Concepts for designing stiffer structures

Tianjian Ji
PhD, CEng, MIStructE
Manchester Centre for Civil and Construction Engineering, University of Manchester Institute of Science and Technology, Manchester M60 1QD UK

Received: 08/02
Modified: 04 03
Accepted: 05/03
Keywords: Stiffness, Design examples, Structures, Internal forces, Frames, Laboratory tests

Synopsis

The paper demonstrates concepts for designing stiffer structures. They are: (a) the more direct the internal force path, the stiffer the structure; (b) the more uniform the internal force distribution, the stiffer the structure; and (c) the smaller the internal forces, the stiffer the structure. These concepts are applicable to the design of many structures. Two ways of implementing the concepts into practice are provided. Simple examples are given to illustrate the implementation and the efficiency of the concepts. Laboratory tests and the demonstration of two physical models further confirm the findings. Several practical designs are also provided to show the applicability and significance of these concepts. An alternative definition of structural stiffness is given which complements the existing definition and allows for designing stiffer structures. It is interesting to note that using the concepts may lead to not only stiffer but also more economical and elegant designs.

Introduction

Buildings have become taller, floors wider and bridges longer in recent years. It is expected that the trend of increasing heights and spans will continue in the future. How can engineers cope with the ever-increased heights and spans, and design structures with sufficient stiffness? The basic theory of structures provides the conceptual relationships between span (L), deflection (Δ), stiffness (K) and natural frequency (f) for a single-span structure carrying distributed loads as follows:

$$\Delta = \frac{c_1}{K} = c_2 L^4 \qquad ...(1)$$

$$f = c_3 \sqrt{K} = \frac{c_4}{L^2} \qquad ...(2)$$

where c_1, c_2, c_3, c_4 are dimensional coefficients. The two equations state that:

- The deflection of the structure is proportional to the span powered four.
- The natural frequency of the structure is proportional to the inverse of the span squared.
- Both deflection and natural frequency are related to the stiffness of the structure.

The limitations on displacement and/or natural frequency of a structure specified in design codes actually require that the structure possess sufficient stiffness. Adding supports, reducing spans or increasing sizes of cross-sections of members can effectively increase structural stiffness. However, these measures may not always be applicable for practical designs due to aesthetic, structural or service requirements. What are the concepts behind these measures? Are there any other measures that can use these concepts to make a structure stiffer? This paper aims to investigate these concepts and develop new measures for designing stiffer structures.

Concept is defined, according to the Longman Dictionary of English Language and Culture[1], as a *principle* or *idea* whilst principle is defined as *a truth or belief that is accepted as a base for reasoning or action*. Structural concepts thus form the foundation of the study and design in civil and structural engineering.

The stiffness of a structure is generally understood to be *the ability of a structure to resist deformation*. Structural stiffness describes the capacity of a structure to resist deformations induced by applied loads. If a discrete model is considered, the structural stiffness of a structure can be completely described by its stiffness matrix. However, it may be difficult to be able to sense how stiff a structure is from its stiffness matrix. According to the McGraw-Hill's Dictionary of Engineering[2], stiffness (K) is defined as *the ratio of a steady force (P) acting on a deformable elastic medium to the resulting displacement (Δ)*. It can be expressed as

$$K = \frac{P}{\Delta} \qquad ...(3)$$

This definition of stiffness provides a means of calculating or estimating the stiffness of a structure, but does not suggest how to find a stiffer structure. How to design a stiffer structure (the form and pattern of a structure) is a fundamental and practical question and may be more important and challenging than how to analyse the structure. It is noted that university courses provide methods for calculating structural responses to different loads, but give little guidance on designing stiffer structures.

The work described in this paper is a further development of Ji and Ellis' earlier work[3] on effective bracing systems for temporary grandstands. In the paper the point stiffness and static stiffness of a structure were defined by which the stiffness of a structure can be evaluated. Two concepts for designing stiffer structures based on pin-jointed structures were derived and applied to the arrangement of bracing systems for temporary grandstands. The present work extends the previous work to beam types of structure and provides an additional concept. The paper also focuses on implementing, verifying and demonstrating the concepts, which can be used for designing stiffer structures. The concepts, which are simple but useful in design, are derived based on truss and beam types of structures in the next section The following two sections implement the concepts into possible applications. Simple examples, experiments, demonstration models and engineering cases are provided to show the efficiency of the concepts and the application of the criteria derived. An alternative definition of structural stiffness is given and the applications of the concepts are discussed. The final section summarises the main results and conclusions obtained from this study.

Concepts for achieving a stiffer structure

Pin-jointed structures

Consider a pin-jointed structure that consists of m bars and n joints, with no limitation on the layout of the structure and the arrangement of bars. To evaluate its static stiffness, a unit force in the required direction is applied to the critical point of the structure which is where the maximum displacement of the structure occurs due to the load. The critical point is not difficult to be identified for many structures. For a horizontal cantilever the critical point for a vertical load would be at the free end of the cantilever. For a simply supported rectangular plate it would be at the centre of the plate. For a plane frame supported at its base the critical point would be at the top of the frame.

The displacement at the critical point and inner forces of bars can be obtained by solving the static equilibrium equations and expressed in the following form:

$$\Delta = \sum_{i=1}^{m} \frac{N_i^2 L_i}{E_i A_i} \qquad ...(4)$$

where N_i is the ith internal force induced by a unit load at the critical point; L_i, E_i and A_i (i=1, 2,..., m) are the length, Young's modulus and area of the ith member respectively. Eq 4 provides the basis of a standard method for calculating the deflection of pin-jointed structures, and can be found in many textbooks, such as Gere and Timoshenko[4] and Coates *et al*[5]. According to the definition (eq 3), the stiffness is the inverse of the displacement, i.e.

$$K_s = \frac{1}{\sum_{i=1}^{m} \frac{N_i^2 L_i}{E_i A_i}} = \frac{1}{\sum_{i=1}^{m} N_i^2 f_i} \qquad ...(5)$$

where $f_i = L_i / E_i A_i$ is known as the flexibility of the ith member. Three concepts embodied in eq 4 or eq 5 are to be explored.

The force N_i in eq 5 is a function of structural form and material properties (not for statically determinate structures). Therefore, finding the largest stiffness of a pin-jointed structure may be considered as an optimisation problem of structure shapes. As eq 5 forms an incompletely defined optimisation

problem, optimisation techniques may not be applied directly at this stage.

Maximising K_s is achieved by minimising the summation,

$$\sum_{i=1}^{m} N_i^2 f_i$$

The characteristics of typical components of the summation are:

(i) $f_i > 0$
(ii) N_i can be zero.
(iii) $N_i^2 \geq 0$ regardless whether the bar is in tension or compression.

Therefore, to make the summation $\sum_{i=1}^{m} N_i^2 f_i$ as small as possible, three conceptual solutions relating to the internal forces can be derived from eq 5 as follows:

i) As many force components as possible are zero.
ii) No one force component is significantly larger than the other non-zero forces.
iii) The values of all non-zero force components should be as small as possible.

The three conceptual solutions, which are inter-related and not totally compatible, correspond to three structural concepts:

1. Direct force path: If many members of a structure subjected to a specific load are in a zero force state, the load is transmitted to the supports of the structure without passing through these members, i.e. the load goes a shorter distance or follows a more direct force path to the supports. This suggests that *a shorter or more direct force path from the load to the structural supports yields a bigger stiffness of a pin-jointed structure*.

2. Uniform force distribution: If the largest value, $|N_j|$, is not significantly bigger than other values of non-zero forces, it means that the absolute values of the internal forces N_i (i =1,2,…, m) should be similar. In other words, *more uniformly distributed internal forces result in a bigger stiffness of a pin-jointed structure.*

3. Smaller force components: If the values of N_i^2 (i = 1, 2,…, m) are small, it means that the force components, either compression or tension, are small. In other words, *smaller internal forces lead to a bigger stiffness of a pin-jointed structure*.

Beam type of structure

For beam types of structure in which bending is dominant, an equation, similar to eq 4, exists[4]:

$$\Delta = \sum_{i=1}^{m} \int_0^{L} \frac{M_i^2(x)}{E_i I_i} dx \qquad ...(6)$$

where $M_i(x)$, L_i, E_i, I_i are the bending moment, length, Young's modulus and the second moment of area of the cross-section of the ith member respectively. Consider E_iI_i is a constant for the ith member, the integral

$\int_0^{L_i} M_i^2(x)\, dx$ can be expressed by $\overline{M_i^2} L_i$ where $\overline{M_i^2}$ is the mean value of $M_i^2(x)$. Thus eq 6 becomes

$$\Delta = \sum_{i=1}^{m} \frac{\overline{M_i^2} L}{E_i I_i} \qquad ...(7)$$

Eq 7 has the same format as eq 4 where the numerator contains the square of the internal force. Thus the three concepts derived for pin-jointed structures can also be abstracted to structures associated with eq 7.

Expression of the concepts

As the derivation has not been related to any particular material properties and structural forms, the three concepts are valid for any bar and beam types of structure and can be used for designing stiffer structures. The three concepts may be summarised in a more concise form as follows:

Concept 1: The more direct the internal force paths, the stiffer the structure.
Concept 2: The more uniform the internal force distribution, the stiffer the structure,
Concept 3: The smaller the internal forces, the stiffer the structure.

The concepts are general and there are no limitations in terms of applications. Two ways of implementing the three concepts are discussed in the following two sections.

Implementation (1) – direct force path

Five criteria

Bracing systems may be used for stabilising structures, transmitting loads and increasing lateral structural stiffness. There are many options to arrange bracing members and there are unlimited numbers of bracing patterns, as evidenced in tall buildings, scaffolding structures and temporary grandstands. Five criteria, based on the first concept derived in the last section, have been suggested for arranging bracing members for temporary grandstands[3]. In fact these criteria are valid for other types of structure, such as tall buildings and scaffolding structures. The five criteria can be presented as follows:

- Criterion 1: Bracing members should be provided in each story from the support (base) to the top of the structure.
- Criterion 2: Bracing members in different stories should be directly linked.
- Criterion 3: Bracing members should be linked in a straight line where possible.
- Criterion 4: Bracing members in the top story and in different bays should be directly linked where possible.
- Criterion 5: If extra bracing members are required, they should be linked following the above four criteria.

The first criterion is obvious since the critical point for multistory structures is at the top and the load at the top should be transmitted to the supports of the structures. If bracing is not arranged over the height of a structure, its efficiency will be significantly reduced. There are a number of ways to achieve this criterion; but the second and the third criteria suggest a way using *a shorter force path*. The first three criteria mainly concern the bracing arrangements in different stories. For a temporary grandstand structure, however, the number of bays is usually larger than the number of the stories. To create a shorter force path, or more zero force members in a structure, the fourth criterion gives a means to consider the relationship of bracing members across the bays of the structure. The fifth criterion suggests that when extra bracing members are required, usually to reduce bracing member forces and distribute them more uniformly, they should be arranged using the previous criteria.

Numerical verification

To demonstrate the efficiency of the concepts, a pin-jointed plane frame, consisting of four bays and two stories with six different bracing arrangements, is considered (Fig 1). All frame members have the same Young's modulus E and area of cross-section A with EA equal to 1000N. The vertical and horizontal bars have unit lengths (1m). A concentrated horizontal load of 0.2N is applied to each of the five top nodes of the frame. The lateral stiffness can be calculated as the inverse of the averaged displacement of the top five nodes in the horizontal direction.

The bracing members in the six cases are arranged in such a way that the efficiency of each criterion given in the previous section can be identified. The features of the bracing arrangements can be summarised as follows:

a) The bracing of example (a) satisfies the first criterion.
b) The bracing of example (b) satisfies the first two criteria.
c) The bracing of example (c) satisfies the first three criteria.
d) The bracing of example (d) satisfies the first four criteria.
e) Two more bracing members are used on the basis of example (c), but the added bracing members do not follow the criteria suggested.
f) Four more bracing members are used and the bracing of

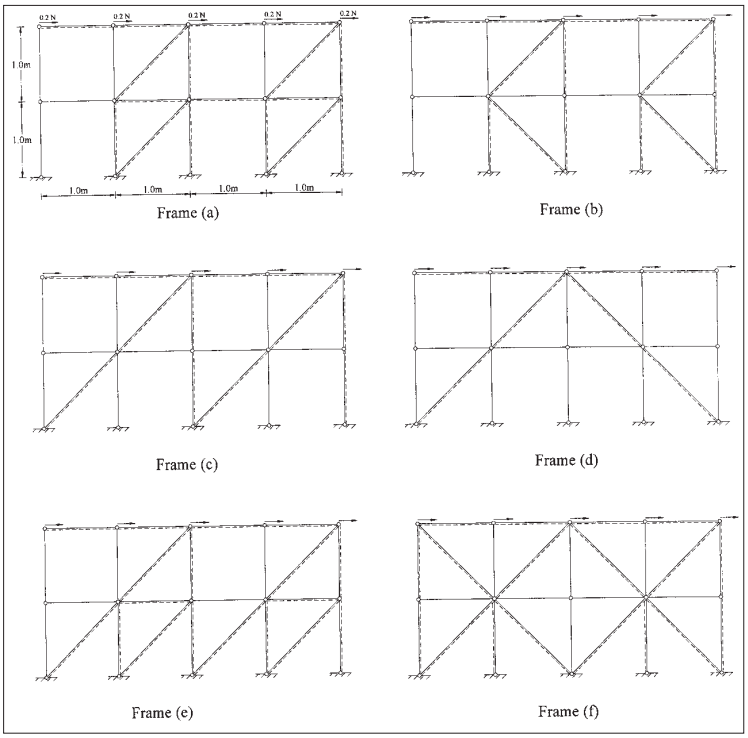


Fig 1. Force paths (dashed lines) and frames with different bracing arrangements

example (f) satisfies all the five criteria suggested.

Table 1 lists the total numbers of members, bracing members and zero-force members, the five largest absolute values of member forces and the average horizontal displacements of the five top nodes of the six cases. The relative stiffnesses of the six frames are also given for comparison.

The force paths, which transmit the loads from the tops to the supports of the frames, are indicated by the dashed lines in Fig 1. To emphasise the main force paths, forces, of less than 3% of the maximum force in each of the first three examples, have been neglected in Fig 1.

Following the concepts suggested on the basis of eq 5, it can been seen from Table 1 and Fig 1 that:

- Frame (a) has a conventional bracing form and the loads at the top are transmitted to the base through the bracing, vertical and horizontal members. There are five members with zero force.
- In Frame (b) the forces in the bracing members in the upper storey are directly transmitted to the bracing and vertical members in the lower storey without passing through the horizontal members that link the bracing members in the two storeys. Thus Frame (b) provides a shorter force path with three more zero force members than example (a) and yields a higher stiffness.
- In Frame (c) a more direct force path is created with two vertical members in the lower storey, which have the largest forces in Frame (b), becoming zero force members. The shorter force path produces an even higher stiffness, as expected.
- To balance the lateral loads at the top nodes where bracing members are involved, forces in vertical members have to be generated to balance the vertical components of the forces in the bracing members in Frame (c). In Frame (d) two bracing members with symmetric orientation are connected at the same node, with one in compression and the another in tension. The horizontal components of the forces in these bracing members balance the external loads while the vertical components of the forces are self-balanced. Therefore, the vertical members are in a zero force state and Frame (d) yields the highest stiffness among Frames (a) – (d).
- Two more members are added to Frame (d), but comparison between Frame (d) and (e) indicates that bracing following the criteria set out can provide a higher stiffness than more bracing members which do not fully follow the criteria.
- Frame (f) shows the effect of the fifth criterion. Four more bracing members are added to Frame (d) and arranged according to the first three criteria. Now the lateral loads are distributed between more members, creating a smaller and more uniform force distribution, which results in an even higher stiffness.

It can be seen from Table 1 that the structure is stiffer when the internal forces are smaller and more uniformly distributed although the first four criteria are derived based on the concept of direct force path. In addition, it is ideal to have smaller compressive forces to avoid buckling of individual members. These examples are simple and the variation of bracing arrangements is limited, but they demonstrate the efficiency of the concepts and the criteria.

The lateral stiffnesses of the frames are provided by the bracing members. It is interesting to examine the ratio of the relative lateral stiffness to the total area of bracing members. In this way, Frame (d) has the highest ratio.

Experimental verification

The three aluminium frames shown in Fig 2, with different bracing arrangements, were constructed with the same overall dimensions of 1025 × 1025mm. All members of the frames had the same cross-section. The frames were tested to determine the static stiffness experimentally[6].

- Frame A is traditionally braced with eight members which satisfies the first criterion.
- Eight bracing members are again used in Frame B but are arranged to satisfy the first three criteria.
- A second traditional bracing pattern is used for Frame C with 16 bracing members arranged to satisfy the first two criteria.

The three frames were tested using a simple arrangement. The frames were fixed at their supports and a hydraulic jack was used to apply a horizontal force at the top right joint of the frame. A micrometer gauge was used to measure the displacement at the left top joint of the frame. A lateral restraint system was provided to prevent out-of-plane deformations.

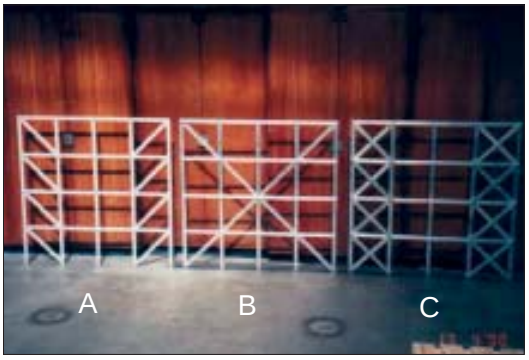


Fig 2. Aluminium test frames

Table 1: A summary of the results for the six frames (see Fig 1)

	(a)	(b)	(c)	(d)	(e)	(f)
No. of elements	22	22	22	22	24	26
No. of bracing elements	4	4	4	4	6	8
No. of zero-force elements	5	8	10	14	6	8
The absolute values of the five largest element forces (N) (b – bracing member) (v – vertical member) (h – horizontal member)	1.04 (v) 0.96 (v) 0.78 (b) 0.72 (b) 0.69 (b)	1.03 (v) 0.97 (v) 0.74 (b) 0.71 (b) 0.71 (b)	0.75 (b) 0.72 (b) 0.69 (b) 0.67 (b) 0.53 (v)	0.71 (b) 0.71 (b) 0.71 (b) 0.71 (b) 0.40 (h)	0.74 (b) 0.67 (b) 0.64 (v) 0.59 (b) 0.58 (v)	0.40 (b) 0.40 (b) 0.37 (b) 0.37 (b) 0.33 (b)
The average horizontal displacement of the five top nodes (mm)	6.60	6.12	4.12	3.23	3.93	1.69
The relative stiffness	1.00	1.08	1.60	2.04	1.68	3.91
The relative ratio of stiffness to the total area of bracing members	1.00	1.08	1.60	2.04	1.12	1.95

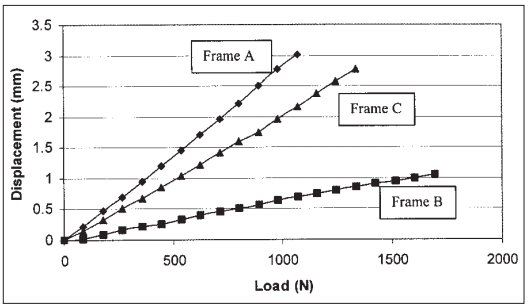


Fig 3. Load–deflection curves for frames A, B and C

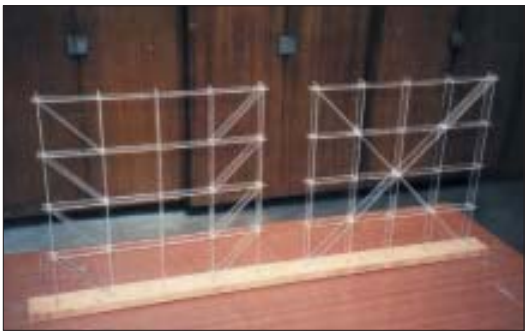


Fig 4. Demonstration: braced frame models

The horizontal load – deflection characteristics of the three frames are shown in Fig 3. It can be seen that the displacements of Frame B, which satisfies the first three criteria, are about a quarter of these of Frame A due to the same loads. Frame C with eight more members but not satisfying the third criteria is obviously less stiff than Frame B.

The experiment confirms the significance of the direct force path, which is implemented by the third criterion.

Physical demonstration

In order to 'feel' the effect of the force paths, two plastic frames, with the same overall dimensions and member sizes, were built as shown in Fig 4. The only difference between the two frames was the arrangement of bracing members. The relative stiffnesses of the two frames can be felt by pushing a top corner joint of each frame horizontally. The frame on the right side feels much stiffer than the one on the left. In fact, the stiffness of the right frame is about four times that of the left frame. The load applied to the right frame is transmitted to its supports through a direct force path while the frame on the left, the force path is zigzag[6, 7].

Practical examples

The collapse of a scaffolding structure: The scaffolding structure shown in Fig 5 collapsed in Manchester in 1993[8], although no specific reasons were given. Using the concept of direct force paths and the understanding gained from the above examples, the cause of the incident may be explained. It can be seen that in this scaffolding structure no diagonal (bracing) members were provided, i.e. no direct force paths were provided, which produced a structure that did not have enough lateral stiffness and it collapsed under wind loads alone.

Fig 5. Collapse of a scaffolding structure (photo: courtesy of Dr John Anderson[8])

The bracing systems of tall buildings: The 100-storey, 344m tall building, the John Hancock Center in Chicago, has an exterior-braced frame tube structure. An advance on the steel-framed tube, this design added global cross-bracing to the perimeter frame to increase the stiffness of the structure as shown in Fig 6a. A saving of $15m was made on the conventional steelwork by using these huge cross braces[9].

It was regarded as an extremely economical design which achieved the required stiffness to make the giant stable. One of the reasons for the success was, as can be seen from Fig 6a, that the direct force path was archived by using the cross-braces, which resulted in a stiffer structure and smaller internal forces according to the concepts. The bracing arrangement of the building satisfied the first three criteria discussed.

Similar buildings using the same bracing arrangements include the Landmark Tower, Yokohama and the Bank of China, Hong Kong (Fig 6b).

Implementation (2) – smaller internal forces

The criterion

In a structure increasing the size of cross-section of a member will effectively reduce its stress level but not necessarily its internal force. This measure usually increases the use of materials and the mass of the structure. Following the third concept given previously, if internal forces can be partly balanced by introducing a structural element in a structure, smaller internal forces will be created and the structure will be stiffer. This can be presented by a criterion as follows:

If a device, or a structural element, can be incorporated into a structure to offset some of the effects of the external loads or balance some of the internal forces before the forces are transmitted to the structural supports, the structure will be stiffer.

Theoretical verification

Considering a beam type of structure with m members, its maximum displacement at the critical point to a given loading P can be expressed as:

$$\Delta = \sum_{i=1}^{m} \int_0^{L_i} \frac{M_i^{L}(x)\, M_i^{U}(x)}{E_i I_i}\, dx \qquad ...(8)$$

where $M_i^L(x)$ and $M_i^U(x)$ are the bending moments of the ith member induced by the applied load and the unit force on the critical point where the displacement is to be calculated. If a structural member is added into the structure, all internal forces are consequently changed. The changes of the internal forces are denoted as M_i^D (i = i, 2, ...m) due to the given loading. Thus, the displacement becomes by superposition

$$\Delta = \sum_{i=1}^{m} \int_0^{L_i} \frac{M_i^{L}(x)\, M_i^{U}(x)}{E_i I_i}\, dx + \sum_{i=1}^{m} \int_0^{L_i} \frac{M_i^{D}(x)\, M_i^{U}(x)}{E_i I_i} = v_1 + v_2 \qquad ...(9a)$$

$$v_1 = \sum_{i=1}^{m} \int_0^{L_i} \frac{M_i^{L}(x)\, M_i^{U}(x)}{E_i I_i}\, dx \qquad v_2 = \sum_{i=1}^{m} \int_0^{L_i} \frac{M_i^{D}(x)\, M_i^{U}(x)}{E_i I_i} \qquad ...(9b)$$

If many of the terms $M_i^L(x)$ and $M_i^D(x)$ in eq 9a have opposite signs, the displacement will be smaller than that in eq 8. The function of the added member is to create the moments that are opposite to those induced by the load, thus it reduces the magnitudes of the forces and yields a stiffer structure.

This criterion can be illustrated in a simple manner. A ring, with radius R, is subjected to a vertical force P at point B as shown in Fig.7a and the deformation of the ring is dominated by bending. Due to symmetry, only the right top quarter of the ring needs to be considered for analysis and the moments at any cross section of the portion due to the load P and the unit load are, respectively

$$M_1^L = PR\left(0.5\cos\theta - 1/\pi\right) \qquad ...(10a)$$

Fig 6. Bracing systems used in buildings satisfying the first three criteria
a) (left) The John Hancock Center in Chicago
b) (above) The Bank of China, Hong Kong

$$M_1^U = R\left(0.5\cos\theta - 1/\pi\right) \quad ...(10b)$$

From eq 8, the vertical deflection of point B due to the load is

$$v_1 = \frac{4}{EI}\int_0^{\pi/2} M_1^L M_1^U R d\theta = \frac{PR^3}{EI}\left(\frac{\pi}{4} - \frac{2}{\pi}\right) = 0.1488\frac{PR^3}{EI} \quad ...(11)$$

If horizontal forces, H, are applied at points A and C of the ring as shown in Fig.7b, the moments of the right top quarter are given by

$$M_1^D = HR\left(0.5\sin\theta - 1/\pi\right) \quad ...(12)$$

The vertical displacement at point B due to the pair of horizontal force (H) can also be evaluated using the second term in eq 9a as follows:

$$v_2 = \frac{4}{EI}\int_0^{\pi/2} M_1^D M_1^U R d\theta = \frac{HR^3}{EI}\left(\frac{1}{2} - \frac{2}{\pi}\right) = -0.1366\frac{HR^3}{EI} \quad ...(13)$$

It can be seen that the forces, H, produce an upward displacement at point B, which can offset the displacement induced by the vertical load P. The negative value is due to the fact that $M_1^L(M_1^U)$ and M_1^D have opposite signs. Fig 8 compares the normalised moments, M_1^L/PR (solid line) and M_1^D/HR (dashed line) between 0 and $\pi/2$, and confirms that if a horizontal wire is introduced between the points A and C, the modified ring shown in Fig 7c will become much stiffer than the original ring shown in Fig 7a. This is because the moment of the modified ring and the vertical displacement at point B become

$$M = PR\left(0.5\cos\theta - 1/\pi\right) + HR\left(0.5\sin\theta - 1/\pi\right)$$
$$= 0.5R\left(P\cos\theta + H\sin\theta\right) - \frac{R}{\pi}\left(P + H\right) \quad ...(14)$$

$$\Delta = 0.1488\frac{PR^3}{EI} - 0.1366\frac{HR^3}{EI} = \left(0.1488P - 0.1366H\right)\frac{R^3}{EI} \quad ...(15)$$

where the force H can be determined using the compatibility of horizontal displacement at point C.

Physical demonstration

Fig 9 shows two rubber rings, one without and one with a wire tied across the diameter. The same weight is placed on the top of each ring and the reduced deformation of the tied ring is apparent and its increased stiffness can be seen and felt. This may be explained by that the force in the wire increases as the applied load increases, and produces a bending moment (eq 12) in the ring in the opposite direction to the bending moment (eq 10a) caused by the external load (Fig 8). In this way the force in the wire 'balances' some of the internal forces in the ring, reducing the internal forces in the ring, thus making it stiffer.

Examples of this criterion in practice are tied arches and tied-pitched roofs. The ties help to balance horizontal forces and reduce horizontal displacements, thus increasing the structural stiffness effectively.

Practical example: Raleigh Stadium

The roof structure of the Raleigh Stadium shown in Fig 10 consists of carrying (sagging) cables, stabilising (hogging) cables supported by a pair of inclined arches[10]. It can be seen that the structure forms, at least in part, a self-balanced system, which effectively reduces the internal forces in the arches, which have a similar function to the wire tie in the demonstration rings in Fig 9. The reduced internal forces not only allow the use of less material but also lead to a stiffer structure as demonstrated in the previous sections.

An alternative definition of structural stiffness

The stiffness of a structure is understood from textbooks[4,5] as *the structural ability to resist deformation*. This definition provides a means to calculate the stiffness of a structure. The structural ability of a structure can be evaluated *quantitatively* as the ratio of the applied load to the resulting structural deformation, as given in eq 3. However, this definition does not provide a way to design a stiffer structure.

Based on the foregoing, the stiffness of a structure may be alternatively defined as *the structural efficiency to transmit the loads applied on the structure to its supports*. This definition, however, may provide a way to find stiffer structures. The structural efficiency of a structure can be assessed qualitatively using the three concepts given in the paper.

Efficiency is defined as *the ratio of the useful work done by a machine, engine, device, etc., to the energy supplied to it*, and is

Table 2: Comparison of the efficiency of two braced frames (see Fig11)

	Fundamental frequency (Hz)	Static stiffness (MN/m)	No. of bracing members used
Original frame (OF) (Fig 11a)	1.95	3.16	64
Improved frame (IF) (Fig 11b)	3.28	8.96	52
(IF)/(OF)	169%	284%	81%

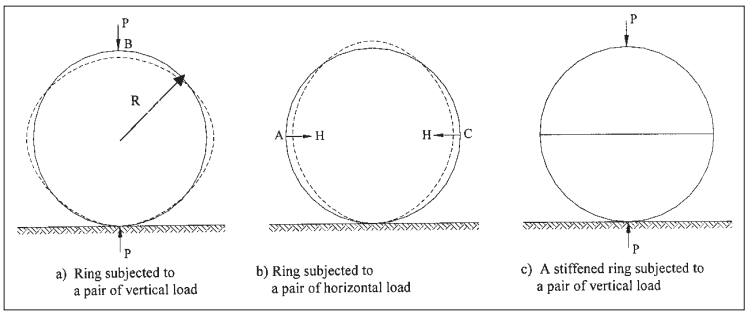


Fig 7. (above) Loading and deformation of rings

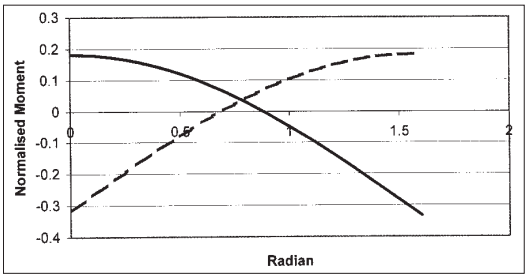


Fig 8. (right) Comparison of normalised moments

Fig 9. Demonstration: ring models with and without a tie

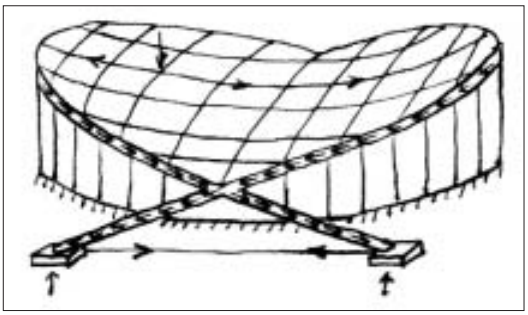


Fig 10. Raleigh Stadium

often expressed as a percentage. Efficiency may also be expressed as the work done by a device or a system to complete a given task or reach a specified purpose.

A structure should be designed to transmit the loads on the structure safely to its supports. To transmit the loads to the supports, many elements of the structure experience forces and deformations, which contribute to the strain energy in the structure. The transmission of the loads on a structure to its supports is a specified task and creates strain energy in each element due to bending, tension, torsion and shearing. Therefore, the smaller the energy (work), the more efficient the structure is. (Eqs 4 and 7 also indicate that the less the energy stored in (or the work done by) the structural elements, the stiffer the structure. Thus the structural stiffness reflects the structural efficiency to transmit the external loads on a structure to its supports.

Work, different from energy, is distinguished as positive and negative. The transmission of the loads on the structure does large positive work. If negative work can be introduced into the structure, as demonstrated in the last section, the structure will do less work to transmit the load to its supports and experience less deformation following the alternative definition. Thus it leads to a stiffer structure.

The three concepts and the alternative definition may explain why reducing spans and adding supports can effectively increase structural stiffness. This is because these measures create more direct force paths, more uniformly distributed and/or smaller internal forces, and make the structure more efficient in transmitting the loading to its supports.

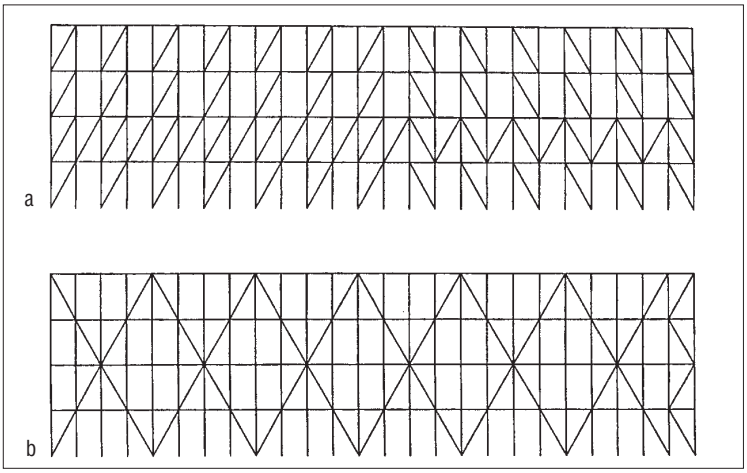


Fig 11. Bracing arrangements for a temporary grandstand

Discussion

Design of a structure needs to consider several aspects and the stiffness requirement is one of them. The concepts and the ways for implementation may be useful for these structures where increasing stiffness is a major factor to be considered.

Safety, economy and elegance

A useful definition of structural engineering was given in the *Journal of Structural Engineer*[11] as follows:

'Structural engineering is the science and art of designing and making, with economy and elegance, buildings, bridges, frameworks, and other similar structures so that they can safely resist the forces to which they may be subjected.'

The disciplines of structural engineering allow structures to be produced with satisfactory performance at competitive costs. Elegance, which is not particularly related to safety and economy, is probably regarded as the architect's contribution.

However, the beauty of the three structural concepts lies in integrating safety, economy and elegance of a structure as a whole. This may be demonstrated by the following example.

Fig 11a shows the bracing arrangement at the back of part of a temporary grandstand, which satisfies the first criterion discussed. Without considering the safety, economy and elegance of the structure but following the concept of direct force path or the first four criteria, the bracing members are re-arranged as shown in Fig 11b. The calculated static stiffnesses and fundamental frequencies of the two frames are summarised in Table 2[3].

It can be concluded from the comparison that the improved structure is safer as the lateral stiffness is increased by 184% and the fundamental frequency by 69%, and is more economical as the number of bracing members is reduced by 19%. Looking at the appearance of the two frames, one would probably feel that the frame with the improved bracing arrangement is more elegant.

The concepts lead to structures with larger stiffness and smaller and more uniformly internal forces as demonstrated, meeting the requirements of safety and economy. It is difficult to show that the concepts also lead to elegant designs. However, the examples, the John Hancock Tower, the Bank of China and the Raleigh Stadium, discussed in the previous sections, are all well-known safe, economical and elegant structures.

The design of structural forms

Choosing a suitable structural form may be far more important and challenging than analysing the structure. The successful design of a structural form requires the choice of a force path, which can safely and effectively transmit the loads on the structure to its supports. The three inter-related concepts demonstrated in this paper provide an aid to selecting, improving and assessing structural forms before calculations.

When designing structures or selecting structural forms, following the alternative definition, one can focus on how to create a more direct force path, more uniformly distributed and/or smaller internal forces. This not only effectively assesses different structural forms but also may imaginatively lead to new and innovative designs.

Optimum design

The three concepts are derived based on making the displacement (eqn. 4 or eqn. 7) as small as possible, which is relating to optimum design of structures. It is useful to compare the general characteristics between optimum methods and the concepts given in Table 3.

Compared to optimum design of structures, design using the concepts does not include an analysis for choosing cross-sections of members, does not seek for the stiffest structure and is not subjected to explicitly applied constraints. Therefore, the design using the concepts becomes simple and many engineers will be able to use the concepts in their designs. It is useful and reasonable that choosing structural forms and selecting the sizes of cross-sections are conducted separately.

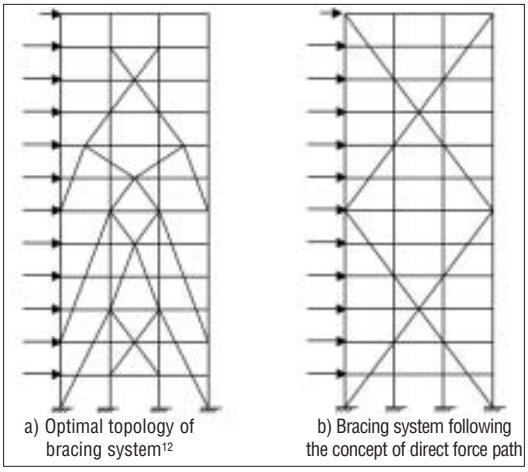
As the concepts are fundamental and they are applied to

Table 3: Comparison of design methods

	Optimum methods	Design using the concepts
Objective	Seek for a maximum or minimum value of a function, such as cost, weight or energy	Seek for a stiffer structure rather than the stiffest structure
Constraints	Explicitly applied	Implicitly applied
Solution method	Computer based mathematical methods	The concepts and derived criteria
Loading	The optimum design depends on loading conditions	The design is independent of loading conditions
The cross-sectional sizes of members	Provided	To be determined
Design	The optimum design may not be practical	The design is practical
Main users	Specialists and researchers	Engineers

Table 4: Displacements of the two designs

	Frame without bracing members	Frame with bracing suggested in Ref 12	Frame with bracing following the first concept
Displacement at the top of the frame	630 mm	87.4 mm	78.5 mm
Maximum displacement & its location	630 mm Top floor	87.4 mm Top floor	84.1 mm 11th floor
Bracing members used		100%	67.5%



a) Optimal topology of bracing system[12]

b) Bracing system following the concept of direct force path

Fig.12: (left) Comparison of bracing systems

a structure globally, it is not surprised that the design using the concepts may be more optimal than some optimum designs. Fig 12a shows the optimal topology of bracing system for a steel building framework with overall stiffness constraint under multiple lateral loading conditions[12]. The solution was obtained by gradually removing the elements with the lowest strain energy from a continuum design domain, which implied creating a direct force path in an iterative manner. Fig 12b shows the design of the bracing system using the concept of direct force path, i.e. the first three criteria. The design of the bracing system takes a few minutes. Using the dimensions, cross-sectional sizes and load conditions given in ref 12, the maximum displacements of the two frames are calculated and listed in Table 4.

It can be seen that the design following the first concept is more practical, economical and is stiffer than the optimal design.

Teaching structural concepts at university

Students are taught many hand and computer based methods at universities for calculating structural response, such as bending moments and displacements. However, less is taught on how to design stiffer structures and how to consider structures as a whole rather than as assemblies of element[13]. These are related to teaching of structural concepts.

The understanding of structural concepts should be a key objective in the teaching and learning of civil and structural engineering. Today many hand calculations are replaced by the use of computers. However, the understanding of structural concepts cannot be replaced by computers. It is also an understanding of such concepts that allows correct computer modelling and quick checking of results from calculations. Thus some contents developed in this paper have been converted into new teaching materials[7].

Conclusions

The paper explores the concepts for designing stiffer structures and the following three simple but effective concepts are suggested:

- Concept 1: The more direct the internal force path, the stiffer the structure
- Concept 2: The more uniform the internal force distribution, the stiffer the structure
- Concept 3: The smaller the internal forces, the stiffer the structure

The derivation of the above concepts is based on the general energy equations for bar and beam types of structure and these can be applied to the design of many structures. Two ways of implementation of the first and third concepts have been considered as:

- create more direct force paths using five suggested criteria for arranging bracing members.
- create a partly self-balanced system to reduce internal forces.

These measures and their effectiveness have been demonstrated by numerical examples, experiments, physical models and engineering cases. It is expected that other measures, based on the three concepts, for designing stiffer structures can be developed.

Stiffness of a structure is alternatively defined as *the structural efficiency to transmit the loads applied on the structure to its supports*, which helps to find a stiffer structure in design. This definition complements the existing definition of stiffness as *the structural ability to resist deformation*, providing a means to calculate the stiffness of a structure.

Acknowledgment

The author is very grateful to his colleagues, Dr A. J. Bell, Prof. F. M. Burdekin and Prof. K. Johnson, UMIST, and Prof. K. Zelka, Szent Istvan University in Budapest, for their constructive comments on the paper. Mr El-Dardiry calculated the examples in Table 4. Permission for reprinting Fig 5 and Fig 6a are given by Thomas Telford Ltd, UK, and Dr Mary Ann Sullivan, Bluffton College, USA, respectively.

REFERENCES

1. Longman: *Dictionary of English Language and Culture*, (1992), Longman Group UK Ltd, ISBN 0582 08676 0
2. Parker, S. P.: *Dictionary of Engineering,* 5th Edition, (1997), McGraw-Hill, ISBN 0-07-052435-1
3. Ji, T. and Ellis, B. R.: 'Effective bracing systems for temporary grandstands', *The Structural Engineer*, (1997), **75**/6, p 95-100
4. Gere, J. M. and Timoshenko, S. P.: *Mechanics of Materials*, (1990), PWS-KENT Publishing Company, ISBN 0 534 92174 4
5. Coates, R., Coutie, M. and Kong, F.: *Structural Analysis*, (1998), Chapman & Hall
6. Roohi, R.: *Analysis, testing and model demonstration of efficiency of different bracing arrangements*, Investigative Project Report, (1998), UMIST
7. Ji, T. and Bell, A. J.: 'Seeing and touching structural concepts in class teaching', *Proc. of the Conf. on Civ. Eng. Education in the 21st Century*, Southampton, UK, 26-28 April 2000
8. Anderson, J.: 'Teaching health and safety at university', *Proc. of the Inst. of Civ. Eng., Journal of Civil Engineering*, (1996), **114**/2, p 98-99
9. Bennett, D.: *Skyscrapers – Form & Function*, (1995), Simon & Schuster ISBN 0-684-80318-6
10. Bobrowski, J.: 'Design philosophy for long spans in buildings and bridges', *Journal of Structural Engineer*, (1986), **64A**/1, p 5-12
11. *The Structural Engineer*, **72**/3, 1994
12. Lian, Q., Xie, Y. and Steven, G.: 'Optimal topology design of bracing systems for multistorey steel frames', *J. of Structural Engineering*, ASCE, **126**/7, p 823-829
13. Zalka, K. A.: *Global Structural Analysis of Buildings*,(2000), E & FN Spon, London, ISBN 0-415-23483-2