

# Cost Padding in Regulated Monopolies

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## Abstract

This paper considers the regulated monopoly that pads or falsifies its costs to increase the cost reimbursement it receives from the regulator. Contrary to the standard literature on cost regulation, the firm engages in cost reducing investment before it enters into a regulatory contract. This pre-contractual investment in cost reduction determines the firm type at the contracting stage. The paper derives both the optimum incentive compatible falsification contract and the equilibrium type distribution. With the distribution of cost types determined endogenously by the pre-contractual investment choice, an increase in the cost of falsification has two effects. First, there is a direct effect that reduces cost padding because it becomes more expensive to do so. Second, there is an indirect effect that increases cost padding because the firm responds by choosing lower investments, and lower investments are associated with more cost padding. It is demonstrated that the direct effect will dominate and both expected levels of cost padding and expected costs for falsification will be reduced. However, the indirect effect increases real costs and, despite the reduction in cost padding, the net effect can reduce welfare. It is determined that these conclusions are significantly different from those obtained when the distribution of cost types is exogenously fixed.

*Keywords:* cost padding, costly state falsification, endogenous screening

*JEL:* D82, L43, L52

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## 1. Introduction

One of the most serious concerns of regulators is that regulated firms may attempt to raise the reimbursement they receive from the regulator by engaging in cost padding or accounting contrivances that inflate observed costs. Firms have many ways of padding costs such as: increasing salaries and expense claims, “gold-plating” expenditures, charging other equipment to project costs, advertising for corporate image, charging for depreciated assets, and not reporting cost reducing improvements. While cost padding is often identified as a serious concern, its extent is difficult to quantify because, by definition, much of it is hidden from the regulator.<sup>1</sup> There is, however, evidence from the classic

studies of Berliner (1957), Schiff and Lewin (1968) and Schiff and Lewin (1970) that the exaggeration of costs in regulated systems is widespread.<sup>2</sup> There are also numerous case studies that show how, and to what extent, regulated firms pad their costs.<sup>3</sup>

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lic utilities, the reader is referred to McAfee and McMillan (1988; government contracting - North America), Quiggin (1998; electricity - Australia); Kerr (water - New Zealand); Department of Transportation and Regional Services (2000; transport - Australia); Ontario Federation of Agriculture (1999; energy - Canada); and Watson (public utilities - Australia). In addition, the OECD study by Gönenç et al. (2001) compares the incentives that price-cap regulation provides for cost padding in electricity and telecommunications. For examples in procurement contracts, the reader is referred to Manoj (2000; Shipping - India) and Higgs (1998; military - US).

<sup>2</sup>Berliner interviewed former managers of Soviet firms and found, as one manager reported, “an enormous amount of falsification in all branches of production and in their accounting systems...” (p.161). Schiff and Lewin studied the efficiency of divisions within three large U.S. corporations and produced estimates of the size of cost padding within divisions of the same company to be between 20% and 25%.

<sup>3</sup>For example, the report by the Godbole committee into the Enron-owned Dabhol Power Company in Maharashtra

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<sup>1</sup>There are many reports of regulators’ concerns about cost padding. For examples of such reports related to pub-

Such cases may represent the tip of the iceberg with much of the padded costs either tolerated or hidden.

In this paper, we introduce endogenous screening and costly state falsification into a modification of the Laffont and Tirole (1986) regulatory model. The modified model allows for a difference between real and observed costs at the contracting stage and for the firm to undertake a cost-reducing investment at a pre-contractual stage. Padded costs, the difference between real and observed costs, are a post-contractual hidden action, chosen by the firm to increase its cost reimbursement. In contrast, the pre-contractual investment in cost reduction is the hidden information of the firm at the contracting stage. In treating cost padding as a hidden action of the firm, we follow the costly state falsification approach initially proposed and analyzed by Lacker and Weinberg (1989).<sup>4</sup> The treatment of investment in cost reduction at the pre-contractual stage follows the endogenous screening approach of González (2004) and Gul (2001).<sup>5</sup>

The objectives of the paper are to explain how the endogenization of the distribution of types affects the optimum contract – that is, the extent of cost padding and the level of welfare – and to compare the solution to the case where the distribution of types is given exogenously, (as in the model of Laffont and Tirole 1986).

Investment by the firm at the pre-contractual stage generates a hold-up problem, as in the model of González (2004), because some of the benefits of cost reduction can be appropriated by the regulator during the contracting stage. Consequently, pre-contractual investment in cost reduction is inefficiently low. In particular, the optimum response of the firm is a mixed strategy that places positive

weight on less than efficient investments in cost reduction.

When the distribution of pre-contractual investment is determined endogenously, changes in parameter values will have both a direct and an indirect effect on cost padding. Consider, for example, an increase in the cost of falsification. Such an increase may correspond to a policy that tightens accountancy standards or requires more intensive monitoring of the firm's costs by the regulator. The direct effect reduces cost padding, because it has become more expensive to pad costs. There is, however, an indirect effect because the firm responds by choosing lower pre-contractual investment.<sup>6</sup> This indirect effect will offset the direct effect. Nevertheless, it will be shown that at the optimum, the direct effect will dominate and expected levels of both cost padding and expected falsification costs will be lower (Proposition 3). However, the effect on overall welfare will be ambiguous, except in some special cases, because the indirect effect increases real costs.

Consider, by way of contrast, the effect of an increase in the cost of falsification when the distribution of types is exogenously fixed. In this case, an increase in the cost of falsification does not affect cost padding, which depends only on the curvature of the cost falsification function (at least for the parameterizations of falsification costs we consider), and expected falsification costs will increase, causing a reduction in welfare. Furthermore, the increase in cost of falsification will reduce marginal falsification costs and therefore, the rent the regulator must to give to all firm types, thereby causing the opposite effect on welfare. Consequently, the overall effect on welfare will be ambiguous, except in special cases. We provide an example to illustrate that the policy implications of the two cases, the exogenous distribution of types and the endogenously determined distribution of types, are completely different. In the example, an increase in the cost of falsification reduces welfare in the endogenous case and increases welfare in the exogenous case.

The paper that is most closely related to ours is González (2004). He examines the implications of endogenizing the type distribution in a buyer-seller model in which the buyer cannot observe the seller's costs. In his model, the pre-contractual investment

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State, India found cost padding of Rs 930 crore (about \$200 million) and in the case of the US government versus the defense contractor Sundstrand in 1989, a sum of \$200 million was recovered because the court found that Sundstrand had co-mingled commercial and government costs.

<sup>4</sup>Subsequently, the costly state falsification approach has been used and extended in a number of other papers. For example, Maggi and Rodríguez-Clare (1995) consider a general agency model with a risk neutral principal and agent, Crocker and Morgan (1998) examine falsification and fraud in insurance contracts under risk aversion and Crocker and Slemrod (2007) analyze the relationship between earnings manipulation and executive compensation.

<sup>5</sup>There are older studies, such as (Albon and Kirby 1983) and (Daughety 1984), that examines cost padding when there is an exogenous regulatory constraint. See Waterson (1988) for a summary of this type of model.

<sup>6</sup>Lower in the sense of first-order stochastic dominance.

determines the seller's costs.<sup>7</sup> There are two main differences between our model and his. First, in González (2004), the main objective is to understand how the agent's pre-contractual investment choice under imperfect information mitigates the hold-up problem relative to the case of perfect information. For that reason, and because there is no hold-up problem in the incomplete information case with exogenously given types, González (2004) discusses welfare comparisons between the perfect information case and the endogenous screening case. In contrast, we focus on the differences in welfare implications between the two incomplete information cases. In particular, we compare welfare in the case where the type distribution is given exogenously with the case where the type distribution is endogenously determined.

Second, the comparative static analysis in González (2004) considers a change whereby both the direct and indirect effect work in the same direction. In particular, he indicates that a reduction in the marginal cost of production unambiguously improves welfare. The reason is that both the direct effect (lower costs lead to an increase in the quantity) and the indirect effect (shift of the distribution of investment to the right) are welfare enhancing. In our case, the indirect and direct effects work in opposite directions. For example, an increase in falsification costs has a direct effect that is similar to reducing production costs in González (2004): that is, it will lower total costs and hence, raise welfare. However, the indirect effect will lead to a decline in welfare because firms will shift the distribution of pre-contractual investment to the left. Thus, it is difficult to know a priori which effect will dominate.

While cost padding is an important practical consideration for regulators, most of the main theoretical models of regulation are not well suited to address this question.<sup>8</sup> In Baron and Myerson (1982), for example, it is assumed that the regulator is unable to observe the firm's costs, and therefore, because the payment to the firm does not depend upon costs, the firm has no incentive to pad them. Equally, in the standard model of Laffont and Tirole (1986), it is assumed that the regulator perfectly observes the firm's costs but is unable to observe the firm's effort in cost reduction. Again,

<sup>7</sup>See also Gul (2001), who studies a sequential bargaining problem in which types are endogenously determined.

<sup>8</sup>For a survey of these studies, see, for example, Laffont (1994).

the firm has no incentive to pad costs because the true cost is perfectly and costlessly monitored.<sup>9</sup> Although the standard model does not explicitly deal with cost padding, cost padding activity can be interpreted as a negative effort in cost reduction as evidenced formally in footnote 22.<sup>10</sup> We choose the model with cost padding because it provides a better representation of the regulatory issues that we wish to explore. Having investment determined at a prior, pre-contractual stage and cost padding chosen after the contract has been agreed upon, seems a natural ordering of the firm's choices and provides a simple, tractable model of the regulatory problem.<sup>11</sup>

## 2. The model

In this section we describe the regulatory model. The regulator wishes to engage a firm to undertake a project of social benefit. Before the firm contracts with the regulator, the firm chooses the amount of capital to invest and this investment determines the cost of undertaking the project. Once the firm has contracted with the regulator, the firm is able to pad its costs. The regulator observes the firm's total cost for the project but does not observe how much capital the firm has invested, or how the total cost is divided between the real and the padded costs. The regulator has to design a contract for the firm that discourages cost padding and encourages pre-contractual investment in cost reduction. Thus, we change the order of the standard Laffont and Tirole (1986) regulatory model and assume that the effort or investment in cost reduction is pre-contractual and that the firm can artificially inflate or pad its cost at the post-contractual stage after production is undertaken. At the contractual stage, the regulator treats the firm's investment in cost reduction as *hidden information* and the padded costs of the firm as a *hidden action*.

<sup>9</sup>More precisely, in Laffont and Tirole (1986), the regulator observes a noisy signal of cost. However, this forecast error is independent of the parameters and choice variables of the model; thus, it has no effect on its solution.

<sup>10</sup>Laffont and Tirole (1993; Chapter 12) consider cost padding by introducing two post-contractual choice variables: one activity reducing costs and another padding costs. Our formulation is similar except for the timing.

<sup>11</sup>The paper by Maggi and Rodríguez-Clare (1995) is the one most closely related to ours in terms of cost padding, but they consider the case in which the initial distribution is fixed exogenously. They also consider padding of marginal costs while we consider padding of total costs because we restrict attention to fixed size projects.

The size of the project is fixed and yields social benefit  $V > 0$ . There is a shadow cost of public funds of  $1 + \lambda$  where  $\lambda > 0$ . The firm chooses its capital from an interval  $\mathcal{K} \subset \mathbb{R}_+$  before the project contract is signed. The market for capital is perfect, and the price of capital is  $\rho > 0$ . The cost to the firm of undertaking the project depends on the amount of capital  $k$  it invests. Denote the firm's cost as  $g(k)$  where  $g: \mathcal{K} \rightarrow \mathbb{R}_+$  is  $\mathbb{C}^3$  and satisfies  $g'(k) < 0$ ,  $g''(k) > 0$  and  $-g'(0) > \rho$ . The last assumption guarantees that a positive investment in cost reduction is efficient. We assume that there is a finite value,  $k_{max}$  that is the maximum investment a firm will choose, and let  $\mathcal{K} = [0, k_{max}]$ .<sup>12</sup> In addition, we shall assume that  $V$  is large enough such that the equilibrium outcome is non-trivial.

The regulator cannot observe the true cost but instead observes total cost  $C$ . Total cost includes the true cost  $g(k)$  and the level of cost padding  $x \geq 0$  undertaken by the firm; that is,  $C = g(k) + x$ . Thus, the amount  $x$  represents the extent to which the firm falsely reports its costs and can be considered as an accounting contrivance that raises the costs as seen by the regulator.<sup>13</sup> We follow the standard costly state falsification model of Crocker and Morgan (1998) and assume that there is a cost to falsifying the reported costs of  $\phi(x)$  dependent on the extent of cost padding  $x$ . We make the following straightforward assumptions about the falsification function:  $\phi: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is  $\mathbb{C}^3$ ,  $\phi(0) = 0$ ,  $0 \leq \phi'(x) < 1$ ,  $\phi'(0) = 0$ , and  $\phi''(x) > 0$ . These assumptions are straightforward. The marginal cost of falsification is positive and increasing such that more exaggerated costs are increasingly costly to falsify. Assuming that  $\phi'(x) < 1$ , the marginal cost of falsification is always less than the amount falsified. This is assumed here, but it could easily be imposed as a constraint because it will never be profitable for the firm to falsify the extra unit of

<sup>12</sup>A simple assumption to guarantee that  $k_{max} < \infty$  is to assume  $g(0) < \infty$ . Then, because  $\rho > 0$  and  $g$  is non-negative, decreasing and strictly convex, there will be some  $k_{max}$  such that for all  $k > k_{max}$ ,  $g(k) + \rho k > g(0)$ . In this case, it is clear that the firm will never choose  $k > k_{max}$ .

<sup>13</sup>There is another interpretation for  $x$ , which will be discussed at a later point, as additional or unnecessary expenditures undertaken by the firm. As noted in the introduction, there are many ways in which firms can pad costs: advertising and sponsorship, transfer of funds across divisions, unnecessary remuneration increases, larger than normal allowances for depreciation, not reporting on cost-saving improvements, and various other perks as well as other costly accounting contrivances.

cost if the cost of the falsification is greater than the potential benefit.

We now consider the sequential-move game in which the firm first chooses its capital input, and then the regulator chooses an optimum incentive contract for the firm. We suppose that the firm can choose a mixed strategy and represent its mixed strategy choice by a distribution function  $F: \mathcal{K} \rightarrow [0, 1]$ .<sup>14</sup> The distribution function is, by definition, non-decreasing and right-continuous.<sup>15</sup> We shall denote the support of  $F$ , that is, the set of all the points where  $F$  is strictly increasing, as  $\mathcal{S} \subseteq \mathcal{K}$ .<sup>16</sup>

Let  $\mathcal{C}$  be the set of possible costs. The regulator observes total costs and can offer an incentive contract conditional on  $C \in \mathcal{C}$ . The contract will specify a transfer to the firm of  $t \in \mathbb{R}$ . Furthermore, we suppose, as a convention, that the regulator reimburses the firm's total cost  $C$ . Note that as the reimbursed cost includes padded costs, the transfer  $t$  may be, and in some cases will be, negative. The rent  $r$  that the firm receives from the contract is given by the transfer  $t$  plus the reimbursed costs observed by the regulator  $C$ , less the true cost of production  $g(k)$ , less the cost of falsifying the accounts  $\phi(C - g(k))$ .<sup>17</sup> That is:

$$r = t + C - g(k) - \phi(C - g(k)) \quad (1)$$

The objective of the regulator is to maximize social benefit  $V$ , less the sum of real and falsification costs  $g(k) + \phi(C - g(k))$ , less the shadow cost of the transfer  $\lambda(t + C)$ , less the capital cost  $\rho k$ .<sup>18</sup> Using (1) to substitute  $r + g(k) + \phi(C - g(k))$  for  $t + C$  allows the regulator's objective to be written as:

$$V - (1 + \lambda)(g(k) + \phi(C - g(k))) - \lambda r - \rho k$$

<sup>14</sup>A mixed strategy is a probability function on  $\mathcal{K} \subset \mathbb{R}$ , and this is uniquely related to its distribution function.

<sup>15</sup>The distribution may, however, have positive density, zero density or positive probability.

<sup>16</sup>The support of a distribution is the minimal closed set whose complement has probability zero. Every element of the support is a point of increase of  $F$ . See, for example, Shorack (2000; p.110).

<sup>17</sup>A participation constraint will be imposed such that the rent is non-negative. Hence, even when the transfer  $t$  is negative, the total transfer  $t + C$  is sufficient to cover costs.

<sup>18</sup>The cost of capital is included in the total welfare for later comparisons even though  $k$  is fixed at the time of contracting and therefore, will not affect the regulator's optimization problem. We assume here that the regulator has a simple utilitarian objective function.

The second term,  $(1+\lambda)(g(k)+\phi(C-g(k)))$ , is total social cost and the third term,  $\lambda r$ , is the social cost of giving a rent to the firm.

We denote the costs that the firm incurs as  $v(C, k) = g(k) + \phi(C - g(k))$ . Given the restrictions on the functions  $g(k)$  and  $\phi(x)$ , the function  $v$  is strictly decreasing and strictly convex in  $k$ , strictly increasing and strictly convex in  $C$ , and satisfies a strict Spence-Mirrlees *single crossing property*,  $v_{Ck} > 0$ . Furthermore, to ensure the regulator's optimization problem is concave and, has a global interior solution and to ensure that random contracts are sub-optimal, we make the additional assumption that the marginal cost function  $v_k(C, k)$  is concave in  $C$ . This is satisfied when  $\phi'''(x) \leq 0$ ; thus, the marginal falsification costs are also concave. We gather these conditions in the following assumption.

**Assumption 1.** The cost function  $v : \mathcal{K} \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$  is  $\mathbb{C}^3$ . It is strictly increasing and strictly convex in  $C$ . It is strictly decreasing and strictly convex in  $k$  with  $v_C(C, k) < 1$ . The marginal cost  $v_k(C, k)$  is strictly increasing and concave in  $C$ .

As previously stated, the regulator observes  $C$  but not the padded cost  $x$ . Thus, the regulator cannot infer the level of capital  $k$  chosen by the firm. We assume that the regulator forms a probability assessment about the choice of capital input made by the firm and we represent this by a probability distribution  $F^r : \mathcal{K} \rightarrow [0, 1]$  with support  $\mathcal{S}^r$ . Given this probability assessment, the regulator will design the transfer to maximize expected social welfare by making the transfer  $t$  depend upon the observed costs  $C$  to reduce the inefficiencies caused by the firm's potential to pad costs. This is a standard hidden information problem. Therefore, it is possible to apply the revelation principle to restrict attention to direct mechanisms whereby the firm reports its investment in cost reduction  $k'$  and to impose incentive compatibility constraints that the firm has no incentive to misreport its investment level. We posit that the regulator's probability assessment is *consistent* when  $F^r = F$ .<sup>19</sup> Therefore, the regulator's problem can be formulated as the

<sup>19</sup>For an analysis of screening where the beliefs of the principal and agent are not consistent, see Grubb (2009). One rationale for focusing on consistent beliefs is that in the long run, and facing many similar situations, the regulator will correctly anticipate the firm's mixed strategy.

choice of a contract  $\Delta \equiv (C(k), r(k))_{k \in \mathcal{K}}$  that specifies the cost  $C(k)$  and the rent  $r(k)$  as functions of investment  $k$  to maximize expected social welfare:

$$\int_{\mathcal{K}} \{V - (1+\lambda)v(C(k), k) - \lambda r(k) - \rho k\} dF^r(k) \quad (2)$$

subject to the incentive compatibility and participation constraints for the firm. Define the function  $r(k', k) \equiv t(k') + C(k') - v(C(k'), k)$  as the rent that the firm earns from the contract when the actual investment is  $k$  and when the firm reports that it has invested  $k'$  for every  $k' \in \mathcal{S}^r$ . Let  $r(k) = r(k, k)$ .<sup>20</sup> The firm will choose to announce the level of cost reducing investment  $k'$  that maximizes the rent given the contract  $\Delta$  it faces. Therefore, the incentive compatibility constraints are:

$$r(k) \geq r(k', k) = t(k') + C(k') - v(C(k'), k) \quad (3)$$

for all  $k, k' \in \mathcal{S}^r$ . As the investment in cost reduction is a sunk cost, the firm will only participate in the contract provided that its rent is non-negative for every investment level  $k$  in the support of the regulator's beliefs. That is:

$$r(k) \geq 0 \quad \forall k \in \mathcal{S}^r \quad (4)$$

An equilibrium consists of three elements: a regulatory contract that maximizes social welfare subject to the incentive compatibility and participation constraints, a strategy for the firm that maximizes profits, and a consistent probability assessment. If the firm is to adopt a mixed strategy in equilibrium, then each possible choice of investment level  $k$  must generate the same level of profit. Therefore, in equilibrium:

$$r(k) - \rho k = r(k') - \rho k' \quad \forall k, k' \in \mathcal{S} \quad (5)$$

We further assume that if the firm is indifferent between two or more strategies, then it will choose the strategy that maximizes the regulator's objective. Hence, we can define an equilibrium as follows.

**Definition 1.** An equilibrium is a contract  $\Delta$  that maximizes (2) subject to (3) and (4), a strategy for the firm that maximizes profits and a consistent probability assessment  $F^r = F$ . Where the

<sup>20</sup>We assume that if the firm announces a  $k' \notin \mathcal{S}^r$ , then the regulator will make no transfer to the firm. Hence, such announcements will not be optimal for the firm. The firm may, however, choose some  $k \notin \mathcal{S}^r$  provided it announces a  $k' \in \mathcal{S}^r$ .

firm is indifferent between two or more strategies, it chooses the strategy that maximizes the regulator's payoff. In addition, if the firm chooses a mixed strategy, then equation (5) is satisfied.

Before proceeding to the analysis, it is important to realize that although in describing the model we have treated cost padding as an accounting contrivance, there is an equivalent model in which cost padding has an alternative interpretation. In this alternative interpretation, cost padding is an extra but unnecessary expenditure that generates a utility benefit for the firm. To see this, let  $\psi(x)$  denote the benefit to the firm of cost padding an amount  $x$ . In this case,  $\psi(x)$  may be regarded as the benefit of gold-plating expenditures. Assume that the benefit function  $\psi(x)$  satisfies  $\psi(0) = 0$ ,  $0 \leq \psi'(x) \leq 1$ ,  $\psi'(0) = 1$  and  $\psi''(x) < 0$ . With this formulation, an increase in padded costs by one unit generates a positive gain in utility (but not as much as the costs incurred), and the marginal benefit is declining with costs. The rent of the firm  $r$  is given by the transfer  $t$  plus the utility benefit of the padded costs  $\psi(x)$  so that  $r = t + \psi(x)$ . The two alternative interpretations are formally equivalent provided  $\psi(x) = x - \phi(x)$ .<sup>21</sup> Although the two interpretations are formally equivalent when this condition is satisfied, it should be emphasized that they represent very different situations. In one case, there is a real expenditure that generates utility benefits, whereas in the other case, it is an accounting contrivance that has real costs.<sup>22</sup>

### 3. Results

The results are organized as follows. Section 3.1 considers the first-best solution where the regulator can observe the true cost. Section 3.2 develops some preliminary results. Section 3.3 assumes

<sup>21</sup>In the previous case, rent can be rewritten as  $r = t + x - \phi(x)$ , and the assumptions on  $\psi(x)$  imply the restrictions imposed on the function  $\phi(x)$ .

<sup>22</sup>This alternative interpretation also makes it clear that the cost padding formulation is isomorphic to the standard Laffont and Tirole (1993; Chapter 1) regulatory model that has ex post effort in cost reduction. To see this, pick a constant  $a$  and let  $e = a - x$ ,  $g_e(k) = g(k) + a$  and  $\psi_e(e) = -\psi(a - e)$  where  $e$  is effort in cost reduction,  $\psi_e(e)$  is the cost of effort and  $g_e(k)$  is the cost parameter. Then, the cost is  $C = g(k) + x = g_e(k) - e$ ,  $C + \psi(x) = C - \psi_e(e)$  and the incentive constraint (see equation (3') below) is  $\dot{r} = -\psi'(x)g'(k) = -\psi'_e(e)g'_e(k)$ . As costs are non-negative, it is always possible to find an  $a$  such that  $a \geq x$  so that  $e \geq 0$ . We thank the editor for pointing out this equivalence.

that the firm adopts some exogenously given mixed strategy and derives the optimum contract. Section 3.4 extends the analysis by deriving the equilibrium where the distribution of pre-contractual investment is determined endogenously. Section 3.5 considers some comparative statics properties. Finally, Section 3.6 compares the solutions of the exogenous and endogenous cases studied in Sections 3.3 and 3.4. Relevant proofs are in the Appendix.

#### 3.1. The first-best solution

As a benchmark, consider the case where the regulator can observe the true cost, and hence can infer  $k$ . For this benchmark case, there is no hidden action and no hidden information problem. The firm will choose its capital input to equate the marginal benefit of cost reduction to its price:

$$-g'(k) = \rho \quad (6)$$

The restrictions on the  $g$  function imply that there is a unique level of investment that solves the above equation. We let  $k^*$  denote the solution for this first-best level of capital.

#### 3.2. Preliminaries

The support of regulator's beliefs satisfies  $\mathcal{S}^r \subseteq [\underline{k}, \bar{k}]$  where  $\underline{k} \geq 0$  and  $\bar{k} \leq k_{max}$ . From Hellwig (2010; Lemma 2.7), there is no loss of generality in assuming that the contract menu  $(C(k), r(k))$  is defined for all  $k \in [\underline{k}, \bar{k}]$ .<sup>23</sup> In section 3.4, we shall show that  $\mathcal{S}^r = [\underline{k}, \bar{k}]$  and will determine the values of  $\underline{k}$  and  $\bar{k}$ .

Given that the contract can be extended to the interval, we rewrite the regulator's objective function as:

$$\int_{\underline{k}}^{\bar{k}} (V - (1 + \lambda)v(C(k), k) - \lambda r(k) - \rho k) dF^r(k) \quad (2')$$

Moreover, we can use a standard procedure (see, for example, Segal and Whinston 2002) to give necessary and sufficient conditions for incentive compatibility.<sup>24</sup>

<sup>23</sup>Lemma 2.7 of Hellwig (2010) shows that any contract  $\Delta$  satisfies the incentive compatibility and participation constraints on  $\mathcal{S}$  if and only if there is an extension of  $\Delta$  to the interval  $[\underline{k}, \bar{k}]$  that also satisfies the incentive compatibility and participation constraints on  $[\underline{k}, \bar{k}]$ .

<sup>24</sup>As the method is standard, the lemma is stated without proof.

**Lemma 1.** Necessary and sufficient conditions for incentive compatibility are for each  $k$  and  $k'$ .

- (i)  $r(k) = r(\underline{k}) - \int_{\underline{k}}^k v_k(C(\kappa), \kappa) d\kappa$
- (ii)  $-\int_{\underline{k}}^{k'} (v_k(C(k'), \kappa) - v_k(C(\kappa), \kappa)) d\kappa \geq 0.$

As  $v_k(C, k) < 0$ , the first part of Lemma 1 shows that higher rent must be paid to a low cost firm (high  $k$ ) to induce it to report its lower cost. This higher rent exactly reflects the reduction in costs due to higher investment. As  $k$  is increased by one unit, true cost falls by  $g'(k)$ , but for a given  $C$  there is a partially offsetting increase in falsification costs of  $\phi'(C - g(k))g'(k)$ . Consequently, the firm's rent increases by  $v_k(C, k) = g'(k)(1 - \phi'(C - g(k)))$ . Moreover, Lemma 1(i) shows that the derivative  $\dot{r}(k)$  satisfies:

$$\dot{r}(k) = -v_k(C(k), k) \quad (3')$$

at all points where  $r(k)$  is differentiable. The second part of Lemma 1 is the usual second-order condition expressed in integral form. We shall follow the standard procedure of ignoring this condition and impose assumptions (see Assumption 2 below) such that it is satisfied by the solution to the relaxed problem. With this procedure, all the incentive compatibility conditions of equation (3) can be replaced by equation (3').

Similarly, it is possible to follow a standard argument to simplify the participation constraint given in equation (4). Provided  $\lambda > 0$ , rent enters negatively into the objective function (2), and the regulator gains by a downward shift in the rent function  $r(k)$ . Therefore, because it follows from (3') that  $\dot{r}(k) \geq 0$ , the participation constraints can be replaced by the single equality condition:

$$r(\underline{k}) = 0 \quad (4')$$

Taking these simplifications into account, the problem for the regulator is to choose a contract  $\Delta = (C(k), r(k))$  to maximize (2') subject to (3') and (4').

We shall only consider consistent distributions where  $F^r(k) = F(k)$  from this point. Thus, we drop the superscript notation  $r$  and let  $F(k)$  refer to the equilibrium distribution.

### 3.3. Optimum contract with fixed distribution

In this subsection, it is assumed that the firm's mixed strategy is exogenously given. Let

$F(k)$  denote this fixed mixed strategy and let  $H(k) = -\log(1 - F(k))$  denote the hazard function.<sup>25</sup> For the moment, we assume:

**Assumption 2.** The distribution  $F(k)$  is continuously differentiable with a support on the closed interval  $[\underline{k}, \bar{k}]$  and has a positive density  $f(k) = F'(k) > 0$  on  $[\underline{k}, \bar{k}]$  with  $f(\bar{k}) \geq 0$ . In addition,  $H'(k)/g'(k)$  is decreasing in  $k$ .

Since  $F(k)$  is differentiable, it follows that  $H(k)$  is differentiable, and hence, that the hazard rate function is  $h(k) = f(k)/(1 - F(k))$ . We maintain Assumption 2 in this section. In Section 3.4, we shall show that the equilibrium distribution will also satisfy these properties.<sup>26</sup> Maintaining this assumption will allow us to directly compare the two cases. The extra condition, that  $H'(k)/g'(k)$  is decreasing, is imposed so that  $x(k)$ , and, hence  $C(k)$ , are decreasing in  $k$  at the solution of the relaxed problem. With  $C(k)$  decreasing, it is assured that the solution is optimal because it also satisfies the integral condition in Lemma 1(ii).

Given the integral equation in Lemma 1(i) and the fact the distribution function has a density, it is possible to integrate  $r(k)dF(k)$  by parts, using (3') and (4'), to rewrite the objective function (2') as the virtual surplus function:

$$\int_{\underline{k}}^{\bar{k}} (V - (1 + \lambda)v(C, k) - \rho k) dF(k) + \int_{\underline{k}}^{\bar{k}} \lambda v_k(C, k)(1 - F(k)) dk \quad (2'')$$

The objective of the regulator is to choose a cost reimbursement function  $C(k)$  that maximizes this virtual surplus.

Maximizing (2'') with respect to  $C$ , and using  $v_C(C, k) = \phi'(x)$  and  $v_{kC}(C, k) = -g'(k)\phi''(x)$ , yields the following first-order condition for  $k < \bar{k}$ :

$$h(k) = -\frac{\lambda}{1 + \lambda} \frac{\phi''(x)}{\phi'(x)} g'(k) \quad (7)$$

Given the distribution function, this equation determines the cost padding function  $x(k)$ . At the first-best solution  $\phi'(x) = 0$ , there is no cost padding. Equation (7) shows how the optimum

<sup>25</sup>Also known as the log-survival function.

<sup>26</sup>We do not assume that the density has  $f(\bar{k}) > 0$  because we shall show that the equilibrium distribution need not have this property (see Example 2 below).

contract differs from the first-best solution. Reducing cost padding by one unit reduces falsification costs by  $\phi'(x)$ . When cost padding is reduced for  $dF(k)$  cost types, the marginal social benefit is  $(1 + \lambda)\phi'(x)dF(k)dk$  because the shadow cost of the funds saved is  $(1 + \lambda)$ . The cost of this one unit reduction is, however, the extra rent that must be paid to all types in the interval  $[k, \bar{k}]$  to induce them to report lower costs. This extra rent is determined from equation (3). The change in the rent for a firm with capital  $k$  that follows from a one unit reduction in padded costs is  $\phi''(x)g'(k)dk$ . The social cost of an extra rent payment of one unit is  $\lambda$ , and, as  $(1 - F(k))$  cost types are affected, the social cost of the extra rent payments is  $\lambda(1 - F(k))\phi''(x)g'(k)$ . Equating this marginal social cost to the marginal social benefit and using the definition of the hazard rate function  $h(k)$  gives equation (7). Since there is no extra social cost at  $k = \bar{k}$ , there is a classic “efficiency at the top” result. Thus, there is no cost padding for the type with the lowest true costs,  $x(\bar{k}) = 0$ . We summarize this discussion in the following proposition.

**Proposition 1.** Under Assumption 2 and for a given mixed strategy  $F(k)$ , costs are padded in the optimum contract:  $x(k) = C(k) - g(k) > 0$  for all  $k < \bar{k}$  and  $x(\bar{k}) = 0$ .

It should be noted that the regulator will take into account that costs will be padded when setting the contract. Therefore, a firm that reports a high cost (low  $k$ ) may be required to make a transfer to the regulator to offset the fact that the regulator reimburses full costs. On the other hand, a firm that reports a low cost (high  $k$ ) may well receive a positive transfer from the regulator.

Equation (7) can also be used to understand when no cost padding will occur. It follows from (7) that there is no cost padding if  $g'(k) = 0$  or  $\phi''(x) = 0$ . If  $g'(k) = 0$ , then the unobserved investment does not influence cost. The regulator knows the true cost and there is no room for cost padding. Equally, there will be no cost padding if the costs of falsification are linear, that is, if  $\phi''(x) = 0$ . This result was shown by Lacker and Weinberg (1989), who assume a cost function of the type  $\phi(x) = \alpha|x|$ . The intuition is that if falsification costs are linear, then the cost of deterring cost padding is the same irrespective of its amount, and therefore, no extra rent has to be offered to lower cost types. Thus, if cost padding is deterred at all, even very small amounts of cost padding will be deterred as well.

We conclude this subsection with a short example to illustrate the nature of the optimum contract.

**Example 1.** Assume that the cost function satisfies  $g(k) = 2(2 - \sqrt{k})$ . Assume that falsification costs are quadratic  $\phi(x) = (\alpha/2)x^2$  for  $\alpha \leq 1$  and that the distribution function satisfies  $F(k) = \sqrt{k}$  on  $[0, 1]$ . We consider a special case where  $\rho = \lambda = 1$ . Then  $x(k) = 1 - \sqrt{k}$ , and rent is  $r(k) = 2(1 - \alpha)\sqrt{k} + \alpha k$ .<sup>27</sup> The total cost is  $C(k) = 5 - 3\sqrt{k}$ , which because it is decreasing in  $k$ , means that the second-order condition for incentive compatibility is satisfied. The transfer function is  $t(k) = 3(1 - \alpha)\sqrt{k} + (\alpha/2)(1 + 3k) - 1$ . The expected amount of cost padding is  $E[x] = 1/2$  and the expected falsification cost is  $E[\phi(x)] = \alpha/6$ . The expected total cost is  $E[C] = 7/2$ . The expected rent paid to the firm is  $E[r] = (1 - \alpha) + (\alpha/3)$ , and the expected transfer is  $E[t] = (1 - \alpha)/2$ . Investment is undertaken provided that  $V > 7 - (\alpha/3)$ .

### 3.4. Equilibrium distribution

In this section, we solve for the equilibrium mixed strategy. As discussed in Section 2, if the firm is to adopt a mixed strategy, its profits must be the same for each level of investment in the support. That is, the rent of the firm satisfies  $r(k') = r(k) + \rho(k' - k)$  for every  $k, k'$  in the support. As the investment  $k$  determines the firm type and the distribution of types is endogenous, we adopt a procedure similar to González (2004), but we must solve the contract for an arbitrary type set.<sup>28</sup> To do this, we use results from Hellwig (2010) that demonstrates how to solve optimum incentive schemes for arbitrary type distributions.<sup>29</sup>

We first characterize the support of the regulator’s beliefs.

<sup>27</sup>The assumption  $\alpha \leq 1$  guarantees that  $r(k)$  is increasing for each  $k \in [0, 1]$ . We show in the next subsection that the equilibrium distribution for this example is  $F(k) = 1 - (1 - \rho\sqrt{k})^{(\alpha/\rho)}$ . Thus, the fixed distribution in this example is an equilibrium distribution for  $\rho = \alpha = 1$ .

<sup>28</sup>Our model is not directly comparable to González (2004). His model has a variable quantity but no cost padding. In our model, if there were no cost padding, the regulator could implement the first-best, and hence, the distribution of types would be degenerate. González (2004) assumes that the type set is continuous.

<sup>29</sup>An alternative approach applied to auction theory can be found in Monteiro and Svaiter (2010). Hellwig (2010), however, provides a characterisation of the optimum that is useful for our purposes.

**Lemma 2.** In equilibrium, the support of the regulator's beliefs  $\mathcal{S}^r$  is the closed interval  $[0, k^*]$ .

The intuition for why the support is an interval is straightforward. If the support is not an interval, then the firm can choose an intermediate investment level not in the support, report a lower investment level that is within the support and increase profits. This shows that the regulator's beliefs are defined on an interval  $[\underline{k}, \bar{k}]$ . The property that the lower endpoint  $\underline{k} = 0$  is a consequence of the hold-up problem for the firm's investment in cost reduction. In particular, once the firm has invested in cost-reducing activity at the pre-contractual stage, the regulator can extract the entire rent from the highest cost firm. Therefore, if the firm is to have non-negative profits ex ante, a zero investment in cost reduction must be feasible. Therefore, it follows from the participation constraint (4') that the rent function satisfies  $r(k) = \rho k$ .<sup>30</sup> Hellwig (2010; Corollary 2.2) shows that provided the strict single crossing property is satisfied, there is downward distortion for all types except at the upper endpoint, where there is no distortion. Given this property of no distortion at the upper endpoint, costs are minimized by setting  $\bar{k} = k^*$ .

We now proceed to describe the equilibrium distribution and optimum contract. Before proceeding, however, we show that the equilibrium distribution is continuous and has no mass points.

**Lemma 3.** In any equilibrium in which the firm adopts a mixed strategy, there are no mass points.

The idea of the proof is to consider a distribution with mass points and show that a change in the distribution that smooths out the discontinuity by appropriately shifting the distribution function while maintaining the original contract can increase the virtual surplus. Clearly, changing the contract to the optimum for the new distribution will increase the virtual surplus further, or at least, not decrease it.

It follows from the integral equation of Lemma 1(i) and the equilibrium condition  $r(k) = \rho k$  that

$$-v_k(C(k), k) = \rho \quad (8)$$

<sup>30</sup>As the rent just covers the investment cost, it may be viewed as a rent to the quasi-fixed factor at the contractual stage rather than an information rent.

This condition, together with the regulator's first-order condition (7) can be used jointly to determine the properties of the optimum contract  $\Delta$  and equilibrium distribution  $F(k)$ .

**Proposition 2.** In equilibrium, cost padding  $x(k)$  is determined by equation (8). The cost padding function  $x(k)$  is continuous, differentiable and decreasing with  $x(k^*) = 0$ . The equilibrium distribution  $F(k)$  is determined by  $F(k) = 1 - \exp(-\int_0^k h(\kappa) d\kappa)$  where  $h(k)$  is determined by equation (7) with  $x = x(k)$ . The distribution function is continuously differentiable with a support on the closed interval  $[0, k^*]$  and has a positive density  $f(k) = F'(k) > 0$  on  $[0, k^*)$  with  $f(k^*) \geq 0$ . The hazard function  $H(k)$  is differentiable and  $H'(k)/g'(k)$  is decreasing in  $k$ .

Proposition 2 demonstrates that the equilibrium distribution satisfies the properties assumed in Assumption 2. In particular, the equilibrium distribution satisfies the requirement which is sufficient for the incentive compatibility constraints of the regulator's problem to be satisfied.

### 3.5. Comparative statics

As a first comparative static exercise, consider how the solution changes as the shadow cost of public funds  $\lambda$  changes. Suppose that the shadow cost increases from  $\lambda$  to  $\lambda'$ . This leads to an increase in the hazard rate function. Abusing the notation slightly, we have from equations (8) and (7) that  $h_{\lambda'}(k) > h_{\lambda}(k)$  for all  $k$ . Integrating the hazard rate, therefore, shows that the distribution generated with  $\lambda$ , *first-order stochastically dominates* the distribution generated with  $\lambda'$ . That is, the distribution with  $\lambda'$  puts more weight on low values of  $k$  (higher costs) than does the distribution generated with  $\lambda$ . Hence,  $F_{\lambda'}(k) > F_{\lambda}(k)$  for  $k \in (0, k^*)$ . It follows from equation (8) that  $C(k)$  is decreasing in  $k$ . Thus, first-order stochastic dominance implies that expected costs are higher with the distribution generated with  $\lambda'$  than the one generated with shadow cost  $\lambda$ . We write this as  $E_{\lambda'}[C(k)] > E_{\lambda}[C(k)]$  where the subscript indicates that the distribution over which the expectation is taken depends on  $\lambda$ . Likewise, as  $x(k)$  is decreasing in  $k$ , a higher value of the shadow cost of public funds will be associated with higher expected cost padding. Using equation (8) and given that  $C(k)$  is decreasing in  $k$ ,  $v(C(k), k) + \rho k$  is decreasing in  $k$ ,  $v_k(C, k) = -\rho$  and  $v_C(C, k) > 0$ . Therefore,  $(1 +$

$\lambda')E_{\lambda'}[v(C(k), k) + \rho k] > (1 + \lambda)E_{\lambda}[v(C(k), k) + \rho k]$ . Thus, expected welfare falls as the shadow cost of public funds rises.

Consider the case in which  $\phi(x) = (\alpha/\beta)x^{\beta}$  where  $\alpha > 0$ ,  $\beta > 1$  and  $x \in [0, \alpha^{1/(1-\beta)}]$  and consider a change in the falsification cost parameter  $\alpha$ . Let  $x_{\alpha}(k)$  denote cost padding with parameter  $\alpha$  and  $x_{\alpha'}(k)$  denote cost padding with parameter  $\alpha'$ . From equation (8),  $x_{\alpha'}(k) = \delta x_{\alpha}(k)$  where  $\delta = (\alpha/\alpha')^{1/(\beta-1)}$ . With an increase in  $\alpha$  to  $\alpha'$ , and because  $\delta < 1$ , the change will lower the amount of cost padding for each given value of  $k < k^*$ . This is the direct effect on cost padding of an increase in falsification costs. However, there is an indirect effect on cost padding because the distribution of investment changes. Equations (7) and (8) show that  $h_{\alpha'}(k) = (1/\delta)h_{\alpha}(k)$ . Thus, for  $\alpha' > \alpha$ , the distribution with parameter  $\alpha$  first-order stochastically dominates the distribution with parameter  $\alpha'$ . Hence,  $F_{\alpha'}(k) > F_{\alpha}(k)$  for  $k \in (0, k^*)$ . In particular, integrating the hazard functions yields  $F_{\alpha'}(k) = 1 - (1 - F_{\alpha}(k))^{1/\delta}$ . Thus, although cost padding falls for all values of  $k$ , the distribution changes to put more weight on lower values of  $k$ . As  $x(k)$  is decreasing in  $k$ , the effect of the change in the distribution is to increase expected cost padding. It is shown in the following proposition, however, that the direct effect dominates and the net effect of the increase in the falsification cost is to reduce expected levels of cost padding. Proposition 3 also indicates that expected falsification costs  $E[\phi(x)]$  decrease in  $\alpha$  and that expected costs  $E[g(k) + \rho k]$  increase in  $\alpha$ .

**Proposition 3.** An increase in the falsification cost from  $\alpha$  to  $\alpha'$  reduces expected cost padding:  $E_{\alpha'}[x_{\alpha'}] < E_{\alpha}[x_{\alpha}]$ ; reduces falsification costs:  $E_{\alpha'}[\phi(x_{\alpha'})] < E_{\alpha}[\phi(x_{\alpha})]$ ; and increases expected costs (net of falsification costs):  $E_{\alpha'}[g(k) + \rho k] > E_{\alpha}[g(k) + \rho k]$ .

The net effect on welfare is generally ambiguous. Although expected falsification costs fall with an increase in  $\alpha$ , the leftward shift of the distribution increases expected real costs.<sup>31</sup>

As another comparison, consider a change in the cost of capital  $\rho$ .<sup>32</sup> Since the upper endpoint of the distribution  $k^*$  depends on  $\rho$ , an increase in  $\rho$  to

<sup>31</sup>A simple example of the effect of a change in  $\alpha$  on welfare is given below.

<sup>32</sup>An essentially similar comparative static exercise is to introduce a parameter to measure the effectiveness of pre-

$\rho'$  will lower the upper endpoint of the distribution from  $k^*(\rho)$  to  $k^*(\rho')$ . Also, from equation (8), the increase in  $\rho$  will reduce cost padding for each  $k > 0$ . Thus, the direct effect of an increase in  $\rho$  is to reduce cost padding. The indirect effect of a change in  $\rho$  through the change in  $x$  again shifts the distribution function leftward so the distribution with parameter  $\rho$  first-order stochastically dominates the distribution with parameter  $\rho'$ . However, the net effect on cost padding will, in general, depend on parameters of the cost function. The effect on the expected capital of the firm is unambiguously negative because the distribution has shifted leftward. However, the effect on rent is ambiguous because, although capital is decreased, the cost of capital has increased. Thus, the overall effect on welfare will be ambiguous, in general.

Finally, we give a simple example to illustrate how the solution depends on the cost of falsification  $\alpha$  and the cost of capital  $\rho$ .

**Example 2.**  $g(k) = 2(2 - \sqrt{k})$ ;  $\phi(x) = (\alpha/2)x^2$ ,  $\lambda = 1$ . Equation (8) indicates that cost padding is  $x(k) = (1 - \rho\sqrt{k})/\alpha$ . The distribution function is defined on  $[0, k^*]$  where  $k^* = 1/(\rho^2)$ .<sup>33</sup> Equations (7) and (8) determine the hazard rate function  $h(k)$  and hence the distribution function  $F(k) = 1 - (1 - \rho\sqrt{k})^{(\alpha/\rho)}$ . The density function,  $f(k) = F'(k)$ , has  $\lim_{k \rightarrow k^*} f(k) = 0$  for  $\alpha > \rho$ ,  $\lim_{k \rightarrow k^*} f(k) = \rho^2/2$  for  $\alpha = \rho$ , and  $\lim_{k \rightarrow k^*} f(k) = \infty$  for  $\alpha < \rho$ . The transfer function is  $t(k) = \rho k + (\rho^2 k)/(2\alpha)$ . Expected cost padding is  $E[x] = 1/(\alpha + \rho)$ , expected cost is  $E[g] = 4 - (2/(\alpha + \rho))$ , expected falsification cost is  $E[\phi(x)] = 1/(2(\alpha + 2\rho))$  and expected rent is  $E[r] = \rho E[k] = 2\rho/((\alpha + \rho)(\alpha + 2\rho))$ . Expected welfare is  $E[W] = V - 8 + (3/(\alpha + 2\rho))$ . It can be seen that in this example, expected cost padding is decreasing in  $\alpha$  and  $\rho$  so that the direct effect dominates. Expected costs increase in  $\alpha$  and  $\rho$ . The expected rent decreases with  $\alpha$  but is non-monotonic in  $\rho$ . Expected welfare decreases with  $\alpha$  and  $\rho$ .

### 3.6. Endogenous versus exogenous distributions

The solution when the distribution of pre-contractual investment choices is determined en-

contractual investment in reducing costs. For example, consider replacing  $k$  in the production function by  $k/\theta$ . In this case, an increase in  $\theta$  decreases the effectiveness of pre-contractual investment and will have exactly the same effect as an increase in  $\rho$ .

<sup>33</sup>Therefore, it follows that  $\phi'(x) = \alpha x < 1$  for all  $k \in [0, k^*]$ .

dogenously might be quite different from the solution when the distribution of investment choices is exogenously given, (as in the standard model Laffont and Tirole 1986). This section will illustrate some of these differences.

Consider the case in which the cost of falsification is  $\phi(x) = (\alpha/\beta)x^\beta$  where  $\alpha > 0$ ,  $\beta > 1$  and  $x \in [0, \alpha^{1/(1-\beta)}]$ . Fix some level of  $\alpha$ , say  $\bar{\alpha}$ . Suppose that in the exogenous case, the hazard rate function  $h^e(k)$  is set equal to the hazard rate in the endogenous case with  $\alpha = \bar{\alpha}$ . With a fixed distribution, cost padding is determined directly from equation (7) and the amount of cost padding is independent of  $\alpha$ . Denote the level of cost padding in this exogenous case to be  $x^e(k)$ . From Proposition 3, cost padding  $x(k)$  in the endogenous case is inversely related to  $\alpha$ . It is easy to check that  $x(k) = \delta x^e(k)$  and  $h(k) = (1/\delta)h^e(k)$  where  $\delta = (\bar{\alpha}/\alpha)^{(1/(\beta-1))}$ . Thus, for  $\alpha < \bar{\alpha}$ , we have  $x(k) > x^e(k)$  for each value of  $k$  and cost padding is raised relative to the exogenous case. However, as  $h(k) < h^e(k)$  for  $\alpha < \bar{\alpha}$ , integration shows that  $(1 - F(k)) = (1 - F^e(k))^{(1/\delta)}$  so that the distribution function shifts rightward and the endogenous distribution first-order stochastically dominates the exogenous distribution. Nevertheless, using Proposition 3, it follows that the direct effect dominates and  $E[x(k)] \geq E[x^e(k)]$  as  $\bar{\alpha} \geq \alpha$ .

**Table 1**

The comparative statics of changes in  $\alpha$  in the exogenous (X) and endogenous (N) cases.

	X	N
Expected Cost Padding ( $E[x(k)]$ )	0	-
Expected Costs ( $E[g(k) + \rho k]$ )	0	+
Expected Falsification Costs ( $E[\phi(x)]$ )	+	-
Expected Welfare ( $E[W]$ )	?	?

In the exogenous case, expected falsification costs  $E[\phi(x^e(k))]$  increase linearly in  $\alpha$ . In contrast, Proposition 3 shows that because the distribution adjusts to an increase in  $\alpha$  by reducing cost padding, the expected falsification costs  $E[\phi(x(k))]$  decrease even though the distribution shifts rightward. Likewise, in the exogenous case, an increase in  $\alpha$  leaves true costs unchanged whereas, the distribution shifts to the right and thus raises expected

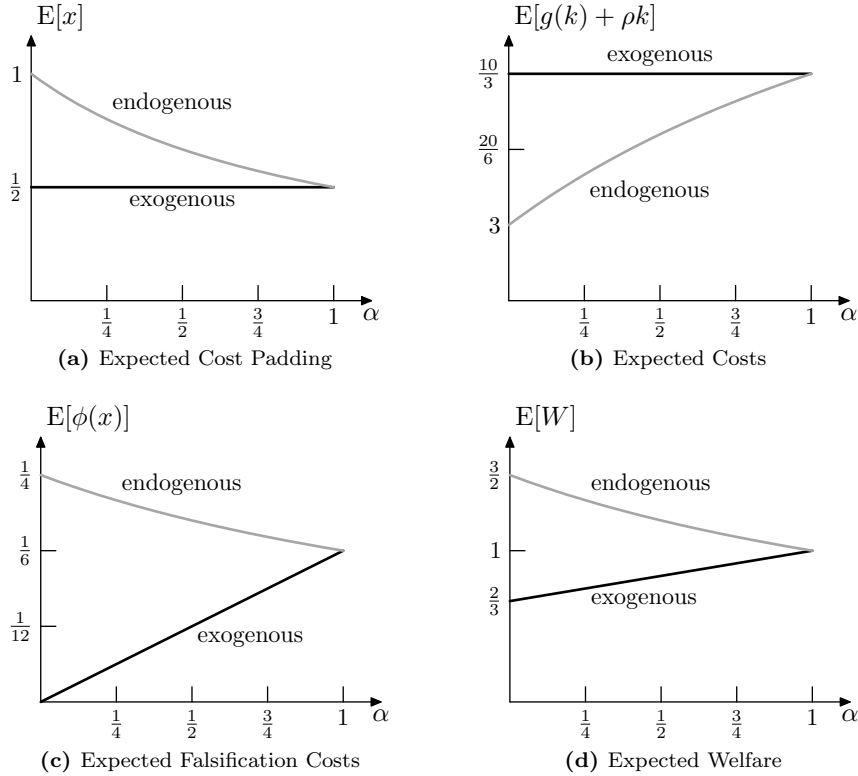
costs in the endogenous case. In both the exogenous and endogenous cases, the expected rent falls with an increase in  $\alpha$  but for rather different reasons. In the exogenous case, an increase in  $\alpha$  raises marginal falsification costs and thus reduces the increase in rent that is offered to lower cost types. In the endogenous case, rent just covers capital costs, but the shift in the distribution towards lower investment reduces the expected rent. In both cases, the effect of a change in  $\alpha$  on welfare is generally ambiguous. These effects are summarized in Table 1.

Although the effect of a change in  $\alpha$  on welfare is generally ambiguous, the effect may be quite different in the two cases.<sup>34</sup> Figure 1 provides an illustration comparing Example 1, where the distribution function is exogenously given by  $F(k) = \sqrt{k}$  on  $[0, 1]$ , and Example 2 (setting  $\rho = 1$ ), where the distribution function endogenously adjusts to the change in  $\alpha$  and is given by  $F(k) = 1 - (1 - \sqrt{k})^\alpha$  on  $[0, 1]$ . For  $\alpha = 1$ , the two distribution functions are equal, and the solutions are identical. However, as  $\alpha$  is lowered below one, the two solutions will diverge.<sup>35</sup> Figures 1a–1d plot expected cost padding, expected total costs, expected falsification costs and expected welfare.<sup>36</sup> The darker line depicts the solution in the exogenous case, indicates and the lighter line the solution when the distribution adjusts endogenously to a change in  $\alpha$ . As can be seen from the diagram, expected cost padding and expected falsification costs are higher in the endogenous case for  $\alpha < 1$  (Figure 1a and Figure 1c). However, real costs are lower in the endogenous case, as shown in Figure 1b. The net effect on welfare is illustrated in Figure 1d. Welfare is higher in the endogenous case but is decreasing in  $\alpha$ . In the exogenous case, in contrast, welfare increases with  $\alpha$ . Figure 1 illustrates the distinction between the exogenous case of a fixed distribution and the endogenous case where the distribution is determined in equilibrium by pre-contractual investment in cost reduction. It shows that this distinction can be critical for welfare, and hence, have potentially important policy implications. For example, consider a policy that intends to raise falsification costs through improved accountancy regulations or

<sup>34</sup>In what follows, recall that expected welfare is given by  $E[W] = V - (1 + \lambda)(E[v(C(k), k)]) - \lambda E[r(k)] - \rho E[k]$ .

<sup>35</sup>The solution in Example 1 is only defined for  $\alpha < 1$ .

<sup>36</sup>The value of the project has been set at  $V = 8$ . This value of  $V$  is sufficient for the project to be worthwhile.



**Figure 1.** Comparison of Example 1 (exogenous case) and Example 2 (endogenous case). Parameters are  $\rho = \lambda = 1$  and  $V = 8$ .

closer monitoring (raise  $\alpha$ ). This would lead to an increase in welfare if the distribution is fixed. However, in the endogenous case, although this will lead to a fall in falsification costs, it will also lead to a shift in the distribution toward lower investment and higher expected costs. Overall, such a policy would, in this example, reduce welfare.

#### 4. Conclusion

This paper has adapted the standard model of procurement in regulated monopolies to allow for both cost padding and incentives for cost reduction at the pre-contractual stage. By allowing firms to undertake a pre-contractual investment in cost reduction, the distribution of cost types is derived endogenously. Furthermore, with this change, the properties of the optimum cost reimbursement contract depend only on the fundamental technology and preference parameters of the model and not on the distribution of cost types. The paper has shown the importance of studying this pre-contractual

choice of firms for welfare. The welfare implications of the two cases, where the distribution of cost types is given exogenously and where the distribution of cost types is determined endogenously by pre-contractual investment in cost reduction, can be diametrically opposed.

#### Appendix

**Proposition 1.** Under Assumption 2 and for a given mixed strategy  $F(k)$ , costs are padded in the optimum contract:  $x(k) = C(k) - g(k) > 0$  for all  $k < \bar{k}$  and  $x(\bar{k}) = 0$ .

*Proof.* The virtual surplus function of equation (2'') is strictly concave in  $C$  because  $v(C, k)$  is strictly convex in  $C$  and  $v_k(C, k)$  is concave in  $C$  from Assumption 1. Thus, the first-order condition (7) describes a global maximum. For  $k < \bar{k}$ ,  $dF(k) > 0$  and  $F(k) < 1$ . As  $\phi''(x) > 0$  and  $g'(k) < 0$ , it follows from the first-order condition (7) that  $\phi'(x) > 0$  and  $x(k) > 0$ , so costs are padded. If  $dF(\bar{k}) > 0$ , then (7) shows  $x(\bar{k}) = 0$ . If

$dF(\bar{k}) = 0$ , then, as  $\lim_{k \nearrow \bar{k}} (1 - F(k))/dF(k) = 0$ , it follows that  $\lim_{k \nearrow \bar{k}} x(k) = 0$ , where the notation  $\lim_{k \nearrow \bar{k}}$  indicates that the limit of a sequence converging to  $\bar{k}$  from below. Rewrite (7) as

$$-\frac{\phi''(x)}{\phi'(x)} = \frac{1 + \lambda}{\lambda} \frac{H'(k)}{g'(k)} \quad (\text{A.1})$$

The left-hand-side is increasing in  $x$  from Assumption 1, and the right-hand-side is decreasing in  $k$  by Assumption 2. Thus,  $x(k)$  is decreasing in  $k$ , and hence, because  $g(k)$  is decreasing,  $C(k) = g(k) + x(k)$  is decreasing in  $k$ . Therefore, it follows from  $v_{kC}(C, k) > 0$  that  $v_k(C(\kappa), \kappa) \geq v_k(C(k'), \kappa)$  for  $k' > \kappa$ . Hence, the condition in Lemma 1(ii) is satisfied, and the contract is incentive compatible.  $\square$

**Lemma 1.** In equilibrium, the support of the regulator's beliefs  $\mathcal{S}^r$  is the closed interval  $[0, k^*]$ .

*Proof.* The proof proceeds in four parts. We first show that if beliefs are consistent, then the set  $\mathcal{S}^r$  is non-degenerate and there cannot be a pure strategy equilibrium where one investment level is chosen. Next, we show that if there are isolated points in the belief set, then the downward incentive compatibility constraints bind for any two isolated points that are adjacent. Next, we show that in equilibrium, the belief set is an interval as otherwise, it would be profitable for the firm to choose a higher investment not in the set while reporting a lower investment. Finally, we determine the endpoints of the distribution.

Part a: We first rule out the existence of a pure strategy. Suppose the regulator were to believe that the firm chooses some  $\hat{k} > 0$  with probability one. In this case, the regulator will reimburse the cost  $C(\hat{k}) = g(\hat{k})$  and make no transfer to the firm. This, however, cannot be an equilibrium because, were the firm to choose  $\hat{k}$ , it would pad no costs and receive no rent from the contract, thereby generating profits of  $-\rho\hat{k} < 0$ . Thus, a deviation to choosing zero investment would increase profits. The only other candidate for a pure strategy equilibrium is where the regulator believes the firm chooses  $k = 0$  with probability one. In this case, the regulator reimburses costs  $C(0) = g(0)$ . Consider then the deviation where the firm marginally increases investment by  $dk > 0$  and pads costs so that total costs remain at  $C(0)$ . Real costs decrease by  $g'(0)dk$ , and investment costs increase by  $\rho dk$ . The cost of padding costs is zero on the margin (as  $\phi'(0) = 0$ ). Hence, the increase in profits from the deviation is  $(-g'(0) - \rho)dk$ , which is positive by assumption. Therefore, the deviation is profitable and a pure strategy where  $k = 0$  is not an equilibrium.

Part b: The support of a distribution is the mini-

mal closed set whose complement has probability zero. Suppose, contrary to the lemma, that the support  $\mathcal{S}^r \subseteq [0, k_{max}]$  is not an interval. Then, there are two values  $k_0$  and  $k_1$  such that  $k_0 \in \mathcal{S}^r$  and  $k_1 \in \mathcal{S}^r$  and  $(k_0, k_1) \cap \mathcal{S}^r = \emptyset$ . We show that an optimum contract has  $r(k_1, k_1) = r(k_0, k_1)$ . Suppose, to the contrary, that  $r(k_1, k_1) - \xi = r(k_0, k_1)$  where  $\xi > 0$ . That is, suppose:

$$\begin{aligned} t(k_1) + C(k_1) - v(C(k_1), k_1) - \xi \\ = t(k_0) + C(k_0) - v(C(k_0), k_1) \end{aligned}$$

By incentive compatibility,  $r(k_0, k_0) \geq r(k^-, k_0)$  for any  $k^- < k_0$ . In addition,  $C(k^-) \geq C(k_0)$  for any  $k^- < k_0$ . Using  $C(k^-) \geq C(k_0)$  and the single-crossing property  $v_{Ck} > 0$ , it follows that  $r(k_0, k_1) \geq r(k^-, k_1)$ . Thus,  $r(k_1, k_1) - \xi \geq r(k^-, k_1)$ . Now, suppose the regulator lowers  $t(k_1)$  by  $\xi$  and lowers  $t(k^+)$  by the same amount for all  $k^+ > k_1$ . This change preserves incentive compatibility. It will not affect incentive compatibility for pairs of  $k$  above  $k_1$  where both sides are reduced by  $\xi$  or for constraints between pairs of  $k$  below  $k_0$  where there is no change. The upward incentive compatibility constraints between  $k^- \leq k_0$  and  $k^+ \geq k_1$  are relaxed. The downward incentive compatibility constraints between pairs  $k^+ \geq k_1$  and  $k^- \leq k_1$  are satisfied by the above construction. Such a reduction in transfers also preserves the participation constraint. Thus, the change increases the regulator's welfare. Therefore, in any optimum, the adjacent downward incentive constraints bind with  $\xi = 0$ .

Part c: We shall now show that if there are two values  $k_0$  and  $k_1$  such that  $k_0 \in \mathcal{S}^r$  and  $k_1 \in \mathcal{S}^r$  and  $(k_0, k_1) \cap \mathcal{S}^r = \emptyset$ , then, given the corresponding optimum contract, the firm can always deviate and raise profits by choosing some  $k \in (k_0, k_1)$  and reporting  $k_0$ . Abusing the notation, let  $C_0$  denote profit,  $t_0$  denote the transfer and so on when  $k_0$  is reported, and let  $\pi_0$  denote profits when  $k_0$  is reported and chosen. Then

$$\begin{aligned} \pi_0 &= t_0 + C_0 - v(C_0, k_0) - \rho k_0 \\ \pi_1 &= t_0 + C_0 - v(C_0, k_1) - \rho k_1 \end{aligned}$$

where the second equation uses the fact that the downward incentive compatibility constraint binds. Now, suppose that the firm chooses some  $k \in (k_0, k_1)$ . Let  $\pi(k_0, k)$  denote the level of profits from choosing  $k$  but reporting  $k_0$ . For this to be an equilibrium, the original contract must remain incentive compatible,  $\pi(k_0, k) < \pi_0 = \pi_1$ . Using the previous equations

$$\begin{aligned} \pi(k_0, k) - \pi_0 &= v(C_0, k_0) - v(C_0, k) - \rho(k - k_0) \\ \pi(k_0, k) - \pi_1 &= v(C_0, k_1) - v(C_0, k) + \rho(k_1 - k) \end{aligned}$$

Therefore, if the firm is not to gain from deviation, the following conditions must hold:

$$\begin{aligned} v(C_0, k_0) - v(C_0, k) &< \rho(k - k_0) \\ v(C_0, k) - v(C_0, k_1) &> \rho(k_1 - k) \end{aligned}$$

Taking any  $k$  such that  $k_0 < k \leq \frac{1}{2}(k_0 + k_1)$  and given that  $v$  is strictly decreasing, this implies that

$$\begin{aligned} 0 < v(C_0, k_0) - v(C_0, k) &< \rho(k - k_0) \leq \rho(k_1 - k) \\ &< v(C_0, k) - v(C_0, k_1) \end{aligned}$$

However, for  $k = \frac{1}{2}(k_0 + k_1)$ , this contradicts the strict convexity of  $v$  in  $k$  posited in Assumption 1. Thus, if  $\mathcal{S}^r$  is not an interval, the firm will always gain by deviating and choosing some investment level not in  $\mathcal{S}^r$  between two adjacent points in  $\mathcal{S}^r$  and reporting the lower investment level (higher cost).

**Part d:** At the pre-contractual stage, the firm has the option not to undertake any extra investment in cost reducing activity. With  $k$  known, the regulator will leave the firm with no profits. The firm can do no worse than this, so there is an ex ante constraint that  $r(k) - \rho k \geq 0$ . As  $r(\bar{k}) = 0$  from equation (4'), this implies  $\bar{k} = 0$ , such that the lower endpoint of the distribution represents no ex ante investment in cost reducing activity. Now, consider the upper endpoint. If the firm were to choose a  $k > k^*$  with positive probability, there would be an advantage to reduce  $\bar{k}$  by  $dk$  thus leading to a fall in  $g(\bar{k}) + \rho\bar{k}$ . The only efficiency benefit from not doing so would be if there were additional efficiency savings from a reduction in falsification costs. Applying Corollary 2.2 of Hellwig (2010), there is no cost padding at  $\bar{k}$ . Thus, there are no such benefits and it is better to reduce  $\bar{k}$  to  $k^*$ . Equally, if  $\bar{k} < k^*$ , there is an efficiency benefit from increasing  $\bar{k}$  as  $g(\bar{k}) + \rho\bar{k}$  falls.  $\square$

**Lemma 2.** In any equilibrium in which the firm adopts a mixed strategy, there are no mass points.

*Proof.* We let  $F(k-) \equiv \lim_{h \searrow 0} F(k - h)$  be the left-hand limit where the notation  $\lim_{h \searrow 0}$  indicates that the limit is approached as  $h$  tends to zero for  $h > 0$ . Because the distribution is right continuous,  $F(k+) \equiv \lim_{h \searrow 0} F(k + h) = F(k)$ . Also as the distribution function is monotonic, it can have, at most, a countable set of discontinuities. We let  $f(k_i) \equiv F(k_i) - F(k_i-) > 0$  denote a mass or saltus at  $k_i$ . Suppose there is a mass point at  $k_i$ . Thus we define a new distribution  $F_\epsilon(k)$  such that

$$F_\epsilon(k) = \begin{cases} F(k) + \frac{f(k_i)}{\epsilon}(k - (k_i + \epsilon)) & \text{if } k \in [k_i, k_i + \epsilon] \\ F(k) & \text{otherwise} \end{cases}$$

where  $\epsilon > 0$ . We can choose  $\epsilon$  small so that  $F_\epsilon(k)$  is continuous on  $[k, k + \epsilon]$ . Note the  $F_\epsilon(k_i) = F(k_i) - f(k_i) = F(k_i-)$  and  $F_\epsilon(k_i + \epsilon) = F(k_i + \epsilon)$ . As  $F$  is continuous on  $(k_i, k_i + \epsilon]$ , we have  $dF_\epsilon(k) = dF(k) + (f(k_i)/\epsilon)$  on this interval. As  $r(k) = \rho k$ , it is possible to integrate  $r(k)dF(k)$  by parts to rewrite the regulator's objective as:

$$\begin{aligned} W &= \int_0^{k^*} (V - (1 + \lambda)v(C(k), k)) dF(k) \\ &\quad - (1 + \lambda)\rho \int_0^{k^*} (1 - F(k)) dk \end{aligned}$$

Consider a change in the distribution from  $F(k)$  to  $F_\epsilon(k)$ , where the contract remains unchanged. The contract remains incentive compatible but may no longer be optimal. For notational simplicity, we write  $C = C(k)$ ,  $C_i = C(k_i)$ ,  $f_i = f(k_i)$  and so on. Using the above equation, the effect of such a change in distribution on the surplus is

$$\begin{aligned} \Delta W &= \int_{k_i}^{k_i + \epsilon} (V - (1 + \lambda)v(C, k)) (dF_\epsilon(k) - dF(k)) \\ &\quad - (V - (1 + \lambda)v(C_i, k_i)) f_i \\ &\quad - (1 + \lambda)\rho \int_{k_i}^{k_i + \epsilon} ((1 - F_\epsilon(k)) - (1 - F(k))) dk \end{aligned}$$

Using the definition for  $F_\epsilon$ , this can be rewritten as:

$$\begin{aligned} \Delta W &= (1 + \lambda) \left( \frac{f_i}{\epsilon} \right) \int_{k_i}^{k_i + \epsilon} (v(C_i, k_i) - v(C, k)) dk \\ &\quad + (1 + \lambda) \left( \frac{f_i}{\epsilon} \right) \int_{k_i}^{k_i + \epsilon} (\rho(k - (k_i + \epsilon))) dk \end{aligned}$$

We first show that  $v(C_i, k_i) - v(C, k) > 0$ . First, as  $v(C, k)$  is strictly decreasing in  $k$  and  $k > k_i$ ,  $v(C, k_i) > v(C, k)$ . Next, as  $-v_k(C, k) = \rho$ , then  $v_k(C_i, k_i) = v_k(C, k)$ . Since  $v_{kk}(C, k) > 0$ ,  $v_k(C, k) > v_k(C, k_i)$ , and hence,  $v_k(C_i, k_i) > v_k(C, k_i)$ . This last inequality implies that  $C_i > C$  as  $v_{kC}(C, k) > 0$ . Hence, as  $v_C(C, k) > 0$ ,  $v(C_i, k_i) > v(C, k_i)$  so that  $v(C_i, k_i) > v(C, k)$  as required. As  $\rho > 0$  and  $k > k_i$ , it follows that we can find a number  $\gamma > 0$  such that  $v(C_i, k_i) - v(C, k) + \rho(k - k_i) \geq \gamma$  for each  $k \in [k_i, k_i + \epsilon]$ . Then, computing the change in welfare, it is easily confirmed that  $\Delta W \geq (1 + \lambda)f_i(\gamma - \rho\epsilon)$ . Thus, provided  $\epsilon < \gamma/\rho$ , then  $\Delta W > 0$ . As this improvement is achieved without changing the contract as the distribution is changed, it is clear that the optimum contract in the new distribution cannot lower welfare. Hence, the original distribution could not have been an equilibrium. Finally, note that the above arguments rule out mass points by changing the distribution to the right

of the mass point. Thus, it does not work when there is a mass point at  $k^*$ . When there is a mass of  $f_*$  at  $k^*$ , a similar argument works however, by defining a distribution

$$F_\epsilon(k) = \begin{cases} F(k) + \frac{f_*}{\epsilon}(k - (k^* - \epsilon)) & \text{if } k \in [k^* - \epsilon, k^*] \\ F(k) & \text{otherwise} \end{cases}$$

that smooths out the mass point to the left. Reapplying the same argument as above also shows that such a change increases welfare, and thus the original distribution with mass at  $k^*$  could not have been an equilibrium.  $\square$

**Proposition 2.** In equilibrium, cost padding  $x(k)$  is determined by equation (8). The cost padding function  $x(k)$  is continuous, differentiable and decreasing with  $x(k^*) = 0$ . The equilibrium distribution  $F(k)$  is determined by  $F(k) = 1 - \exp(-\int_0^k h(\kappa) d\kappa)$  where  $h(k)$  is determined by equation (7) with  $x = x(k)$ . The distribution function is continuously differentiable with a support on the closed interval  $[0, k^*]$  and has a positive density  $f(k) = F'(k) > 0$  on  $[0, k^*]$  with  $f(k^*) \geq 0$ . The hazard function  $H(k)$  is differentiable and  $H'(k)/g'(k)$  is decreasing in  $k$ .

*Proof.* As  $r(k)$  is increasing and because  $\dot{r}(k)$  is integrable,  $r(k)dF(k)$  can be integrated to show that the regulator's objective function may be rewritten in the same form as equation (2'). Therefore, the first-order condition for the regulator's problem is given by equation (7). Equation (8) can be solved for  $x$  to give  $x(k) = \phi^{-1'}(1 + (\rho/g'(k)))$ . By the implicit function theorem,  $x(k)$  is continuous and differentiable. Given  $g''(k) > 0$ , it then follows that  $x(k)$  is decreasing in  $k$ . Using  $-g'(k^*) = \rho$  and  $\phi'(0) = 0$ , it follows that  $x(k^*) = 0$ . As  $x(k)$  is decreasing, so is  $C(k)$ , and therefore, the condition in Lemma 1(ii) is satisfied and the contract is incentive compatible. Substituting  $x(k)$  into equation (7) shows that  $h(k)$  is continuous and  $h(k) > 0$ . Lemma 2 indicates that the distribution is defined on  $[0, k^*]$ . As  $h(k)$  is continuous,  $F(k)$  is differentiable, and hence, has a density of  $F'(k) = f(k) = (1 - F(k))h(k)$ , which is positive for  $k \in [0, k^*]$ . As  $x(k)$  is decreasing in  $k$  and because  $-\phi''(x)/\phi'(x)$  is increasing in  $x$ , it follows from (A.1) that  $H'(k)/g'(k)$  is decreasing in  $k$ .  $\square$

**Proposition 3.** An increase in the falsification cost from  $\alpha$  to  $\alpha'$  reduces expected cost padding:  $E_{\alpha'}[x_{\alpha'}] < E_\alpha[x_\alpha]$ ; reduces falsification costs:  $E_{\alpha'}[\phi(x_{\alpha'})] < E_\alpha[\phi(x_\alpha)]$ ; and increases expected costs (net of falsification costs):  $E_{\alpha'}[g(k) + \rho k] > E_\alpha[g(k) + \rho k]$ .

*Proof.* (i)  $x_{\alpha'}(k) = \delta x_\alpha(k)$ ,  $h_{\alpha'}(k) = (1/\delta)h_\alpha(k)$  where  $\delta = (\alpha/\alpha')^{1/(\beta-1)}$ . Integrating  $h(k)$  gives  $-\log_e(1 - F_{\alpha'}(k)) = -(1/\delta)\log_e(1 - F_\alpha(k))$  and therefore,  $F_{\alpha'}(k) = 1 - (1 - F_\alpha(k))^{(1/\delta)}$ . As there is no cost padding at  $k^*$ ,  $x_{\alpha'}(k^*) = x_\alpha(k^*) = 0$  and thus, integrating by parts, expected cost padding is given by:

$$\begin{aligned} E_{\alpha'}[x_{\alpha'}] &= - \int_0^{k^*} x'_{\alpha'}(k) F_{\alpha'}(k) dk \\ &= - \int_0^{k^*} x'_\alpha(k) \delta \left(1 - (1 - F_\alpha(k))^{1/\delta}\right) dk \end{aligned}$$

as  $x'_{\alpha'}(k) = \delta x'_\alpha(k)$ . Likewise:

$$E_\alpha[x_\alpha] = - \int_0^{k^*} x'_\alpha(k) F_\alpha(k) dk$$

We show that if  $\alpha' > \alpha$ , then  $x'_{\alpha'}(k)F_{\alpha'}(k) > x'_\alpha(k)F_\alpha(k)$  for each  $k < k^*$ . We define the function  $u(F_\alpha) := \delta(1 - (1 - F_\alpha)^{1/\delta})$  where  $F_\alpha \in [0, 1]$ . Then  $u(0) = 0$  and  $u(1) = \delta$ . Using the mean value theorem, there is a  $\hat{F}_\alpha \in (0, F_\alpha)$  such that we have  $u(F_\alpha) = u'(\hat{F}_\alpha)F_\alpha$ . Hence, as  $u'(\hat{F}_\alpha) = (1 - \hat{F}_\alpha)^{-(1/\delta)}$ , we have  $u(F_\alpha) \geq F_\alpha$  as  $\delta \geq 1$ . For  $\alpha' > \alpha$ ,  $\delta < 1$ , and hence,  $\delta(1 - (1 - F_\alpha)^{1/\delta}) < F_\alpha$ . Therefore, as  $x'_\alpha(k) < 0$ , we have  $E_{\alpha'}[x_{\alpha'}] < E_\alpha[x_\alpha]$ .

(ii) We have:

$$\begin{aligned} \frac{E_{\alpha'}[\phi(x_{\alpha'})]}{\alpha} &= - \int_0^{k^*} \left(\frac{\alpha'}{\alpha}\right) (x_{\alpha'}(k))^{\beta-1} x'_{\alpha'}(k) F_{\alpha'}(k) dk \\ &= - \int_0^{k^*} \delta (x_\alpha(k))^{\beta-1} x'_\alpha(k) \left(1 - (1 - F_\alpha(k))^{1/\delta}\right) dk \end{aligned}$$

as  $x'_{\alpha'}(k) = \delta x'_\alpha(k)$  and  $\delta = (\alpha/\alpha')^{1/(\beta-1)}$ . Likewise

$$\frac{E_\alpha[\phi(x_\alpha)]}{\alpha} = - \int_0^{k^*} (x_\alpha(k))^{\beta-1} x'_\alpha(k) F_\alpha(k) dk.$$

From part (i) for  $\delta < 1$ ,  $\delta(1 - (1 - F_\alpha)^{1/\delta}) < F_\alpha$  and therefore, since  $x'_\alpha(k) < 0$ ,  $E_{\alpha'}[\phi(x_{\alpha'})] < E_\alpha[\phi(x_\alpha)]$ .

(iii) We have, upon integrating by parts:

$$\begin{aligned} &E_{\alpha'}[g(k) + \rho k] - E_\alpha[g(k) + \rho k] \\ &= \int_0^{k^*} (g'(k) + \rho) (F_\alpha(k) - F_{\alpha'}(k)) dk \end{aligned}$$

As  $g'(k) + \rho < 0$  for  $k < k^*$  and  $F_{\alpha'}(k) > F_\alpha(k)$  for  $k \in (0, k^*)$ , we have proved that  $E_{\alpha'}[g(k) + \rho k] > E_\alpha[g(k) + \rho k]$  as required.  $\square$

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