} \begin{bmatrix}
    T_w(t) \\
    T_r(t)
\end{bmatrix} + \begin{bmatrix}
    \frac{1}{(R_C C_r)} & \frac{1}{(R_C C_r)} & \frac{1}{(R_C C_r)} \\
    0 & 0 & 0
\end{bmatrix} \begin{bmatrix}
    T_a(t) \\
    q_i(t) \\
    q(t)
\end{bmatrix}
\]

Model variables and parameters correspond to a 2R2C electrical equivalent representation of the system which is depicted in Figure 5.2. In particular, \(T_w\) and \(T_r\) are the system states with the former representing the temperature of the building mass (i.e. the walls) and the latter (the output state) representing the temperature of the air inside the room(s). Also, \(R_w\) and \(R_r\) represent the thermal resistance of the wall and air mediums, respectively, with \(C_w\) and \(C_r\) relating to the corresponding thermal capacities. Three inputs to the system are considered. The first, \(T_a\), is an uncontrollable input and results in heat flow due to temperature difference with external/ambient environment. The second, \(q_i\), is another uncontrollable input which represents the internal heat contributions of the building such as occupant body heat or heat producing equipment. The final input, \(q\), is the controllable input which represents the heat produced by the heating system and which, relating to the flexible demand model, is equal to \(L_n\). Assuming zero-order hold for the input vector, the continuous time model in (5.21) can be discretised according to a sampling time \(t_s\) and written in the following form:

\[
T(k+1) = A_d T(k) + B_d q(k)
\]

where \(A_d\) and \(B_d\) are the discrete time state transition and input matrices respectively, obtained in the following way:

\[
A_d = \mathcal{L}^{-1}\{sI - A_e\}_{s = t_s}
\]

\[
B_d = \left(\int_{\tau = 0}^{t_s} \mathcal{L}^{-1}\{sI - A_e\}_{s = t_s} d\tau\right) B_e
\]

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5.4.1 Comfort-based Pliable Demand

The demand related to temperature introduced in the previous section has \textit{pliable} flexibility if temperature is allowed to fluctuate between some upper and lower comfort levels set by the energy user, relaxing the instantaneous energy matching constraint. In particular temperature must satisfy the inequalities in (5.25) where $T_r(k)$ and $\bar{T}_r(k)$ correspond to the upper and lower temperature comfort limits.

\begin{align*}
T_r(k) &\leq T_r(k) \leq \bar{T}_r(k) \quad \forall k 
\end{align*}

Let each period in the scheduling horizon correspond to a single energy segment, i.e. $N^i_u = N_H$ and $N^p_i = N^p_i = 1 \quad \forall i$. Then, the nominal minimum and maximum energy requirements relate to the temperature in the following way:

\begin{align*}
E_{n,i+1} &= (T_w(k)a_{d(2,1)} + T_r(k)a_{d(2,2)} - \bar{T}_r(k+1))C_a && (5.26a) \\
E_{n,i+1} &= (\bar{T}_r(k+1) - T_w(k)a_{d(2,1)} + T_r(k)a_{d(2,2)})C_a && (5.26b)
\end{align*}

where $a_{d(i,j)}$ denotes the element of the discrete time state transition matrix $A_d$ with row index $i$ and column index $j$. As is shown, the baseline minimum and maximum energy requirements for each segment/scheduling period are a function of the temperature limits and dynamics.

5.4.2 Comfort-based Adjustable Demand

If it is determined that $T_r$ must follow a particular pre-defined profile (i.e. $T_r = \bar{T}_r = T_r$) then a temperature comfort-based demand has \textit{adjustable} flexibility since limits on temperature are not, physically speaking, hard constraints.

Let $\tau^+$ and $\tau^-$ denote slack variables in the optimisation which represent the amount by which the nominal temperature is exceeded ($\tau^+$) or not met ($\tau^-$). The constraints (5.25)
can then be written as:

\[ \begin{align*}
T_r(k) - \tau^+(k) &\leq \overline{T}_r(k) \\
- T_r(k) - \tau^-(k) &\leq - \underline{T}_r(k) \\
\tau^+(k) &\geq 0 \\
\tau^-(k) &\geq 0
\end{align*} \] (5.27a-b-c-d)

The slack variables are related to the FDC in the following way:

\[ \begin{align*}
\Delta l^-(h) &= \tau^-(k)C_r \\
\Delta l^+(h) &= \tau^+(k)C_r
\end{align*} \] (5.28a-b)

In order to prevent the mathematically feasible but practically impossible situation where both \( \tau \) variables become positive at the same sampling instant, it is necessary to employ binary decision variables (in a similar manner seen in Section 3) to ensure mutual exclusivity, i.e.

\[ \begin{align*}
\delta \tau^+(k) &= 1 \iff \tau^+(k) > 0 \\
\delta \tau^-(k) &= 1 \iff \tau^-(k) > 0
\end{align*} \] (5.29a-b)

where \( \delta \tau^+ \) and \( \delta \tau^- \) are the binary decision variables for upper and lower violations of the soft constraints. The conditions of (5.29) are implemented by including the constraints of (5.30).

\[ \begin{align*}
\varepsilon \delta \tau^+(k) &\leq \tau^+(k) \leq \tau^+ \delta \tau^+(k) \\
\varepsilon \delta \tau^-(k) &\leq \tau^-(k) \leq \tau^- \delta \tau^-(k) \\
\delta \tau^+(k) + \delta \tau^-(k) &\leq 1
\end{align*} \] (5.30a-b-c)

where \( \tau^+ \) and \( \tau^- \) are upper bounds on the values of \( \tau^+ \) and \( \tau^- \) respectively. In a similar fashion to Section 3 the lower bound on the \( \tau \) slack variables is a small number \( \varepsilon \) (machine precision), to make infeasible the case where \( \tau = 0 \) and \( \delta \tau = 1 \). Note that if, for a particular situation, it is not necessary to distinguish between upper and lower constraint violations then \( \tau^+(k) = \tau^-(k) \) can be considered true and the binary decision variables are not required.

### 5.4.3 Comfort Based Adjustable Pliable Demand

Combining the comfort-based models in the previous two sub-sections yields a adjustable pliable demand where the temperature is allowed to vary between the limits of \( \overline{T}_r(k) \) and \( \underline{T}_r(k) \) under ‘normal’ operation but suitably chosen cost-function coefficients can facilitate demand response resulting in the possibility of the temperature comfort bounds being violated if necessary/incentivised.
Chapter 6

Case Study

The MES used to demonstrate the modelling framework presented in Sections 2-5 is schematically shown in Figure 6.1. It represents energy infrastructure for three buildings located at the University of Manchester (UoM) North Campus, namely Barnes Wallis Building (BWB), Sackville Street Building (SSB) and Ferranti Building (FB). The transportation networks interconnecting the energy infrastructure of the three buildings is assumed to be non-dynamic and, therefore, is considered as a single EEZ introduced in Section 2. It can be seen that multiple facets of MES are included within the EEZ: multiple energy vectors (electricity, gas, heat and cooling), multi-generation (CHP), multi-mode operation (heat pump), energy storage (battery and thermal store), bi-directional flow (grid connection and heat network between FB and BWB), prosumer (FB both consumes electricity and produces it from a PV installation) and IDSM with a shiftable-interruptible (SI) electricity demand in SSB, and an adjustable-pliable (AP) heat demand in FB. Not all of the technology shown in Figure 6.1 currently exists within the actual buildings. In particular, the CHP, heat pump, heat storage, chiller and FDC have all been added to the simulation for demonstration purposes.

The graph diagram that represents the ECM is shown in Figure 6.2. With reference to Section 3, the terminal nodes are represented as squares, sum nodes as circles, transmitter nodes as triangles and switch nodes as irregular pentagons. The alphabetic characters in the node identifiers seen in Figure 6.2 relate nodes to specific energy carriers (i.e. e is electricity, g is gas, c is cooling and h is heat) which, combined with the numeric characters, generate unique indices for each node. For example, employing the node identifiers in Figure 6.2, the arc power flow variable from node g2 to c1 is denoted $P_{(g2\rightarrow c1)}$ and the associated arc weighting is denoted $\eta_{(g2\rightarrow c1)}$. In Figure 6.2, where conversion devices exist in the ECM and consequently the arc weightings are not equal to 1, the $\eta$ symbols associated with particular devices have been labelled. For example, $\eta_{(g2\rightarrow c1)} = \eta_{gc}^C$ represents the gas-to-cooling conversion efficiency of the chiller. The values associated with these conversion factors used for the simulations can be seen in Table 6.1. The full ECM requires a single equation for each non-transmitter node and multiple equations for each transmitter node corresponding to the number of its outgoing arcs. Therefore, 21 (19+2) equations represent
The nodes $h_5$ and $e_4$ in Figure 6.2 are the dedicated terminal nodes for the battery and thermal storage devices with the weightings of their incoming and outgoing arcs representing the respective charging and discharging efficiencies of the particular devices. For the case study, simple single state models have been adopted to represent the storage devices. Hence, the $A, B, C$ and $D$ state-space matrices introduced in Section 4 become scalars and equal to
$S_{n_i}, 1,1$ and 0 respectively. For each storage $S_{n_i} < 1$ represents the standby storage efficiency of the device connected to node $n_i$. Also, $u(k) = Q_{n_i}(k)$ is a scalar quantity representing the charging/discharging power for each storage device and $y(k) = R_{n_i}(k)$ is equal to the stored energy in each device. The particular parameter values for the case study are displayed in Table 6.2.

Table 6.2: Additional Simulation Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{g_{2 \rightarrow 3}}$</td>
<td>CHP input capacity</td>
<td>340 kW</td>
<td>$P_{e_{2 \rightarrow 3}}$</td>
<td>HP input capacity</td>
<td>20 kW</td>
</tr>
<tr>
<td>$H_{5}/R_{5}$</td>
<td>Min./max. heat energy storage</td>
<td>30/150 kWh</td>
<td>$Q_{h_{5}}/Q_{h_{5}}$</td>
<td>Max. heat store charge/discharge</td>
<td>100 kW</td>
</tr>
<tr>
<td>$E_{4}$</td>
<td>Min./max. battery storage</td>
<td>27/180 kWh</td>
<td>$Q_{e_{4}}/Q_{e_{4}}$</td>
<td>Max. battery charge/discharge</td>
<td>225 kW</td>
</tr>
<tr>
<td>$S_{h_{5}}$</td>
<td>Heat storage standby loss factor</td>
<td>0.95</td>
<td>$S_{e_{4}}$</td>
<td>Battery standby loss factor</td>
<td>0.96</td>
</tr>
</tbody>
</table>

6.1 Simulation Data

The UoM Estates department operates a building management system (BMS) that, among other functions, logs energy usage data at half-hourly intervals. Data relating to a particular energy vector for a particular building may be logged using a variety of physical meters (e.g. an electricity meter might be present at each switchboard) but the data logging software includes *virtual meters* for each building that logs total energy consumption for each separate energy vector. Therefore, data was collected for the total electricity and heat demands of both buildings, in addition to the amount of energy produced by the PV installation on FB. Because the heat pump and chiller were simulated additions to the case study, it was necessary to synthesise a data set for the cooling demand. Also, to match the frequency of the data logging, the duration of each scheduling period $h$ is set equal to each sampling period $k$ to result in a half-hourly sampling frequency.

For the purposes of the case study, it was assumed that 60% of heat energy consumed in the FB was fixed with the remaining 40% assumed to be adjustable and pliable (AP). A set of energy demand segments was then computed using the 40% of AP demand by summing the energy consumption over every 2hr period. Hence the total *baseline* amount of heat energy delivered to the building remains equal to the original demand profile but the controller has freedom to manipulate that portion of the energy consumption profile according to its objective. This configuration could represent a situation where thermal modelling of a building has previously been conducted and, using historic data, it produced a predicted *baseline* energy consumption profile over bi-hourly periods. It is also assumed that the AP heat demand, denoted $L_{AP}$, is a continuously running demand. Therefore, the minimum and maximum number of scheduling periods during which each energy segments must be processed are both equal to 4 (2hr time interval) i.e. $N_{AP,i}^{P} = N_{AP,i}^{P} = 4 \forall i$. The maximum amount of energy the AP demand can consume over one sampling period is set to
300 kWh (i.e. $\bar{U}_{AP,i}(k + h) = 300 \forall i, k, h$). Also, the maximum deviation from the baseline requirement during a sampling period is ±20 kWh.

The shiftable and interruptible (SI) electricity demand, denoted $L_{SI}$, represents the energy consumption of a machine located within SSB that has a fixed demand profile each time it runs. The example profile used for the case study is equal to $D_{SI} = [E_{SI,1}, E_{SI,2}, E_{SI,3}, E_{SI,4}, E_{SI,5}, E_{SI,6}, E_{SI,7}]^T = [5, 15, 10, 15, 15, 5]^T$. Each element of $D_{SI}$ has kWh units. Also, $N_{SI,i} = N_{SI,i}^p = 1 \forall i$. Since the demand is interruptible, between any energy segment in the profile the controller is able to delay the commencement of the next segment by up to a maximum of 3 scheduling periods (i.e. $N_{SI,i}^w = 3 \forall i$). Further, as the demand is shiftable, the particular period during which the machine commences its operation can be determined by the controller.

Other parameter values related to the case study are presented in Table 6.2 along with their units where applicable.

### 6.2 Control Scheme

In order to exemplify the control-oriented modelling framework presented in this report, a certainty equivalent model predictive controller (CEMPC) is applied to the case study introduced earlier in this section. The CEMPC is introduced in this section whilst the results of its application are provided in Section 6.3.
The CEMPC solves the deterministic multi-period optimisation problem given in (6.1) during each sampling period $k$. The problem (6.1) is defined over a user-specified future control horizon, which is without loss of generality set equal to the scheduling horizon, $N_H$, introduced in Section 5.2. For the purposes of the case study a 48 period (24hr) control horizon is employed, i.e. $N_H = 48$. The objective function $J(k)$ is predominantly economic, aiming to minimise the cost of purchasing energy from external entities, e.g. the power grid. Additional terms are included in the objective function that relate to control performance as well as incentives and/or penalties imposed by external entities. The solution of the optimisation problem produces a schedule defined over the length of the control horizon for the system’s manipulating variables, which are namely the amounts of electrical energy and gas purchased from the external entities, the dispatch of energy to various conversion devices, as well as the schedule of the flexible demand characterised by the various binary decision variables introduced in Section 5 over the length of the horizon. The control actions determined by the schedule for the current sampling period are applied to the system which then evolves based on its inherent dynamics and any associated disturbances before the state of the system is observed again at the next sampling instant and the optimisation problem is resolved.

Since the decision variables specified in (5.5) are calculated repeatedly during each sampling period with the scheduling horizon remaining constant and defined over the next/future $N_H$ scheduling periods, the component elements of the decision variable vectors in (5.5) during online control implementation are time-indexed with $k + h$ instead of $h$.

Furthermore, if the constraints introduced in the previous section, which model the allowable behaviour of a FDC, are part of an optimisation problem solved repeatedly by a predictive controller (i.e. in an MPC scheme), it is then necessary to account for the effect of the control actions determined by the solution of the previous optimisation (at time $k - 1$) on the constraints of the current optimisation (at time $k$). For example, future energy requirements of particular segments reduce, based on how much energy they are scheduled at a given control instant. An interested reader can find a detailed description of the constraint updates in the associated technical report. [43].

Following standard receding horizon implementation, prediction horizon length $N_H$ remains constant throughout the simulation.

$$
\min J(k) = \sum_{h=0}^{N_H-1} [V_{cost}(k+h) + V_A(k+h) + V_{smooth}(k+h)]
$$

(6.1)

$$
s.t. (3.1) - (5.4), (5.6) - (5.11)
$$

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where:

\[
V_{\text{cost}}(k + h) = \left[ P_{(e_1 \to e_2)}(k + h) \pi_{e}^{\text{buy}}(k + h) - P_{(e_2 \to e_1)}(k + h) \eta_{(e_2 \to e_1)} \pi_{e}^{\text{sell}}(k + h) + P_{g1}(k + h) \pi_{g}^{\text{buy}}(k + h) \right]
\]

\[
V_{\text{A}}(k + h) = \sum_{i=1}^{N_n} \left[ \Delta l_{AP,i}^+(k + h) \lambda_{A}^+(k + h) + \Delta l_{AP,i}^-(k + h) \lambda_{A}^-(k + h) \right]
\]

\[
V_{\text{smooth}}(k + h) = \left[ L_{SI}(k + h + 1) - L_{SI}(k + h) \right]^2 \lambda_{SI}^+(k + h)
+ \left[ L_{AP}(k + h + 1) - L_{AP}(k + h) \right]^2 \lambda_{AP}^+(k + h)
\]

In (6.1), \( V_{\text{cost}} \) is the economic term of the objective function with \( \pi_{e}^{\text{buy}} \) and \( \pi_{e}^{\text{sell}} \) representing the purchase price and received revenue for selling electrical energy, respectively. Similarly, \( \pi_{g}^{\text{buy}} \) represents the cost of purchasing gas.

The term \( V_{\text{A}} \) relates to the adjustable portion of demand in the system. Variables \( \Delta l_{AP,i}^+ \) and \( \Delta l_{AP,i}^- \) are the slack variables (introduced in Section 5.2) for the AP FDC. Also, \( \lambda_{A}^+ \) and \( \lambda_{A}^- \) represent a penalty or incentive for respectively exceeding \( (\Delta l_{AP,i}^+ > 0) \) or not meeting \( (\Delta l_{AP,i}^- > 0) \) the apparent energy requirement of the AP energy segment profile. These parameters can be used to implement the provision of services to system operators e.g. frequency response, demand response etc. During ‘normal’ operation, where no additional service is being provided, \( \lambda_{A}^+ \) and \( \lambda_{A}^- \) take positive values, greater than the maximum \( \pi_{e}^{\text{buy}} \) or \( \pi_{g}^{\text{buy}} \). Hence, unless it is necessary to meet an energy balancing constraint imposed on the entire EEZ, the deviations will remain equal to zero. If, however, it becomes desirable to deviate from the baseline energy requirement during some period(s) in the control horizon then the corresponding \( \lambda_{A}^+(k + h) \) or \( \lambda_{A}^-(k + h) \) will become negative, thus influencing the optimisation to maximise the corresponding deviation variable during the period i.e. if curtailment is required then the corresponding \( \lambda_{A}^- \) will be negative and if an increase in demand is required the corresponding \( \lambda_{A}^+ \) will be negative. Also, it is possible to incentivise the operation of a particular energy converter (e.g. the heat pump) by changing the cost coefficient, related to its corresponding variable, into a negative value when, at the same time, the demand response is required.

Finally, the \( V_{\text{smooth}} \) term imposes a control performance penalty on the difference in energy consumption of the FDC between consecutive periods in the control horizon. This ensures that in the presence of multiple optimal scheduling solutions of the FDC, a unique solution that produces the smoothest possible energy consumption profile is selected. The control performance penalty is included based on the assumption that it is typically desirable to avoid
large ramp rates applied to manipulating variables. To ensure that the control performance penalty does not significantly influence the major economic objective of the controller the corresponding objective function coefficients are set such that \( \lambda_{SI}, \lambda_{AP} \ll \pi_{buy}^e, \pi_{buy}^e \).

The certainty equivalence of the CEMPC means that future values of uncertain variables, namely fixed energy demand and PV production, are treated as deterministic values in the corresponding MPC optimisation problem. These future values are obtained by utilising an adaptation of a Self Organising Map (SOM) neural network architecture [44]. Note that exploring forecasting techniques is beyond the scope of this report and the SOM was chosen due to its simplicity in implementation. The SOMs are trained offline using historical data and then, during online implementation, they utilise at each time \( k \) the previous \( N_H - 1 \) observations, along with the observation at \( k \), to predict the future \( N_H - 1 \) values of each uncertain variable. These future values of the uncertain variables are then used to solve the deterministic MPC optimisation problem in order to obtain the optimal schedule of control inputs. For the particular case study introduced in Section 6, collected data spanning the period March 2016 - April 2017 was used to train the SOMs to predict future values of energy demand and PV production.

Due to the presence of the binary decision variables in the EEZ model as well as the quadratic term (6.2c), the corresponding MPC optimisation problem is a mixed integer quadratic program (MIQP) which is solved using the commercial solver CPLEX at each sampling period. For the case study in this paper the resulting optimisation problem is defined with 9790 decision variables of which 3312 are binaries. The average computation time to solve the optimisation problem during each control period is 32 seconds using a general purpose laptop computer.

## 6.3 Results

Simulations were run, using data from a random week in the autumn of 2017. In order to highlight particular controller responses, an artificial time of use tariff was used for the cost of purchasing electricity from the grid. Fixed prices were used for the purchasing of gas and the export of electricity. The daily price profiles are shown in Figure 6.3. The fixed

![Energy Price Profile for One Day](image)

Figure 6.3: Energy Price Profile for One Day
electricity demand for SSB along with the SI demand are shown in Figure 6.4a. It can be seen that the controller schedules the SI demand to run during the period when electricity is least expensive (between 23:00 and 05:30). Also, as there is a slight price increase between 03:00 and 04:00, the controller interrupts the demand for 1.5hrs to avoid buying electricity at the slightly elevated price. Figures 6.4b and 6.4c show the battery charging/discharging profile and the stored energy in the battery respectively. The battery is charged through the periods of relatively inexpensive electricity before being discharged once the price is increased at 03:00, 05:30 and 16:30. Figure 6.5 shows the heat demand of each building (6.5a) along with the CHP operation (6.5b) during an Autumn weekday. As the CHP is sized for base load it runs at capacity throughout the day. For most of the day, the heat produced by the CHP is diverted to SSB as does it not benefit from a heat interconnection with either the heat pump or energy storage. Figure 6.6 shows the heat dispatch for the BWB (6.6a) along with the heat energy storage profile (6.6b). The BWB heat dispatch plot shows the HP being operated at maximum capacity until 07:30. It also shows that an excess in heat production is sent to the FB building where the heat store is situated. This is the case except between 05:00 - 05:30 when, coinciding with the peak in heat demand in BWB, heat flow between BWB and FB is reversed and heat storage discharge is sent from FB to BWB which is combined with the HP heat production to meet the increase in heat demand. Between 07:30 and 12:00 the HP operation is reduced in accordance with the reduction in FB heat demand as, combined with the heat store discharge, it is sufficient to meet the demands of both FB and BWB. The HP is again operated at maximum capacity from 12:00 to 16:30 and heat is largely exported to FB charge the store, except at 12:30 when another relative peak in BWB heat demand causes heat storage discharge and heat to flow from FB to BWB. At 16:30 the electricity price reaches its premium and, after discharging the heat store, it is
more economic to run the boiler in BWB. This also coincides, as shown in Figure 6.5b, with some of the CHP heat output being diverted from SSB to FB, which avoids the use of FB boiler.

Figure 6.6: BWB Heat Dispatch and Ferranti Heat Store Operation

In order to demonstrate the features of the AP heat demand in the FB two separate simulations were run. In the first, no incentive was provided at any point for violating the baseline energy requirement. In the second, an incentive was provided to increase the electricity demand of the heat pump between 17:30 and 19:30. Figure 6.7a shows the fixed
heat demand for the FB along with the AP demand profile for both cases (with and without the incentive). Figure 6.7b shows the BWB heat dispatch for the case when the incentive is applied (the equivalent plot for the case without the incentive is shown in the upper plot of Figure 6.6a). Finally, Figure 6.7c shows the value of the exceeded energy requirement deviation variable $\Delta l_{AP,1}^-$. It can be seen that, due to the incentive, additional energy consumption is scheduled during the incentivised period. In particular, as the maximum allowable deviation from the baseline energy requirement over any sampling period is $\pm 20$ kWh, the controller schedules a maximum increase of 80 kWh over the corresponding period. This additional consumption can also be seen in Figure 6.7a and 6.7b where the HP is operated during the incentivised period and heat is exported to FB. This is in contrast to the ex-incentive case where only the boiler is operated during this period (see Figure 6.6a). Simulations were also run for the scenario where the CHP plan was not present in the system for both Summer and Autumn days. The average cost savings for the two days were approximately 50%.

Figure 6.7: Ferranti AP Heat Demand with Barnes Wallis Heat Dispatch
Chapter 7

Conclusion

This report has presented a novel modelling tool for the representation of multi-energy systems in a predictive control setting. The core value of this contribution lies in the generalised approach which encompasses the various flexibility aspects of MES which can be operated and optimised by an automatic controller. The tool is capable of representing converter topologies of arbitrary complexity containing multiple energy vectors, as well as multi-directional energy flow, multi-generation devices, multi-mode devices, a wide range of controllable producers/consumers, energy storage and dynamic loads. The modelling tool was used to simulate the optimal on-line energy management for a group of university buildings. Where possible, actual energy consumption data was used to ensure a representative case-study. Simulation results highlighted the (economically) desirable responses of the CEMPC.

Future work will focus on utilising the modelling tool to develop sophistication in the control algorithm with regards the uncertain variables.
Bibliography


