MATH 44041/64041 Applied Dynamical Systems, 31 October 2016

Mid-term Test (worth 10% of the course marks)

Attempt **all** three questions. Duration: 50 minutes

1. [30marks] (a) Solve the coupled equations

$$\dot{u} = 4v, \quad \dot{v} = 4u$$

with initial condition $u(0) = u_0$ and $v(0) = v_0$.

(b) If $u_0 = 1$, for what initial values of v_0, does the solution tend to the origin at t goes to $+\infty$?

2. [30marks] There are two straight lines on the phase plane that are invariant under the system

$$\dot{x} = y^2 - x - 1, \qquad \dot{y} = xy.$$

Find these two invariant sets.

3. [40marks] Consider the system

$$\dot{x} = -y - xy, \qquad \dot{y} = x - y.$$

(a) Find all stationary points of the system.

(b) Determine the linear type of the stationary points and sketch the local solutions near each stationary point.

(c) Sketch a consistent global phase portrait for the system.

END OF QUESTION SHEET