BRE digest

The response of structures to dynamic crowd loads

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The 1996 edition of the British Standard BS 6399-1, 'Loading for buildings, Code of practice for dead and imposed loads', included guidance on dynamic loads generated by synchronised crowd movement. This Digest provides information that explains and supports the recommendations in the Standard, and provides a method for calculating structural response to rhythmic crowd loads.

This edition of Digest 426 supersedes the version published in 1997. It includes improved information on the loads generated by crowds.

Dynamic crowd loads are generated by the movement of people. However the largest loads are produced by synchronised rhythmic movements which mainly arise from people dancing or engaging in jumping type movements, usually in response to a musical beat. An example of a dance which involved jumping, and which was popular in the 1980s, was 'pogoing'. Although usually associated with lower numbers of people, activities such as aerobics will give rise to similar dynamic effects.

It should be recognised that a crowd of people jumping rhythmically can generate large loads and this may be a concern for both safety and serviceability evaluations. However this is an extreme case and the rhythmic excitation most frequently encountered does not involve everyone jumping in perfect synchronisation but includes people dancing and clapping, often with some people stationary. This latter situation will produce smaller loads than if everyone was jumping and usually a much smaller structural response. For design it is therefore important to identify an appropriate load scenario.

Two approaches to the design of structures to accommodate dynamic crowd loads are given in British Standards and this Digest. One relies on ensuring that the fundamental natural frequency of the structural system is sufficiently high that resonance generated by the major dynamic components of the load is avoided; the other provides a method for calculating structural response to dynamic crowd loads so that structural safety and serviceability limit states can be checked.

This Digest deals primarily with the modelling of dynamic crowd loads and the calculation of structural response to these loads. It gives the background to the problem and discusses a number of items which must be considered in design.

Background

It is possible that some designers may be unaware of the nature of the potential problems caused by dynamic crowd loads. Therefore this section provides a simple illustration of the problem and records related developments in UK Standards.

A simple illustration

From experience, everyone is aware that the peak load produced by jumping is significantly larger than the load resulting from standing still. This is demonstrated by the action of a child who, when trying to crush a cardboard box, will jump on it. Furthermore, if a person keeps jumping the resulting dynamic load will be cyclic; hence in certain situations it can generate a resonant response of the loaded structure. There are some human activities which involve repeated jumping, but for crowds the dynamic loads will only be significant when the movement is synchronized. In practice this usually occurs in conjunction with a strong musical beat at events like lively pop concerts and aerobic exercise classes.

It is instructive to provide some quantitative values to illustrate the magnitude of the problem. Later in the Digest an equation is given which determines the load produced by one person jumping rhythmically; for one type of jumping, the peak load is 4.7 times the person's weight. With structures which are designed for static loading, there are many safety factors built into the design which allow for most dynamic loads. In fact, if a group of people on a structure were jumping in an uncoordinated manner, the peak displacement induced by the load would not be significantly larger than that induced by the static load; it is only when coordination is achieved that the load becomes significant. In an experiment which is described later, the response of a floor to **64** people jumping on it in time to a musical beat was monitored. The dynamic displacements were 3.5 times the static displacement for the group. It should be appreciated that this is a significant load case.

The real problem, however, arises when a coordinated jumping load induces a resonant response in a structure. If the frequency of the dynamic load occurs at a natural frequency of the structure, a resonant situation would arise whereby the response would be amplified greatly. For example, if a structure with a damping value of 2% critical is subjected to a dynamic load coinciding exactly with the natural frequency of the structure, then the displacement due to resonance would be 25 times the static displacement. Hence it can be appreciated that a dynamic load occurring at a natural frequency of a structure can induce significant vibrations, and is therefore a situation to be considered. It is perhaps wise to recognise that structural resonance produced by dynamic crowd loads will be an unusual event and requires both a specific form of loading and a structure which is vulnerable to such dynamic loads.

The history of dynamic crowd loads and their inclusion in UK Standards

It has been mentioned that BS 6399-1:1996 provides guidance on loads arising from synchronised crowd movement. It is of interest to examine the two versions of the British Standard which preceded it. In the 1967 version of BS CP3, no mention was made of dynamic loads generated by crowds, so at this stage the problem cannot have been considered to be important. In the 1984 edition of **BS** 6399 Part 1, static design loads were detailed but with the warning *'the values for imposed loads given ... allow for small dynamic effects ... The loads do not ... allow for dynamic loads due to crowds'*. Hence at this stage the problem had been recognised although no guidance was provided on how to deal with it. Unfortunately this resulted in the majority of engineers ignoring the problem.

Guidance on dynamic crowd loads was included in BS 6399-1:1996 specifically to take account of loads generated by repetitive jumping during dancing. The guidance considers the loads generated by the activity which are independent of the structure to which they are applied. The relevant section of the Standard states:

'Dynamic loads will only be significant when any crowd movement (dancing, jumping, rhythmic stamping, aerobics etc) is synchronized. In practice this only occurs in conjunction with a strong musical beat such as occurs at lively pop concerts or aerobics. The dynamic loading is thus related to the dance frequency or the beat frequency of the music and is periodical. Such crowd movement can generate both horizontal and vertical loads. If the synchronized movement excites a natural frequency of the affected part of the structure, resonance will occur which can greatly amplify its response.

'Where significant dynamic loads are to be expected the structure should be designed either:

a) to withstand the anticipated dynamic loads; or

b) by avoiding significant resonance effects

*'To avoid resonance effects the vertical frequency should be greater than **8.4** Hz and the horizontal frequencies greater than 4.0 Hz, the frequencies being evaluated for the appropriate mode of vibration of an empty structure.'*

To withstand the anticipated dynamic loads reference is made to published work [1]. It should be noted that the above frequency recommendations relate to safety assessments and not serviceability. Indeed, at that time there was no authoritative guidance on acceptable serviceability levels for this type of vibration.

The Standard, **BS** 6399-1, gave one example of a load model for everyone in a crowd jumping, but did not exclude the use of other load models. Although everyone jumping on a dance floor may be possible, it was questioned whether this model was suitable for grandstands.

In October 2002, BS 6399-1 was amended. While it retains the frequency limits for synchronised excitation, it now refers to the jumping loads through reference to the 1997 edition of Digest 426 where the use of the load model can be explained. This provides an opportunity for specialist documents to be available for specific types of construction and use (eg grandstands).

The response of grandstands to dynamic crowd loads became the focus of attention towards the end of the 1990s – in particular the response of the cantilever tiers of some grandstands. Consequently a joint working group of the Institution of Structural Engineers, the Department for Culture Media and Sport, and the Office of the Deputy Prime Minister was set up to consider dynamic loading of seating decks. As an interim measure the working group suggested limits for vertical frequencies of 3.5 Hz for new grandstands for normal, non-rhythmic loading but 6 Hz for those where pop concerts could be held[2]. The frequency limits were based on the experience of the committee members, but it was recognised that these limits are not a perfect design solution and many items affect a grandstand's performance besides its natural frequency. Even the distinction between stands subject to normal and rhythmic loading is somewhat contentious, because there is always an opportunity for a normal crowd to start jumping rhythmically; alarming vibrations have been observed at three major football stadia in England following rhythmic excitation[3]. These observations resulted in corrective measures being introduced to the stadia.

The logical development, starting from the simple limiting frequency, is to define appropriate load models for an individual and a crowd. These models will then enable responses to be calculated and specific structural configurations to be evaluated.

General considerations

To define crowd loads, several factors relating to the crowd, crowd activity and the structure need to be considered.

Crowd density

With many structures which are subject to crowd loads, a static design load of 5 kN/m^2 is used. This equates roughly to six people per square metre – a relatively dense crowd. However, in many instances the number of people in a given area is limited, possibly due to seating or licensing requirements. Therefore actual static loads appropriate to the activity should be used in determining dynamic loads.

Crowd size

Crowd size and the area a crowd occupies are also important.

First, the structure required to accommodate a large group is likely to consist of many different structural bays and areas. The mode shapes of bays in the structure often show the motion in adjacent bays as being anti-phase; hence loading in one bay may effectively attenuate the response in other bays. The critical situation is often reached by the loading in a single bay, and may well be found with some dance floors.

Second, as already mentioned, synchronised crowd movement is usually coordinated by music. For a large crowd, those furthest from the musical source will hear the music slightly later than those close to the source. This will induce a phase difference in response with distance from the musical source, thereby affecting the overall coordination – a phenomenon which has been observed in a grandstand[4].

Frequency range

The frequency range for an individual person jumping is approximately 1.5 to 3.5 Hz, but, for a crowd, the higher frequency jumping cannot be sustained and an upper limit of 2.8 is more realistic. This is considered in greater detail by Ginty *et al*[5] and Littler[6]. With people dancing or exercising which involves repeated jumping, the energy is not only generated at the dance frequency but is produced at whole number multiples of the dance frequency. This is because the loading is cyclic but is not a simple sinusoid; consequently if the loading function is expressed in Fourier series, the cyclic nature of the activity constrains the Fourier components to be integer multiples of the dance frequency.

Dynamic crowd effect

If a crowd of people all tried to jump at the same frequency, which may be the case for some dances, their coordination will not be perfect. Imperfect coordination can be due either to individuals not jumping at exactly the beat frequency, which may be more common at the extremes of the frequency range, or to individuals dancing with different styles or enthusiasm, or to phase differences between individuals. It means that the loads produced by a group are not a simple summation of the ideal loads produced by individuals. In fact there is an attenuation of the peak load with increasing group size which is greater for the higher Fourier components. This is considered later.

Human–structure interaction

One of the difficulties of modelling structures with crowds of people is how to account for the mass of people since it can affect the structural characteristics and, consequently, the response to any given load. In fact there is a range of different situations of which the two extremes are important. First, a stationary crowd acts as an additional mass–spring–damper system on a structure and should be modelled accordingly[7]. Second, for the case where people are moving (eg walking and jumping) their body mass is not involved in the vibration of the structure and the human involvement is simply as a load. Therefore for evaluation of structural response to dance-type loads, the characteristics of the unloaded structure should be used in the calculations. For the situation where many in the crowd are moving rhythmically but a significant number are stationary, the stationary people will provide a significant damping mechanism and a consequent reduction in any resonant response.

Loading

Load model for an individual jumping

The load–time history for continuous jumping can be described by a high contact force for a certain time t_p (contact period) followed by zero force when the feet leave the floor. It has been proposed that the load–time function can be expressed by a sequence of semi-sinusoidal pulses; this aligns well with measurements for individuals jumping. The load function in one period for a single person $F_s(t)$ is given by:

$$F_s(t) = \begin{cases} K_p G_s \sin(\pi/t_p) & 0 \le t \le t_p \\ 0 & t_p \le t \le T_p \end{cases} \qquad \text{(Equation 1)}$$

where:

K_p = the impact factor (F_{max}/G_s)
F_{max} = the **peak** dynamic load (N)
G_s = the weight of the jumper (N)
t_p = the contact duration (s)
T_p = the period of the jumping load (s)

The contact period t_p can vary from 0 to T_p corresponding to different movements and activities. The contact ratio α is defined as follows:

$$\alpha = \frac{t_p}{T_p} \le 1.0 \qquad \text{(Equation 2)}$$

Therefore different contact ratios characterise different rhythmic activities, which can be seen in Table 1.

It has been observed experimentally[9] that the mean value of the time history of a vertical load corresponding to bouncing to music on toes (jouncing) or to rhythmic jumping is always equal to the weight of the performer. By equating the mean value of Equation 1 over one period T_p to the weight G_s the following relationship is obtained:

$$K = \frac{\pi}{2\alpha} \qquad \text{(Equation 3)}$$

Thus the loads can be determined knowing the weight of the jumper, the period of the jumping and the contact ratio. Figure 1 shows a normalized load–time history for jumping with $\alpha = 1/3$ and $f_p = 1/T_p = 2.0$ Hz. The normalized load of 1.0 corresponds to the static weight of the person. From the figure it can be appreciated that the dynamic load can be significantly larger than the static load. Indeed, from Equation 3 the peak dynamic load can be calculated directly which for these parameters is 4.7 times the static load. It can also be deduced that the higher the jumping, the lower the contact ratio and the higher the peak dynamic load.

Table 1 Typical values of the contact ratio α for various activities[8]

Activity	α
Low impact aerobics	$^2/_3$
Rhythmic exercises, high impact aerobics	$^1/_2$
Normal jumping	$^1/_3$

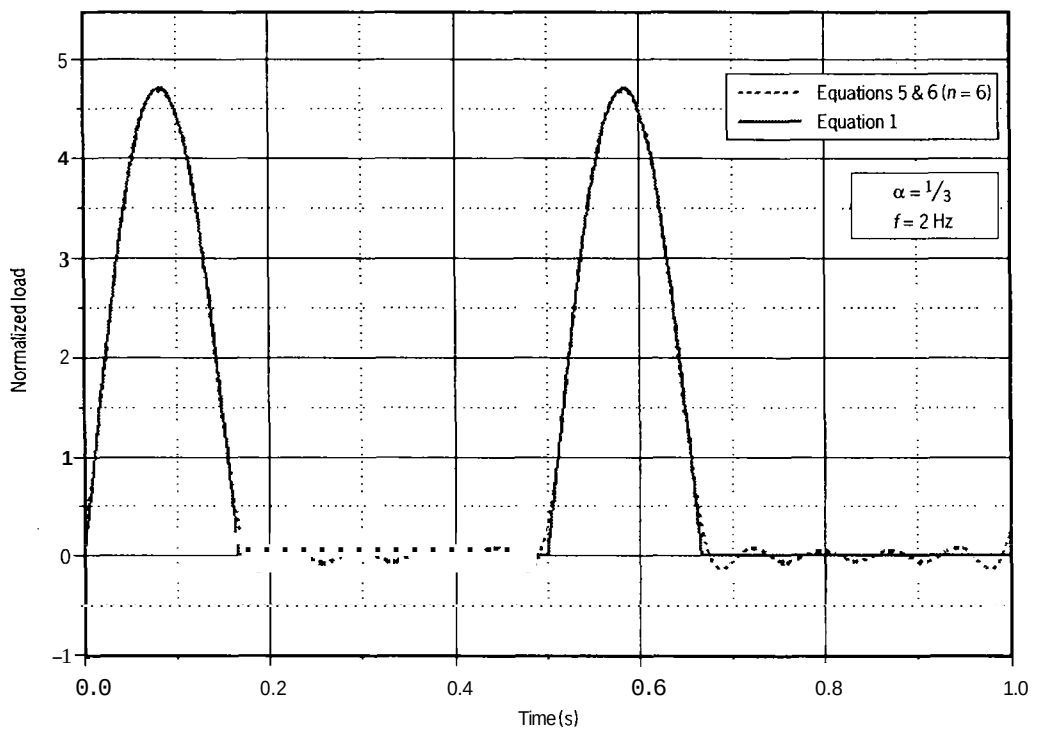

Figure 1 Load–time history for one person jumping

Load model for an individual in terms of Fourier series

For analysis purposes it is useful to express Equation 1 in terms of Fourier series:

$$F_s(t) = G_s\left\{1.0 + \sum_{n=1}^{\infty} r_n \sin\left(\frac{2n\pi}{T_p}t + \phi_n\right)\right\} \qquad \text{(Equation 4)}$$

where:
r_n = Fourier coefficient (or dynamic load factor) of then* term
n = number of Fourier terms
ϕ_n = phase lag of then''' term

Equation 4 can represent the vertical load for an individual for any form of rhythmic movement. In the Equation the terms within the summation sign are the Fourier components of the load and occur at multiples of the load frequency. Hence for each Fourier term the average load per cycle **is** zero. This aligns with the observation upon which Equation **3** was derived. Also if the dancing stopped the dynamic loads would become zero and the resulting force would simply be the weight of the dancer. Based upon Equation 3, the values for the load factors and phase lags can be determined for jumping using the following equations (Ji and Ellis''], and Ji and Wang[10]):

$$r_n = \begin{cases} \pi/2 & \text{if } 2n\alpha = 1 \\ \left|\dfrac{2\cos(n\pi\alpha)}{1-(2n\alpha)^2}\right| & \text{if } 2n\alpha \neq 1 \end{cases} \qquad \text{(Equation 5)}$$

$$\phi_n = \begin{cases} 0 & \text{if } 2n\alpha = 1 \\ \tan^{-1}\left[\dfrac{1+\cos(2n\pi\alpha)}{\sin(2n\pi\alpha)}\right] - \pi & \text{if } \dfrac{\sin(2n\pi\alpha)}{1-(2n\alpha)^2} < 0 \\ -\dfrac{\pi}{2} & \text{if } \sin(2n\pi\alpha) = 0 \\ \tan^{-1}\left[\dfrac{1+\cos(2n\pi\alpha)}{\sin(2n\pi\alpha)}\right] & \text{if } \dfrac{\sin(2n\pi\alpha)}{1-(2n\alpha)^2} > 0 \end{cases} \quad \text{if } 2n\alpha \neq 1$$

(Equation 6)

The normalized load–time history, calculated using Equations 5 and 6, and including the first six Fourier terms, is shown in Figure 1 along with the equivalent load determined using Equation 1. Table 2 lists the first six Fourier coefficients and phase lags for different contact ratios.

Load models for crowds jumping: experimental values

In the previous sections the loads produced by individuals have been considered. This can be extended to cover crowds. The load equation for a crowd jumping at a comfortable frequency becomes:

$$F(x,y,t) = G(x,y)\left\{1.0 + \sum_{n=1}^{\infty} r_{n,v} \sin\left(\frac{2n\pi}{T_p}t + \phi_n\right)\right\} \qquad \text{(Equation 7)}$$

where:
$F(x,y,t)$ = distributed force which varies with time
$G(x,y)$ = density and distribution of human loads
$r_{n,v}$ = n^{th} Fourier (load) coefficient induced by v persons

Here it is necessary to consider the spatial distribution of the people. However there is also a variation in the values of the Fourier coefficients due to the different jumping styles and coordination of the jumpers. This has been determined experimentally and reported in detail in IP 4/02. In the experiments, tests were undertaken on two floors with different sized groups jumping at selected frequencies. The group sizes were selected to be powers of 2 with 64 being the largest group that could be accommodated safely. The jumping was coordinated using a musical beat at selected frequencies. For the experiments only the lowest three Fourier coefficients (FCs) were determined. The FCs were plotted against the number of people in the group on a log/log plot together with regression lines for each FC based on a power relationship. The data are shown in Figure 2 on page **6.**

The regression lines are defined by the following equations which characterise the variation in FCs with the number of people v in the group:

- first FC $r_{1,v} = 1.61 \times v^{-0.082}$
- second FC $r_{2,v} = 0.94 \times v^{-0.24}$
- third FC $r_{3,v} = 0.44 \times v^{-0.31}$

If only one person is considered (ie v = 1), it can be seen that the FCs fall between the values given in Table 2 for normal jumping ($a = 1/3$) and rhythmic exercises ($a = 1/2$).

For calculation purposes both the Fourier coefficients and the phase angles are required. Using the FCs defined above and the theoretical phase angles for normal jumping (Table 2, $a = 1/3$), the load model predicted similar amplitudes of acceleration and displacement to the average measured values for the various group sizes up to **64** people.

Table 2 Fourier coefficients and phase lags for different contact ratios

		$n=1$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
$\alpha = 2/3$	r_n	$9/7$	$9/55$	$2/15$	$9/247$	$9/391$	$2/63$
	ϕ_n	$-\pi/6$	$-5\pi/6$	$-\pi/2$	$-\pi/6$	$-5\pi/6$	$-\pi/2$
$\alpha = 1/2$	r_n	$\pi/2$	$2/3$	0	$2/15$	0	$2/35$
	ϕ_n	0	$-\pi/2$	0	$-\pi/2$	0	$-\pi/2$
$\alpha = 1/3$	r_n	$9/5$	$9/7$	$2/3$	$9/55$	$9/91$	$2/15$
	ϕ_n	$\pi/6$	$-\pi/6$	$-\pi/2$	$-5\pi/6$	$-\pi/6$	$-\pi/2$

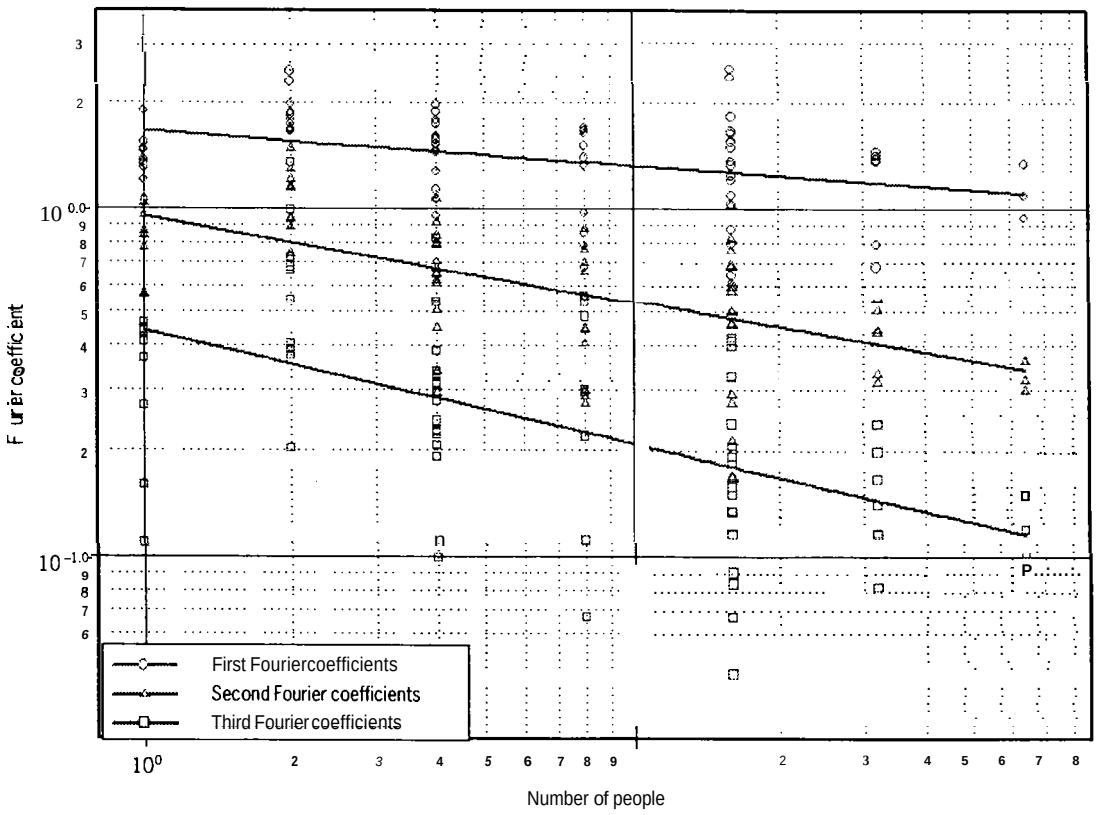

Figure 2 Fourier coefficients determined for all of the tests

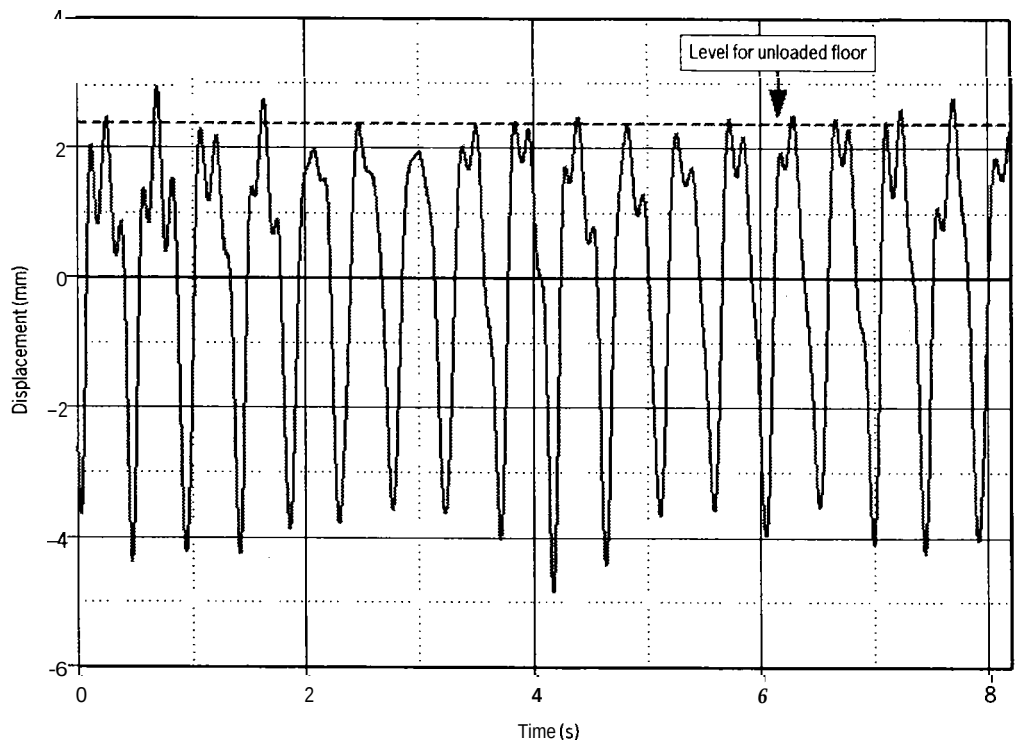

Figure 3 Measured displacements for **64** people on the second floor

Although these FCs reduce with increasing crowd size, it should be recognised that these are still significant loads. To demonstrate this point, it is worth examining the central displacement of the floor measured for 64 people jumping. This is shown in Figure 3.

The floor area was **9 x** 6 m; hence 64 people represent a density of 1.185 people/m^2 which is not densely packed. Taking the average weight of the jumpers gives a distributed load of 0.86 kN/m^2. The measured displacement (static + dynamic) was actually 3.47 times the static displacement

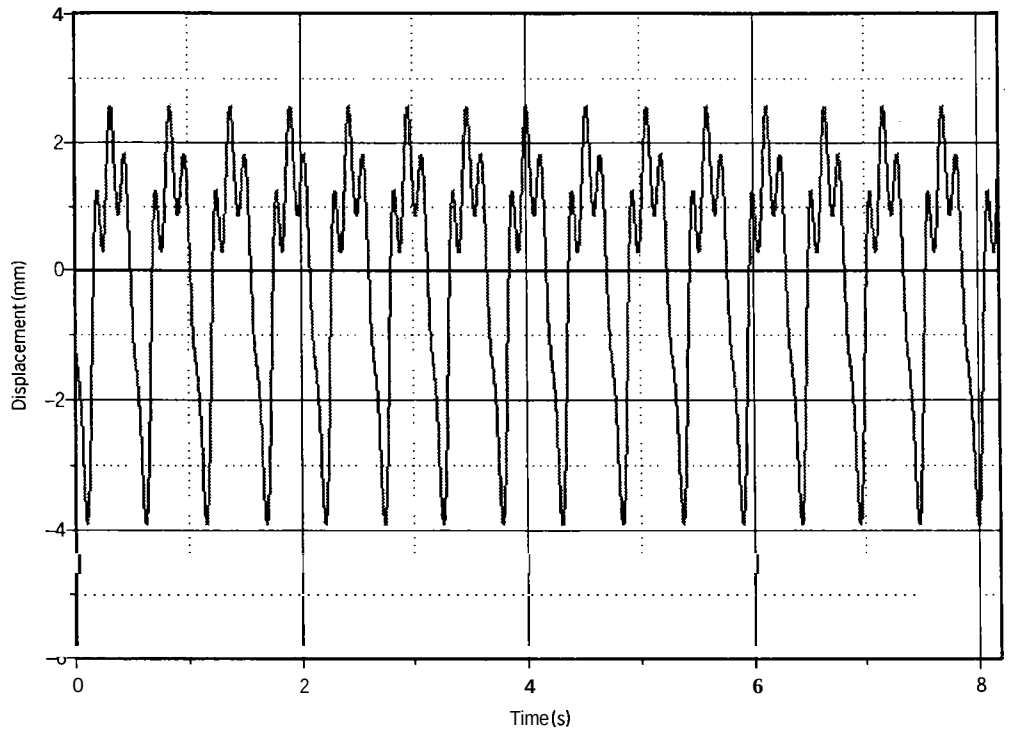

Figure 4 Calculated displacements for 64 people on the second floor

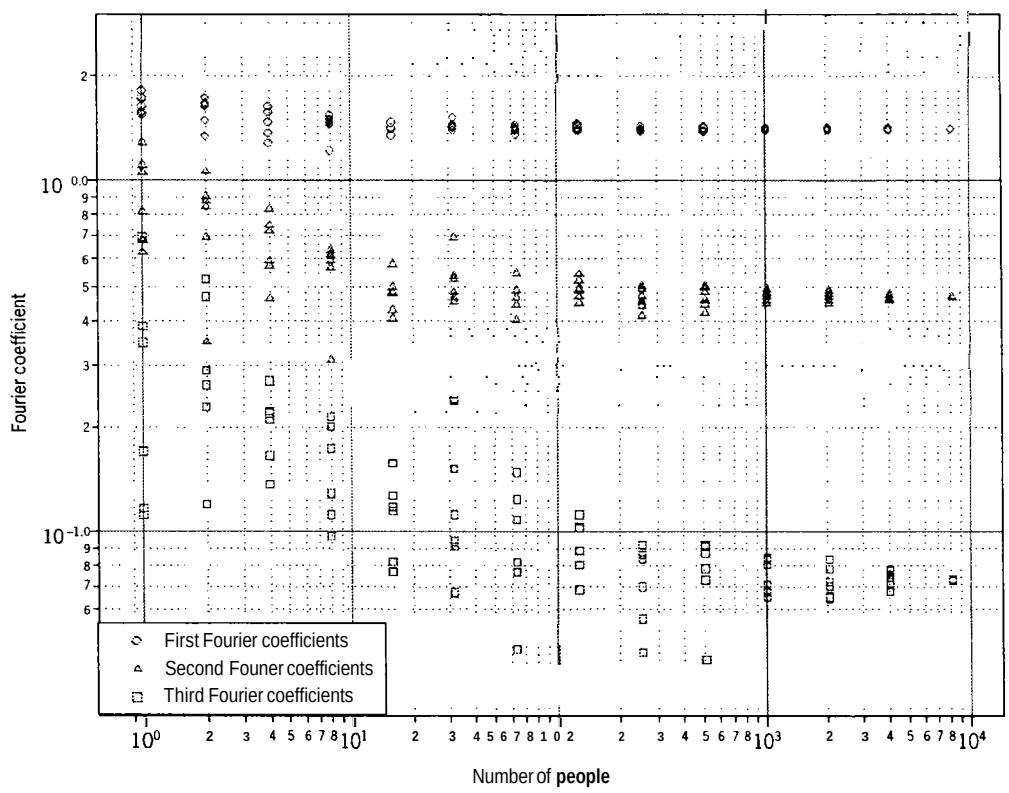

Figure 5 Fourier coefficients against group size: combined model

which, when used to factor the static load, gives an equivalent peak dynamic load of 2.99 kN/m^2. The floor was designed for 3.5 kN/m^2. This load scenario did not include a significant resonant component.

If the FCs for a group of 64 people ($v = 64$) are used in Equation 7 to define the load, then the calculated dynamic displacement–time history for the second floor is as shown in Figure 4.

Load models for crowds jumping: numerical values

Although the experiments provide some useful results they are limited to a relatively small group of people; hence the experiments were modelled numerically, with the objective of replicating the variation in loads observed in the experiments. For an individual jumping in response to an audible prompt at a given frequency, the following variables were considered:

- the contact ratio α and therefore the height of jumping;
- the jumping frequency f which may not be perfectly aligned with the prompt;
- the phase difference between the prompt and the individual jumping.

The load model for an individual jumping (Equation 4) was used as the basis for the numerical modelling with the aforementioned variables introduced into this model in order to calculate a load–time history. The distribution of variables was derived from the experimental measurements. Groups of people were represented by the combination of the appropriate number of individual load–time histories.

The results which were obtained, and which correlate well with the experiments described in the previous section, were used to examine the effects of larger group sizes; the variation in loads with group size is seen in Figure 5 on page 7.

With the results for up to 64 people based upon six calculations for each group size, it was seen that the spread in calculated FCs became less for the larger groups. This was probably due to the statistical variations being smoothed by a larger number of samples. Furthermore, for the final FCs there was no longer a reduction in FC with crowd size, suggesting that the power relationships derived from the experimental data are only appropriate for the smaller groups. The values for the first three FCs for groups of 8,192 people were 1.40, 0.47 and 0.072. Further evaluations are given by Ellis and Ji[11].

It should be remembered that these calculations are based upon experiments with students undertaking the jumping. It is likely that some groups of people (eg professional dancers) will be better coordinated than the students (and therefore produce higher peak loads), whereas in other groups the coordination may be far worse.

Vertical loads for synchronised movement not involving jumping

The previous sections have considered rhythmic jumping which, it is appreciated, can generate high loads. However, in many situations involving rhythmic excitation (eg pop concerts), the situation with everyone jumping does not arise. Usually some people will be jumping or dancing and others standing or sitting. This common scenario leads to lower peak loads than those produced by everyone jumping and a significantly smaller structural response. One example will be given to illustrate this.

In the 1990s, BRE monitored a number of events in cantilever grandstands during concerts. Analysis of recorded data concentrated on the largest responses which were when approximately 90% of the audience was standing, dancing and moving in response to music but not jumping. Similar methods to those developed for the jumping tests were used to determine the loads, although this required several assumptions since the loading on the grandstands was not artificially controlled[12]. Based upon the assumption that the FCs varied with the number of people in a similar fashion to the jumping tests, the first three FCs of the loads were 0.42, 0.087 and 0.017. It can be appreciated that these are much smaller than those for jumping (ie 1.61, 0.94 and 0.44).

There is one further important factor. As everyone was not moving the damping of the occupied structure is increased significantly from that of the bare structure due to human–structure interaction. A damping value of 16% was determined from the analysis of the measurements which would have a significant effect upon any resonant response.

Therefore it should be recognised that it is important to define the appropriate loading scenarios for design. A structure which may perform adequately during concerts may appear to be unacceptable when subjected to jumping loads.

Structural response

Initial considerations

Given the number of people involved and their spatial distribution, and the frequency and type of the coordinated movement, the overall load can be determined. The derivation of the response of a simply-supported floor to dance-type loads is given by Ji and Ellis[1] and extended to cover other structures. This provides formulae for calculating how displacement and acceleration vary with time. These calculations include all the modes of vibration of the system and all the Fourier components of the load. In this Digest the derivation is not given and only the relevant equations are quoted.

Alternatively the loads could be used in a finite element programme to determine the response of a structure.

Selection of the appropriate mode of vibration

For many structures it will be the fundamental mode of vibration which will be important and this term will be used in the rest of this Digest. However, for other structures it may be another mode of vibration which is critical; this is termed the principal mode for that location. For example, if a cantilevered tier of a grandstand is being examined, it would be the vertical mode of vibration of the cantilever, or part of the cantilever, that should be considered and not necessarily the fundamental mode of the whole grandstand. Consequently it is the appropriate mode or modes of vibration for the analysis that must be determined. For simplicity the following sections consider the response in the principal mode which, for a single bay floor, will be the fundamental mode.

Considering a symmetric floor in a single bay under a symmetric load, there will be no vibration of the anti-symmetric modes. Therefore, the first higher mode involved in the vibration is the second symmetric mode which has a frequency considerably higher than the fundamental frequency. Also the modal load for the second symmetric mode will be significantly lower which effectively means that the response will be dominated by the contributions of the fundamental mode. Consequently, only the response from the fundamental mode needs to be considered.

For a more complicated structure it may be necessary to include several of the lower frequency modes to determine displacement, although for acceleration it may be sufficient to consider just one mode excited at resonance.

Number of Fourier components to consider

For calculation purposes, it is not necessary to use all the terms in the Fourier series to describe the load. The number of Fourier terms to be taken into account depends on their contribution to the response, including any possible resonance. It should be noted that resonance can occur when the natural frequency of the structure f_s is equal to the dance frequency f or to integer multiples of the dance frequency. Basically, for serviceability assessments, the first I Fourier terms should be included in the determination of acceleration where I is defined as the first integer bigger than f_s/f. This ensures that any resonant excitation of the fundamental mode will be considered. As resonant excitation may produce the largest response, this is one situation that should be evaluated. For safety assessments, where evaluation of displacement and stresses is appropriate, it is recommended that the first three harmonic terms should be considered.

General solution

The solutions provided by Ji and Ellis[1] can be applied to structures with different boundary conditions. The difficulty encountered is in choosing suitable displacement functions. For a single bay floor with symmetric boundary conditions, under a uniformly distributed load, no anti-symmetric modes are involved in the vibration. Therefore, only the fundamental mode needs to be considered, and the shape of this mode is relatively easy to choose with sufficient accuracy for many common cases. The response of a structure can be approximated by the contribution of the fundamental mode:

$$w(x,y,t) = A(t)\,\Phi(x,y) \qquad \text{(Equation 8)}$$

where $\Phi(x,y)$ is the dimensionless fundamental mode with unit peak value, and $A(t)$ is the amplitude of vibration corresponding to that mode and is a function of time.

The structural factor

The structural factor B relates to the fundamental mode and depends on the type of structure and boundary conditions. If both the structural mass and the loading are uniformly distributed in space, the structural factor can, according to the solution procedure, be defined as follows:

$$B = \frac{\iint_s \Phi(x,y)\,dxdy}{\iint \Phi^2(x,y)\,dxdy} \qquad \text{(Equation 9)}$$

For a simply supported rectangular plate the mode shape would be:

$$\Phi(x,y) = \sin(\pi x/L_x)\sin(\pi y/L_y) \qquad \text{(Equation 10)}$$

where L_x and L_y are the length and width of the plate. The resulting value of B would equal 1.62. This deflected shape may be reasonable for many floors. If the correct mode shape was used to determine B for a rectangular plate with fully fixed boundary conditions, the value of B would be 1.72.

Structural response

For this section it is assumed that the structural mass and loading are uniformly distributed (ie $m(x,y) = m$ and $G(x,y) = G$)

The structural displacement and acceleration can be expressed as:

$$A = B\,\frac{G}{m\,(2\pi f)^2}\,(1.0 + D) \qquad \text{(Equation 11)}$$

$$\ddot{A} = B\,\frac{G}{m}\,D^a \qquad \text{(Equation 12)}$$

D and D^a are defined as dynamic magnification factors for the displacement and acceleration;

$$D = \sum_{n=1}^{3} D_n \qquad D_n = \frac{r_{n,v}\sin(2\pi nft - \theta_n + \phi_n)}{\sqrt{(1 - n_2\beta_2)_2 + (2n\varsigma\beta)^2}} \qquad \text{(Equation 13)}$$

$$D^a = \sum_{n=1}^{I} D_n^a \qquad D_n^a = n^2\beta^2 D_n \qquad \text{(Equation 14)}$$

$$\phi_n = \begin{cases} \tan^{-1}\left(\dfrac{2n\varsigma\beta}{1-n^2\beta^2}\right) & \text{if } 1-n^2\beta^2 > 0 \\ \dfrac{\pi}{2} & \text{if } 1-n^2\beta^2 = 0 \\ \tan^{-1}\left(\dfrac{2n\varsigma\beta}{1-n^2\beta^2}\right) + \pi & \text{if } 1-n^2\beta^2 < 0 \end{cases} \qquad \text{(Equation 15}$$

$$\beta = \frac{f_p}{f_s}$$

Equations 11 and 12 give the steady state response which excludes transient terms; this is because the transient response decays quickly in a damped system and generally is of little interest[13]. At resonance, the acceleration in Equation 14 will be dominated by the term that occurs at the natural frequency of the structure.

Using dynamic measurements in the analysis

For checking the safety and serviceability of existing structures or improving the accuracy of theoretical predictions, it is desirable to perform dynamic tests. This will provide feedback from the actual structure including accurate values of the fundamental frequency, damping, modal stiffness and mode shape. The fundamental frequency of most structures is relatively easy and inexpensive to measure[14]. On the other hand, it may prove difficult or inconvenient to measure the actual response when a crowd of people is involved.

Equations 11 and 12 can be expressed in the following form to accommodate the measurements.

$$A = \frac{G\iint_s \Phi(x,y)\,dxdy}{k^*}\,(1.0 + D) \qquad \text{(Equation 16)}$$

$$\ddot{A} = \frac{G\,(2\pi f_s)^2 \iint_s \Phi(x,y)\,dxdy}{k^*}\,D^a \qquad \text{(Equation 17)}$$

To use the above equations the measured modal stiffness k^*, natural frequency f, damping ratio ζ and mode shape $F(x,y)$ are required. Alternatively, Equations 9 and 10 can be used where the floor mass density needs to be estimated. It is also preferable to use the measured damping value.

If one person with a weight of G_s jumps at the centre of a floor the response is simply:

$$A = \frac{G_s}{k^*}\,(1.0 + D) \qquad \text{(Equation 18)}$$

$$\ddot{A} = \frac{G_s\,(2\pi f_s)^2}{k^*}\,D^a \qquad \text{(Equation 19)}$$

Simple measures to avoid safety problems

As dynamic crowd loading is confined to a narrow frequency range, and as it is only necessary to consider a limited number of Fourier terms to evaluate displacements, it is possible to define minimum frequencies for structures which should avoid safety problems from dynamic loads. This may provide simple criteria which are convenient for the design of many structures; however, it does not mean that all structures with lower frequencies will have problems, and, for some structures, calculation of the actual response will be appropriate. If the calculations show an unacceptable response, then the likely design solution will be to increase the frequencies towards the given limits, or to limit the crowd size or crowd activities. Solutions which seek to increase the damping may have some attraction; however, they recognise that resonance will occur and attempt to limit the resonant response.

The structural frequency above which vertical vibration should not pose a safety problem for jumping on floors is 8.4 Hz (ie 3 x 2.8 Hz, avoiding resonance from the first three Fourier components of the load). For grandstands where concerts may be held, a frequency of 6 Hz[2] has been given as an interim measure (primarily avoiding resonance from the first two Fourier components of the load). As mentioned previously, these frequencies are for the appropriate mode of vibration of an empty structure (ie without a crowd)[7].

Serviceability limits

For serviceability assessments related to human perception of vibrations, it is the acceleration levels which should be determined. However, the question of what are acceptable vibration levels must then be considered. A lot of work has been conducted on what vibration levels can be perceived, and it is known that the frequency of the vibration is important; therefore the UK Standards, BS 6472 and BS 6841, related to perception of vibrations provide frequency weighting recommendations for motion in various directions. The frequency weighting effectively defines a filter that can be applied to the acceleration to compensate for the finding that human perception of vibration varies with frequency. In addition, the number of cycles of the vibration is important when assessing acceptability of such vibrations. Therefore, a vibration level with a duration of one cycle (ie impulsive), will have a different effect to a vibration level which is continuous. These British Standards therefore recommend using vibration dose values (VDVs) which take account of these factors.

For guidance on acceptable vibration levels in structures subject to jumping loads, it seems appropriate to examine data obtained at such events. A number of experiments have been undertaken in Germany[15] on cantilevered grandstands; recommended peak accelerations for low frequency vibration are given in Table 3. Ellis and Littler[16] explain how the acceleration levels given in the table can be linked to the VDVs suggested in the British Standards, and show that they provide a reasonable correlation with the observed audience reaction and measured response on a number of grandstands at a range of events. From the Table it can be seen that, for repetitive jumping, 35% g is a significant limit as above this level panic may occur. However, below this level the vibration levels will certainly be felt by anyone remaining stationary on the structure. The authors experienced vibrations of 15% g at 4 Hz on one structure and this certainly felt uncomfortable, even with knowledge of the source of the vibrations.

It is also likely that visible motion of a structure and items fixed to it would concern those immediately below the structure.

Table 3 Reaction to various peak acceleration levels on grandstands[15] (for a frequency range <10 Hz)

Vibration level	Reaction
<5% g	Reasonable limit for passive persons
<18% g	Disturbing
<35% g	Unacceptable
>35% g	Probably causing panic

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British Standards Institution

BS 6399:Part 1:1984. Loading for buildings. Code of practice for dead and imposed loads.

BS 6399-1:1996. Loading for buildings. Code of practice for dead and imposed loads

BS 6472:1992 Guide to evaluation of human exposure to vibration in buildings (1 Hz to 80 Hz)

BS 6841:1987 Guide to measurement and evaluation of human exposure to wholebody mechanical vibration and repeated shock

CP 3:Chapter V:Part 1:1967. Code of basic data for the design of buildings. Loading. Dead and imposed loads

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