Statistical Shape Models

- Eigenpatches model regions
  - Assume shape is fixed
  - What if it isn’t?
    - Faces with expression changes,
    - Organs in medical images etc
- Need a method of modelling shape and shape variation

Shape Models

- We will represent the shape using a set of points
- We will model the variation by computing the PDF of the distribution of shapes in a training set
- This allows us to generate new shapes similar to the training set

Building Models

- Require labelled training images
  - Landmarks represent correspondences

Suitable Landmarks

- Define correspondences
  - Well defined corners
  - ’T’ junctions
  - Easily located biological landmarks
  - Use additional points along boundaries to define shape more accurately

Building Shape Models

- For each example

\[ \mathbf{x} = (x_1, y_1, \ldots, x_n, y_n)^T \]

Shape

- Need to model the variability in shape
- What is shape?
  - Geometric information that remains when location, scale and rotational effects removed (Kendall)

Same Shape Different Shape
Shape

- More generally
  - *Shape is the geometric information invariant to a particular class of transformations*
- Transformations:
  - Euclidean (translation + rotation)
  - Similarity (translation+rotation+scaling)
  - Affine

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<th>Shapes</th>
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Statistical Shape Models

- Given a set of shapes:
- Align shapes into common frame
  - Procrustes analysis
- Estimate shape distribution $p(x)$
  - Single gaussian often sufficient
  - Mixture models sometimes necessary

Aligning Two Shapes

- Procrustes analysis:
  - Find transformation which minimises $|x_1 - T(x_2)|^2$
  - Resulting shapes have
    - Identical CoG
    - approximately the same scale and orientation

Aligning a Set of Shapes

- Generalised Procrustes Analysis
  - Find the transformations $T_i$ which minimise
    $$\sum |m - T_i(x)|^2$$
  - Where $m = \frac{1}{n} \sum T_i(x)$
  - Under the constraint that $|m| = 1$

Aligning Shapes : Algorithm

- Normalise all so CoG at origin, size=1
- Let $m = x_i$
- Align each shape with $m$
- Re-calculate $m = \frac{1}{n} \sum T_i(x)$
- Normalise $m$ to default size, orientation
- Repeat until convergence
**Aligned Shapes**

- Need to model the aligned shapes

**Statistical Shape Models**

- For shape synthesis
  - Parameterised model preferable
  
  \[ \mathbf{x} = f_{\text{shape}}(\mathbf{b}) \quad \text{e.g.} \quad \mathbf{x} = \bar{\mathbf{x}} + \mathbf{P}\mathbf{b} \]

- For image matching we can get away with only knowing \( p(\mathbf{x}) \)
  - Usually more efficient to reduce dimensionality where possible

**Dimensionality Reduction**

- Co-ords often correlated
- Nearby points move together

\[ \mathbf{x} = \bar{\mathbf{x}} + \mathbf{b}_1 \mathbf{h}_1 \]

**Principal Component Analysis**

- Compute eigenvectors of covariance, \( \mathbf{S} \)
- Eigenvectors: main directions
- Eigenvalue: variance along eigenvector

\[ \mathbf{p}_1 \quad \mathbf{p}_2 \]

\[ \lambda_1 \quad \lambda_2 \]

**Dimensionality Reduction**

- Data lies in subspace of reduced dim.
  \[ \mathbf{x} = \bar{\mathbf{x}} + \mathbf{p}_1 \mathbf{h}_1 + \cdots + \mathbf{p}_t \mathbf{h}_t \]
- However, for some \( t \), \( \mathbf{h}_j \approx 0 \) if \( j > t \)
  (Variance of \( \mathbf{h}_j \) is \( \lambda_j \))

**Building Shape Models**

- Given aligned shapes, \( \{ \mathbf{x}_i \} \)
- Apply PCA
  \[ \mathbf{x} \approx \bar{\mathbf{x}} + \mathbf{P}\mathbf{b} \]
- \( \mathbf{P} \) – First \( t \) eigenvectors of covar. matrix
- \( \mathbf{b} \) – Shape model parameters
Hand shape model
• 72 points placed around boundary of hand
  – 18 hand outlines obtained by thresholding images of hand on a white background
• Primary landmarks chosen at tips of fingers and joint between fingers
  – Other points placed equally between

Face Shape Model
Varying $b_1$  Varying $b_2$  Varying $b_3$

Brain structure shape model

Example : Hip Radiograph

Spine Model
Distribution of Parameters

- Learn $p(b)$ from training set
- If $\mathbf{x}$ multivariate gaussian, then
  - $b$ gaussian with diagonal covariance
    $$S_b = \text{diag}(\lambda_1, \cdots, \lambda_s)$$
- Can use mixture model for $p(b)$

Conclusion

- We can build statistical models of shape change
- Require correspondences across training set
- Get compact model (few parameters)
- Next: Matching models to images