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Spatial Statistics of Stochastic Fiber Networks

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From the known statistics of fiber-fiber contacts in random fiber networks, an analytic estimate is obtained for the variance of local porosity in random fiber suspensions and evolving filtrate networks. The variance of local porosity, and hence the distribution of projected areal density, seem to depend on fiber geometry only through the cube of mean diameter. Also, the coefficient of variation of local flow rate perpendicular to the plane of the pad is, to a first approximation, independent of the mode of flow. Analytic estimates are obtained also for the effect of fiber clumping on the variance of local porosity of pads for small inspection zones.

KEY WORDS: 3D fiber networks; porosity; fiber suspensions; clumping; fiber geometry; network geometry.

1. INTRODUCTION

It is known that the pore size distributions of stochastic fiber networks like paper and nonwoven fabrics are skew and are often of lognormal or gamma shape.[1, 2] Approximate analytic results exist for the cases of random and non-random lines in a plane, where the distributions are represented by structures of rectangles with side lengths drawn from negative-exponential distributions[1] or gamma distributions.[2, 3] These approximations are meaningful because in a planar random array of infinite lines the expected number of sides per polygon is four and the distribution of inter-crossing distances is negative-exponential;[4] the gamma distribution seems to provide the appropriate generalization to non-random situations.[2, 3, 5] Moreover, such approximations give agreement with

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classification schemes based on hydrodynamic measurements of the pore size distribution in real stochastic fiber networks such as paper,[1,2] which are almost planar.

A widely used structural characteristic of planar networks is the distribution of local averages of areal density. From the Central Limit Theorem we expect this to follow a normal distribution with standard deviation decreasing with increasing inspection zone size. This is confirmed by analysis of digitized radiographic images of the areal density distribution in paper and non-woven textiles. The areal density distribution controls also the small-strain behaviour in stochastic bonded fiber networks of elastic fibers.[6]

2. MODELLING

By a fiber we shall mean a rigid rod of length λ and diameter ω. By a stochastic fiber network we shall mean the result of some stochastic process of placing a fixed number of fibers in a bounded rectangular region. A special case is the random network when the fiber centroids follow a Poisson process with uniformly distributed angles for their axes. Moreover, our applications of interest concern the case when the fibers in the network lie close to horizontal planes. This occurs in materials like paper, non-woven textiles and glass fiber mats.

Consider a partition of a stochastic fiber network into congruent regions by a planar square grid of side length x. We shall denote the local averages in zones by placing a tilde ~ over the random variables; thus $\tilde{\beta}$ is the local areal density, $\tilde{c}$ is the local density and $\tilde{\varepsilon}$ is the local porosity. These variables are related through the density, ρ of a fiber and the thickness, $\tilde{z}$ of the network.

$$\tilde{c} = (1 - \tilde{\varepsilon})\,\rho \qquad \text{and} \qquad \tilde{\beta} = \tilde{c}\tilde{z} \tag{1}$$

We wish to estimate the variance of local porosity, $Var_x(\tilde{\varepsilon})$, through the measurable variance of local areal density, $Var_x(\tilde{\beta})$.

There are analytic expressions for $Var_x(\tilde{\beta})$ in the case of *random* networks of arbitrary rectangular fibers[7] or disks.[8] In particular, for a random structure of mean areal density $\bar{\beta}$, made from objects of mean areal density G, the point variance ($x = 0$) is given by the Poisson result

$$Var_0(\tilde{\beta}) = \bar{\beta}G \tag{2}$$

For our fibers of mean width ω and mass per unit length δ, we have $G = \delta/\omega$; this will be the main case that we consider in the sequel.

2.1. 3D Random Fiber Networks

Denote by $A = \lambda/\omega$ the fiber aspect ratio. From ref. 10 the expected number of fiber contacts per fiber in a random 3-dimensional network is given as

$$\bar{n} = 2Ac_{vol} \tag{3}$$

where c_{vol} is the solid volume fraction in the fiber network. Equation (3) is derived by considering the expected number of fibers in a cube of side, λ and the probability of their intersection with a cylinder of diameter, ω and height, λ for all orientations of fibers in three dimensions. In our local zone with network thickness $\tilde{z}$ the local average number of contacts per fiber is

$$\tilde{n} = \frac{2A\tilde{c}}{\rho} = \frac{2A\tilde{\beta}}{\tilde{z}\rho} = 2A(1 - \tilde{\varepsilon}) \tag{4}$$

and the global averages are related by

$$\bar{n} = 2A(1 - \bar{\varepsilon}) \tag{5}$$

It follows from (4) that the required variance of local porosity is

$$Var_x(\tilde{\varepsilon}) = \frac{Var_x(\tilde{n})}{4A^2} \tag{6}$$

We can estimate $Var_x(\tilde{n})$ by observing that it is an average value from the fibers present in the zone. Obviously this number of contributing fibres, $\tilde{N}$ say, will vary from zone to zone but its expected value, N is the ratio of the expected mass of fibers to the mass of one fibre: $N = \bar{\beta}x^2/\delta\lambda$. Now, the variance of averages of samples of size N is $1/N$ times the variance of samples of size 1, for processes that conform to the Central Limit Theorem. So we have

$$Var_x(\tilde{n}) = \frac{1}{N} Var_0(\tilde{n}) = \frac{1}{N}\bar{n} \tag{7}$$

since the underlying process of fiber contacts is supposed to be Poisson. This allows substitution in (6), bearing in mind that we want more than one fiber in the zones

$$Var_x(\tilde{\varepsilon}) = \frac{\bar{n}\,\delta\lambda}{4A^2\bar{\beta}x^2} \qquad \text{for} \quad x > \sqrt{\delta\,\lambda/\bar{\beta}} \tag{8}$$

Using (5), we have, for $x > \sqrt{\delta\lambda/\bar{\beta}}$, the following equivalent expressions:

$$Var_x(\tilde{\varepsilon}) = \frac{(1-\bar{\varepsilon})\,\delta\lambda}{2A\bar{\beta}x^2} \tag{9}$$

$$= \frac{\delta\lambda}{2A\rho\bar{z}x^2} \tag{10}$$

$$= \frac{\pi\omega^3}{8\bar{z}x^2} \tag{11}$$

Equations (10–11) provide an estimate of the variance of local network porosity measured over square zones of side length x when the mean areal density is $\bar{\beta}$ at a depth of $\bar{z}$, in the case of a random network. The important qualitative observation is that it depends on the constituent fibers only through the cube of their diameter.

A projection of the 3D structure onto a plane allows 2D characterisation of the structure. For a network of uniform height $\bar{z}$ the variance of local areal density, $Var_x(\tilde{\beta})$ given by:

$$Var_x(\tilde{\beta}) = \rho^2\,\bar{z}^2\,Var_x(\tilde{\varepsilon}) \tag{12}$$

When the network height varies between zones we have:

$$Var_x(\tilde{\beta}) = \rho^2\,Var_x(\tilde{\varepsilon}\tilde{z}) \tag{13}$$

and if $\tilde{\varepsilon}$ and $\tilde{z}$ are bivariate normally distributed and independent:

$$Var_x(\tilde{\beta}) = \rho^2(\bar{z}^2\,Var_x(\tilde{\varepsilon}) + \bar{\varepsilon}^2\,Var_x(\tilde{z}) + Var_x(\tilde{\varepsilon})\,Var_x(\tilde{z})) \tag{14}$$

2.2. Fluid Flow Rates in 3D Random Fiber Networks

We can estimate also $Var(\tilde{q})$, the variance of local flow rate per unit area through the network. The flow rate depends on the flow mode, and through square zones of side length x, we have the estimate

$$\tilde{q} \propto \frac{\bar{r}^k}{x^2}\tilde{n} \qquad \text{for} \quad \bar{r} \ll x \tag{15}$$

where $\bar{r}$ is the mean pore radius and $k = 4, 3, 2, \frac{1}{2}$, depending on whether the flow is laminar, molecular, turbulent or capillary, respectively. The coefficients of $\bar{r}^k$ are dependent on fluid properties, such as viscosity and surface tension, additional geometric parameters such as capillary length and the driving force; full expressions are given in ref. 9 along with a discussion of the appropriate use of each. It follows immediately from (15) that the mean flow rate through such zones, $\bar{q}$, is proportional to $\bar{r}^k\mu$, where μ is the mean number of pores per unit area, and the variance of the mean flow rate through such zones is

$$Var(\tilde{q}) = \frac{\bar{q}^2}{\mu x^2} \tag{16}$$

Hence the coefficient of variation is expected to a first approximation to be independent of flow mode:

$$CV(\tilde{q}) = \frac{1}{\sqrt{\mu}\, x} \tag{17}$$

This equation has the potential to provide estimates of mean numbers of pores per unit area for fiber networks, by collecting flows in a square grid of receivers. The coefficient of variation of local flow rate through fiber networks has relevance to, for example, the performance of filters in industrial and technical applications, the absorption of inks into paper in printing processes, and the application of mineral slurries to paper in coating processes. We note that a strong relationship has been found between the parameters of the pore radius distribution in paper and the coefficient of variation of local areal density.[2]

2.3. Clumped Networks

Real stochastic fiber networks, such as commercially manufactured papers, nonwoven textiles and glass mats, depart from ideal randomness because fibers are not deposited independently nor with equal likelihood in all positions. The aggregation or "clumping" of fibres, caused by interaction in suspension, manifests itself in increased variability of local porosity and areal density. We seek an estimate for the variance of local porosity, denoted $Var_x^*(\tilde{\varepsilon})$ for such networks, in terms of measurable clumping parameters. Widely used measurements of nonuniformity in planar fiber networks are the variance and coefficient of variation of local areal density, and the ratio of the variance of local areal density to that calculated for a random network of fibers with the same distribution of length, width and linear density at the same mean areal density. More recently,[8] the clumping has been characterised by a technique of stochastic decomposition of radiographs of commercial papers into discs with a lognormal distribution of diameters and uniform mean areal density. Simulated structures generated by deposition of such disks according to a Poisson process in two dimensions yields structures with a decay of variance with increasing zone size close to that seen in commercial papers.[11] For such a structure made from disks of mean diameter D and mean areal density G, the variance of local areal density measured using square zones of side length x is approximated by

$$Var_x^*(\tilde{\beta}) \approx \bar{\beta} G \left(1 - \frac{2\,x}{\pi D} + \cdots\right) \qquad \text{for} \quad x \leqslant D \tag{18}$$

where $\bar{\beta}$ is the mean areal density.[10] We use the superfix * to denote the case of clumped networks.

To obtain an estimate of $Var_x^*(\tilde{\varepsilon})$, we need to make some assumptions about the way that fibers are packed. Accordingly, we look at three special cases, the first being trivial.

Uniform Density Network. Here we have, in Eq. (1), $\tilde{c} = \bar{c}$, so also $\tilde{\varepsilon} = \bar{\varepsilon}$. Then local averages of porosity are constant and $Var_x^*(\tilde{\varepsilon}) = 0$.

Uniform Thickness Network. Here we have, in Eq. (1), $\tilde{z} = \bar{z}$, so $\tilde{c} = \tilde{\beta}/\bar{z} = (1 - \tilde{\varepsilon})\,\rho$. Hence

$$Var_x^*(\tilde{\varepsilon}) = Var_x^*(\tilde{\beta}/\bar{z}\rho) \tag{19}$$

Then it follows that

$$Var_x^*(\tilde{\varepsilon}) = \frac{\bar{\beta} G}{\bar{z}^2 \rho^2} \left(1 - \frac{2x}{\pi D} + \cdots\right) \qquad \text{for} \quad x \leqslant D \tag{20}$$

Variable Thickness Network. Rearranging Eq. (13) for the case of a clumped network gives

$$Var_x^*(\tilde{\varepsilon}) = \frac{1}{\rho^2}\, Var_x^* \left(\frac{\tilde{\beta}}{\tilde{z}}\right) \tag{21}$$

If $\tilde{\beta}$ and $\tilde{z}$ are bivariate normally distributed we have the following

$$Var_x^*(\tilde{\varepsilon}) = \frac{1}{\rho^2} \left(\frac{\bar{\beta}}{\bar{z}}\right)^2 \left(\frac{Var_x^*(\tilde{\beta})}{\bar{\beta}^2} - \frac{2\,Cov(\tilde{\beta}, \tilde{z})}{\bar{\beta}\,\bar{z}} + \frac{Var_x^*(\tilde{z})}{\bar{z}^2}\right) \tag{22}$$

where

$$Cov(\tilde{\beta}, \tilde{z}) = \overline{\tilde{\beta}\tilde{z}} - \bar{\beta}\bar{z} \tag{23}$$

$$= \overline{(1 - \tilde{\varepsilon})\,\rho \tilde{z}^2} - (1 - \bar{\varepsilon})\,\rho \bar{z}^2 \tag{24}$$

and $Cov(\widetilde{\beta}, \tilde{z})$ and is the covariance of $\widetilde{\beta}$ and $\tilde{z}$. For dense networks in suspension, and for clumped fiber networks resulting from the filtration of a suspension, we have the expectation that $\tilde{z}$ is to some extent dependent on $\widetilde{\beta}$ and *vice versa*.

If $\tilde{\varepsilon} \approx \bar{\varepsilon}$:

$$Cov(\widetilde{\beta}, \tilde{z}) \approx (1 - \bar{\varepsilon})\, \rho(\overline{\tilde{z}^2} - \bar{z}^2) \tag{25}$$

$$\approx (1 - \bar{\varepsilon})\, \rho\; Var_x^*(\tilde{z}) \tag{26}$$

$$\approx \frac{\bar{\beta}}{\bar{z}}\; Var_x^*(\tilde{z}) \tag{27}$$

Substition of (27) in (22) yields:

$$Var_x^*(\tilde{\varepsilon}) \approx \left(\frac{\bar{\beta}}{\rho \bar{z}}\right)^2 \left(\frac{Var_x^*(\widetilde{\beta})}{\bar{\beta}^2} - \frac{Var_x^*(\tilde{z})}{\bar{z}^2}\right) \tag{28}$$

$$\approx \left(\frac{\bar{\beta}}{\rho \bar{z}}\right)^2 (CV_x^*(\widetilde{\beta})^2 - CV_x^*(\tilde{z})^2) \tag{29}$$

and when $CV_x^*(\tilde{z}) = 0$ Eq. (29) reduces to Eq. (19). More precise expressions for those enclosed in brackets in Eqs. (18) and (20), valid for all x and lognormal distributions of disk diameters, can be found in the Appendix to ref. 8.

3. APPLICATION

The commercial manufacture of paper, nonwoven fabrics and fibrous filter and barrier media like glass mats involves the continuous filtration of a 3D fiber suspension into an essentially 2D structure. Typically papers are 10–20 fibers deep, in a thickness of 0.1 mm, made from an aqueous suspension of height 10 mm. Here we see two applications of the theory; first, to quantify the potential of a suspension to yield a uniform structure on filtration; and second, to estimate the variance of local density in an almost 2D network, given measures of the local areal density and thickness distributions.

3.1. Suspension Filtration

Equation (14) may be applied to the delivery of a well mixed fiber suspension jet at high speed from the flowspreader of a papermachine to the continuous filtration stage. Typically the fiber volume fraction in such a delivery system is less than 2% and hence, $\bar{\varepsilon} > 0.98$; we have the approximation therefore, as $\bar{\varepsilon} \to 1$:

$$Var_x(\tilde{\beta}) \approx \rho^2(\bar{z}^2\, Var_x(\tilde{\varepsilon}) + (1 + Var_x(\tilde{\varepsilon}))\, Var_x(\tilde{z})) \tag{30}$$

$$\approx \rho^2\bar{z}^2(Var_x(\tilde{\varepsilon}) + (1 + Var_x(\tilde{\varepsilon}))\, CV_x(\tilde{z})^2) \tag{31}$$

where $CV_x(\tilde{z})$ is the coefficient of variation of local suspension height and Eqs. (14, 30, 31) recover Eq. (12) when $Var_x(\tilde{z}) = 0$.

From fiber geometry we can calculate $Var(\tilde{\varepsilon})$ for a given $\bar{z}$ and thus the potential of the delivered suspension to yield a uniform structure on filtration can be estimated for given $Var_x(\tilde{z})$ or $CV_x(\tilde{z})$. Experimentally, $Var_x(\tilde{z})$ and $CV_x(\tilde{z})$ may be determined using video image acquisition and analysis as, for example, by Kiviranta and Paulapuro.[12]

We observe from Eq. (11) however, that for typical papermachine initial suspension heights and fiber diameters, say $\bar{z} = 10$ mm and $\omega = 30\ \mu$m then

$$Var_x(\tilde{\varepsilon}) \approx \frac{10^{-6}}{x^2} \qquad (x \text{ in mm}) \tag{32}$$

and as $\bar{\varepsilon} \approx 1$ then

$$CV_x(\tilde{\varepsilon}) \approx \frac{10^{-3}}{x} \qquad (x \text{ in mm}) \tag{33}$$

Substituting (32) in (31) and assuming the fiber density is that of cellulose, i.e., $\rho = 1.55$ g cm^{-3} allows calculation of $Var_x(\tilde{\beta})$ for a range of $CV_x(\tilde{z})$ and x. A plot of the standard deviation of $\tilde{\beta}$ is given in Fig. 1 for $0 \leqslant CV_x(\tilde{z}) \leqslant 5\%$ and $30\ \mu\text{m} \leqslant x \leqslant 1$ mm.

In a continuous filtration process, such as papermaking or the manufacture of nonwoven textiles, a fiber suspension is delivered as a jet from a flowspreader to a filtration stage. The theory quantifies the importance of the uniformity of the jet in determining the state of the suspension delivered. For the case given above, the suspension required to form a 60 g m^{-2} network has, at the 1 mm scale, a coefficient of variation of local areal density, $CV_1(\tilde{\beta}) = 25.7\%$ when $CV_1(\tilde{z}) = 0$, increasing to 250% when $CV_1(\tilde{z}) = 1\%$. This treatment assumes that the most uniform state achievable by the input of turbulence to the suspension is random and that the probability of fibers being located in surface perturbations is the same as that in the bulk of the fluid.

As filtration of a suspension proceeds, fibers are drawn preferentially towards sparse regions in the evolving structure due to their lower

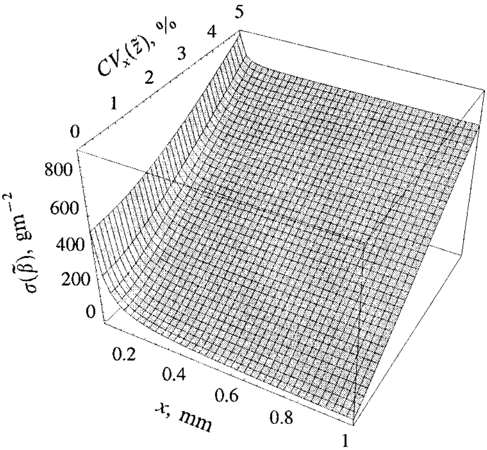

Fig. 1. Relationship between standard deviation of $\widetilde{\beta}_{\text{proj}}$, $CV_x(\tilde{z})$ and zone size, x. Surface plotted is that for fiber diameter, $\omega = 30\,\mu$m; mean suspension height, $\bar{z} = 10$ mm.

resistance to flow. Our model provides an interpretation of this "preferential drainage" effect which has been simulated by Gorres *et al.*[13] and demonstrated experimentally by Sampson *et al.*[14] for laboratory formed networks and by Norman *et al.*[15] for machine made papers. A typical machine made paper will have $CV_1(\widetilde{\beta})$ between 5% and 10%, illustrating substantial preferential drainage in the forming section of a paper machine even when the height of the suspension is uniform. The theory has potential for on-line monitoring of the efficiency of the filtration stage through on-line estimates of $CV_x(\tilde{z})$ and variance of local areal density, which can be acquired by high speed video imaging.

3.2. Variance of Local Density in Paper

Strong correlations between $\widetilde{\beta}$ and $\tilde{z}$ for paper made from three types of pulp are reported by Schultz-Eklund *et al.*[16] who calculated also the coefficient of variation of local density, $CV(\tilde{c})$. Table 1 shows the results of applying their data, for papers made from three pulp types, to Eq. (29). The mean porosity was estimated using the expression, $\bar{\varepsilon} = 1 - \bar{c}/\rho$. For the paper made from Chemi-Thermo-Mechanical Pulp (CTMP), $CV(\widetilde{\beta}) < CV(\tilde{z})$ and therefore the bracketed term in Eq. (29) is negative and $Var(\tilde{\varepsilon})$ cannot be estimated without knowledge of $Cov(\widetilde{\beta}, \tilde{z})$. For the Thermo-Mechanical Pulp (TMP) and the "unbleached Kraft" papers, the model slightly underestimates $CV(\tilde{c})$, though agreement would probably be

Table 1. Variance of Local Density and Porosity in Machine-Made Papers[a]

	Data from ref. 16							Data from model	
Paper type	$\bar{\beta}$ $\mathrm{g\,m^{-2}}$	$CV(\tilde{\beta})$ %	$\bar{z}$ μm	$CV(\tilde{z})$ %	$\bar{c}$ $\mathrm{kg\,m^{-3}}$	$CV(\tilde{c})$ %	$\bar{\varepsilon}$ { }	$CV(\tilde{\varepsilon})$ { }	$CV(\tilde{c})$ { }
CTMP	57.9	13.4	171	13.5	338	9.0	0.78	—	—
TMP	39.7	12.2	51	7.4	778	12.3	0.50	9.8	9.7
Unbleached Kraft	56.8	12.6	92	11.4	617	7.9	0.60	3.6	5.4

[a] Data from ref. 16. Values of $CV(\tilde{\varepsilon})$ calculated from Eq. (29).

improved given data to calculate $Cov(\tilde{\beta}, \tilde{z})$ and a better estimate of ρ for each fiber type.

4. CONCLUSIONS

The theory presented here suggests that the variance of porosity in three dimensional random fiber networks is dependent on fiber morphology only through the cube of fiber diameter. A projection of the network in two dimensions indicates that the variance of local areal density is controlled by the square of the coefficient of variation of local network height. In a practical application, this demonstrates the importance of the uniformity of suspension height when delivering a fiber suspension to a papermachine, in optimising the distribution of fiber in the suspension. An approximate expression has been obtained also for the coefficient of variation of local flow rate perpendicular to the plane of a fiber network which, for sufficiently large zones is independent of the mode of flow. For clumped networks, the variance of local porosity can be expressed in terms of the mean areal density and thickness and their coefficients of variation.

NOMENCLATURE

A	Mean fiber aspect ratio	{ }
$\tilde{\beta}$	Local areal density	$\mathrm{g\,m^{-2}}$
$\bar{\beta}$	Mean areal density	$\mathrm{g\,m^{-2}}$
$\tilde{c}$	Local mass density	$\mathrm{g\,m^{-3}}$
c_{vol}	Solid volume fraction	{ }
D	Mean disk diameter	m
δ	Fibre linear density	$\mathrm{g\,m^{-1}}$
$\tilde{\varepsilon}$	Local porosity	{ }
$\bar{\varepsilon}$	Mean porosity	{ }

G	Mean disk areal density	$\mathrm{g\,m^{-2}}$
k	Exponent determining flow regime	{ }
λ	Fibre length	m
μ	Mean number of pores per unit area	{ }
$\tilde{n}$	Local average number of contacts per fibre	{ }
$\bar{n}$	Global average number of contacts per fibre	{ }
N	Mean number of fibers in a zone of side x	{ }
$\tilde{q}$	Local volumetric flow rate	$\mathrm{m^3\,s^{-1}}$
$\bar{q}$	Mean volumetric flow rate	$\mathrm{m^3\,s^{-1}}$
$\bar{r}$	Mean pore radius	m
ρ	Fibre density	$\mathrm{g\,m^{-3}}$
ω	Fibre width	m
x	Zone size	m
$\tilde{z}$	Local network thickness	m
$\bar{z}$	Mean network thickness	m

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