Final Report on Grant GR/S74069/01

Prediction of Floor Vibration Induced by Walking Loads and Verification Using Available Measurements

Tianjian Ji
The University of Manchester

1. Background

In recent years, there have been an increasing number of problems related to human-induced vibrations of floors, bridges, assembly structures and stairs. Modern structures adopt high-strength and lightweight construction materials resulting in decreased structural mass and damping properties. In addition the need for open and large column-free spaces has resulted in construction of long-span floor systems with relatively low stiffness. Consequently, the natural frequencies of the floor structures are reduced to the point that the human walking may produce resonant or near-resonant structural vibrations that are uncomfortable to the users.

Vibration problems of long-span floors were often noted after construction, but they were not often reported as safety problems are not concerned. Why can such vibration problems not be identified at the design stage? Reasons are that walking loads have not been fully understood and defined, models of floors are inadequate sometime and the methods for predicting floor response may be oversimplified. It is only to be expected that the output (prediction of the responses) cannot be any better than the input (definition of the loads, models of floors and the quality of analysis methods). The problems discussed concern many new and existing long-span floors and footbridges. It is expected that in the future floors and footbridges will be even longer and lighter with lower natural frequencies and human expectation to the quality of living and working environment will be even greater. Therefore, the vibration problem of floors and footbridges due to walking loads addressed here are likely to be even more significant in the future and the design of problem-free long-span floors and footbridges becomes an increasingly important issue in structural engineering.

This project aimed to investigate the models of walking loads induced by an individual, study the dynamic behaviour of floor systems and predict the response of floors induced by the walking loads. The available measurements from our own previous tests and others were used where possible for creating the load models and checking the predictions

2. Key Advance and Supporting Methodology

The main outcome from the project are summarized in five sub-sections on models of walking loads, dynamic behaviour of floors, prediction of floor responses, recommendations and the other related work.

2.1 Models of walking loading

2.1.1 Frequency and velocity of people walking [1]

The project investigated the frequency and velocity of people walking on footbridges and shopping floors. Two footbridges and two shopping floors in Manchester were selected for the tests. 100 men and 100 women were randomly selected for the tests at each of the venues. Therefore, a total of 800 measurements were taken. During the tests, the participants were not aware that they were being observed and walked naturally. The measurements of walking frequency, velocity and step-length were processed using statistical methods. The main conclusions obtained from the study are summarised as follows:

- *People walked faster on the shopping floors, with an average frequency of 2.0 Hz and a velocity of 1.4 m/s, than on the footbridges, with an average frequency of 1.8 Hz and a velocity of 1.3 m/s.*
- *The mean values of the step-length (L_s) on the footbridges and the shopping floors were similar with 0.75m for men and 0.67m for women.*

- *There was a linear relationship between walking velocity (v) and frequency (f_{pv}). The relationship can be expressed as*

$$v = L_s\ f_{p_V} = \begin{cases} 0.75\, f_{p_V} & \text{for men} \\ 0.67\, f_{p_V} & \text{for women} \end{cases} \qquad (1)$$

- *The experiments indicated that men walk with higher velocities than women, while women walk with higher stepping frequencies than men.*
- *The walking frequency and velocity obtained on the footbridges and the shopping floors followed normal distributions with standard deviations of (0.11 Hz, 0.13 m/s) and (0.13 Hz, 0.13 m/s) respectively.*

2.1.2 Load models [6]

A method was developed to define a continuous walking load model in the vertical direction using the available force-time measurements induced by a single foot. The gap between the force-time histories for a single foot and for a continuous walking was bridged using the principle that *the integration of the walking force-time history in a period is equal to the product of the period and the weight of the walker*. It was demonstrated mathematically that *the frequency of walking loads in the vertical direction is twice as that in the lateral directions* and *the walking loads in the lateral directions do not contain the even numbered harmonic terms*. Therefore, the walking frequency in the lateral directions can be deducted from the measurements of that in the vertical direction studied in 2.1.1.

Four types of walking, normal walk, brisk walk, slow walk and fast walk, are considered. All the parameters in the load models were quantified, including Fourier coefficients and phase lags. It was concluded and recommended that *the load model of normal walking should be used for design and analysis as it generates higher dynamic components than slow walk and fast walk and has similar amplitudes to that of brisk walk, and people can comfortably walk at such a pace*. The continuous load model for an individual walking provides a basis to investigate the load model for a crowd walking.

2.2 Dynamic behaviour of floor systems

Understanding the dynamic behaviour of floors is the key for correctly predicting the floor response to walking loads. Both analytical and numerical methods are used to investigate the general characteristics of multi-panel floors.

2.2.1 Modelling of profiled composite floors [3]

A composite floor normally comprises profiled steel decking and a concrete slab lying on the steel decking and spanning between support beams, which are widely used in building and bridge construction and are sensitive to vibration. The appropriate models were investigated for predicting the dynamic behaviour of composite floors simply and reasonably accurately. An isotropic and an orthotropic flat plate models were developed based on the equivalence of the maximum displacement of a sophisticated 3D composite deck panel model. Thin shell elements were used to model the steel sheet and 3D-solid elements to represent the concrete slab. Parametric studies were conducted to examine the effects of boundary condition, loading condition and shear modulus on the equivalent models. The two simplified flat plate models were then applied to studying the dynamic behaviour of a full-scale multi-panel composite floor (45m x 21m) in the Cardington eight-storey steel-framed building. The predicted and measured natural frequencies are reasonably close. The modelling process becomes easier and significant time saving is achieved when either of the two simplified model is used. The variation of floor thickness due to construction can significantly affect the accuracy of the prediction.

2.2.2 Free vibration of rectangular floors with the internal column supports [2]

Following our previous study on modelling of floor-column systems using the finite element method and natural frequency measurements, the project derived the exact solution for the free vibration of a thin rectangular plate with two opposite edges simply supported and with internal column supports. Considering only the compatibility of displacements and rotations between the plate and the columns, the

coupled vibration of plate-column system was derived. Comparison of the results obtained from the proposed method and from finite element analysis demonstrated the accuracy and correctness of the method. The effects of column sizes and of approximate models on the natural frequencies of a plate-column system were investigated in detail. The parametric study was focused on fully simply supported rectangular plates with a single internal column. It was shown that *the effect of column flexibility in plate-column structures should be considered.* The solution provided is general and includes several particular solutions for fixed point-supports, pinned point-supports and elastic point-supports. Results with high accuracy were obtained and these can be used as the benchmark for the further investigation and for other approximate methods. This work provides an improved understanding on modelling of floors where the columns should be included to form a floor-column system.

2.2.3 Effect of eccentricities [4]

The effect of the neutral axis locations of a composite section on the dynamic behaviour of composite floors was also investigated. It was demonstrated that *the second moment of area of a composite section in respect to any assumed location of the neutral axis is always larger than the true value of the second moment of area of the section*. The relative errors on the calculation of the second moment of a composite section between the actual location and four possibly assumed locations of the neutral axis were examined. The calculated natural frequencies of several stiffened plates using the assumed locations of the neutral axis were compared with the available frequency measurements and numerical results. Finally the effect of the assumed eccentricities on the dynamic characteristics of a real composite floor was assessed. It was concluded that *the assumed locations of the neutral axis are not sensitive to the predicted natural frequencies, but exclusion of the eccentricity in calculation would underestimate at least the fundamental natural frequency of a composite floor and alter the orders of mode shapes*.

2.3 Prediction of floor response to walking loading [7]

The project investigated the methods for predicting floor response induced by an individual walking. A single degree of freedom (SDOF) model of a structure was developed considering the action of walking loads that are functions of time and location. The model was verified by FE modelling of a simply supported beam and a simply supported plate where the newly developed model of walking loads was used, confirming a SDOF model is still a useful tool for predicting structural response under walking loads. The SDOF model was further applied to predict the response of a concrete floor and a composite floor. Comparison between predictions of the SDOF model and the FE results and the available dynamic measurements were also given. The dynamic responses of the FE model of the full-scale composite floor matched in both amplitude and pattern with the measured dynamic responses. To predict the dynamic response of a full-scale floor using a SDOF system requires a correct representation of the floor stiffness.

The responses of a structure induced by an individual walking were studied from a simply supported beam to a composite floor based on the load model proposed in 2.1.2. The conclusions obtained are summarised as follows:

- *A SDOF system is introduced to predict the dynamic response of the structures induced by an individual walking. The predictions match in both amplitude and pattern with the FE analysis results.*
- *If the maximum dynamic response of a structure is of interest, the walking load moving on the structure can be replaced by walking at spot on the critical point where the maximum response is likely to occur.*
- *The maximum dynamic response (amplitude and pattern) of a simply supported plate induced by walking load is not affected by the paths of walking if they all pass the critical point of the structure. The implication of the finding is that the walking load can be applied arbitrarily on a floor, providing it passes the centre of the floor.*
- *The dynamic response of the FE model of the full-scale composite floor matches in amplitude and pattern with the experimental dynamic response.*
- *To predict the dynamic response of the full-scale floor using a SDOF system requires a correct representation of the floor stiffness.*

2.4 Summary and Recommendations [8]

The project also summarised the modelling of three types of floor system in order of increasing complexity primarily for providing information for the calculation of their responses to human loading and for a better understanding of the floors. They are the flat floors, flat floors supported by steel beams and profiled composite floors. It also presented the lessons that had been learnt by the modelling of multi-panel floors. It focused on the low damped, medium to long-span floors typically encountered in modern offices. For these floors, the resonant response to rhythmic loading from walking or jumping is the prime consideration.

For numerical modelling the following recommendations are provided:

- *The columns which support or link to the floor should be considered in the model.*
- *For floors supported by beams, the eccentricity of the beams needs to be considered in the analysis. This will provide a realistic representation of the actual structure and increase the stiffness of the floor. The calculated frequencies are not very sensitive to the location of the neutral axis, but it should be considered.*
- *For a composite floor the profiled steel decking should be considered in modelling as it represents a significant part of the stiffness of the system.*
- *Accurately modelling a composite floor can be complex, but relatively simple equivalent flat plate models can be developed. These provide significant savings in modelling and CPU time in comparison* with the detailed model.

The other factors that affect the floor response to walking loads were also considered, such as human-structure interaction, other types of human loading and serviceability requirements.

2.5 Other related studies

The project was conducted simultaneously with another project, human-structure interaction –applying body biodynamics into structural dynamics, awarded by the Leverhulme Trust. An invited lecture, *sports stadia used for pop concerts*, based on the work on the two projects, was presented at the recent IASS conference, New Olympics, New Shells and New Spatial Structures, in Beijing, 16-19 October 2006 [9].

Consultancy was conducted on the measurement of natural frequencies of the Goven stand of the Ibrox stadium before and after its extension [11, 12] and on the vibration assessment of the Wembley National Stadium.

3. Project Planning Review

The project aimed to provide models and methods for predicting floor vibrations induced by walking loads, which will allow vibration problems to be eliminated or minimised by a prediction at the design stage. It also targets to produce practical methods for structural engineers and a technical basis for related building codes. The specific objectives are:

a) to extend the available measurements of walking loads induced by a single foot from measurements to a continuous walking load by two feet and to define the load using Fourier series.
b) to investigate the models and dynamic behaviour of composite floors based on the Cardington steel framed building and available frequency measurements.
c) to predict floor response to walking loads induced by an individual and a group of people using the FE method, based on the two Cardington test buildings and available measurements.
d) to provide an analytical method for evaluating floor vibrations due to walking loads, and to identify the critical situations which must be considered in design, and to verify the proposed load model and the analysis method using the response measurements on the Cardington building floors and numerical results.
e) to evaluate the proposed design method and the existing methods based on the Cardington building floors, and to summarise the results for peer review.

It was also stated in the Dissemination section of the original proposal that

- Four journal/conference papers are expected to be submitted for publication during the two-year period.
    - Modelling of walking loads induced by an individual (through peer-reviewed journal publication)

    - Dynamic behaviour of composite floors (through peer-reviewed journal publication)
    - Floor vibration induced by walking loads (through peer-reviewed journal publication)
    - The dynamic behaviour of building floors (through peer-reviewed conference paper)
- The research output will form a part of the book in preparation, Structural Vibrations Induced by Human Movements.
- The key output from the study will be summarised and published as a BRE Digest that is expected to be widely circulated in construction industry.
- Some research results, such as the model of walking loads, simplified methods for designing structures subject to walking loads, may provide input to related British and European codes.

The investigator is pleased to report that all the listed objectives have been successfully achieved, which is reflected in a series of publications listed on the last page of the report. At present, the project has not presented a paper reporting the floor responses induced by a group of people walking, although some analysis has been conducted. The study of walking loads induced by an individual, dynamic behaviour of floors and the floor responses to the walking loads provides fundamentals for further investigation when a group of people walking is concerned.

Further research beyond the objectives was also conducted, which included the theoretical studies of the dynamic behaviour of a floor supported by columns [2] and on the free vibration of a generalised suspended system [5]. A MSc student was arranged to participate in the project for completing her MSc dissertation. It was beneficial to the student for being involved in the research of a practical problem and to the project for adding a journal publication [1].

The industrial collaborator, Dr Brian Ellis, provided test results for the project, advised the comparison of the measured and predicted results and commented on the research outcome. His contributions are also reflected in the publications [8, 9, 10].

After completion of the project, Dr Ding Zhou was back to China and becomes a special professor at the Nanjing University of Technology and Dr Emad El-Dardiry returned to his university in Egypt as an assistant professor.

4. Research Impact and Benefits to Society

The research impact of the project to the structural engineering communities is multifold. First our papers have been published or accepted for publication by some top journals. Second our research work has been presented at an international conference as an invited lecture [9]. The study of the models of walking loads induced by an individual provides a basis to consider the loads induced by a crowd. The outcome of the project provides an improved understanding of the characteristics of walking loads, dynamic behaviour of multi-panel floor systems and floor response to walking loads. The isotropic and an orthotropic flat plate models will effectively reduce the analysis time of profiled composite floors. A summary paper for engineers is to be completed for submission [8].

The investigator attended the one-day course on Communication Skills at the Royal Society. The course helped to improve his presentation, *What makes buildings stand up*? given to Year 9-11 school students at the National Science and Engineering Week and Engineering Awareness Day.

It was identified that the predicted modal stiffness of the composite floor at the Cardington steel framed building is significantly larger than that measured at site. This observation coincided with what was happened on the most prestigious stand in the UK. This indicates that further research is required for modelling of complex structures.

5. Explanation of Expenditure

The EPSRC project and the Leverhulme project are conducted in the same period and both projects were allocated costs for a part-time technician. As we had a PhD student being involved in the study for conducting tests, it would be ideal if an additional RA post could be created using the costs allocated for a technician. Agreed by EPSRC, Leverhulme Trust and the head of Department, an additional RA (Dr Ding Zhou) was appointed to conduct research on the two projects. The time allocations for the two RAs are:

Dr Emad El-Dardiry	15 Months	100% EPSRC
Dr Ding Zhou	16 Months	75% EPSRC and 25% Leverhulme Trust

A total of £1200.00 was paid to a PhD student who helped to prepare and conduct the experiment. The total grant awarded by EPSRC was £106,932. The total expenditure is close to the budget without overspending.

6. Further Research and Dissemination Activities

A new PhD student has been arranged to study the modelling of complex structures due to the observations from the modelling of the composite floor for the project and of the Wembley National Stadium.

In addition to several journal papers and a conference paper published or submitted, the second edition of the BRE Digest was published which is widely circulated in the engineering communities. The work on the velocity and frequency of walking people [1] is edited into our website [13] to explain the wobbling of the London Millennium Footbridge. A website introducing this project was created when the grant was awarded [14]. Now this final report has been added into the website.

The publications and related outcome from the project are listed as follows:

Journal papers

1. Pachi, A and Ji, T., (2005), Frequency and velocity of walking people, The Structural Engineer, Vol. 83, No.3, pp.36-40.
2. Zhou, D and Ji, T, (2006), Free vibration of a rectangular plate with internal column supports, Journal of Sound and Vibration, Vol. 297, No. 1-2, 146-166.
3. El-Dardiry, E and Ji, T, (2006), Modelling of the dynamic behaviour of profiled composite floors, Engineering Structures, Vol.28, No.4, pp567-579.
4. El-Dardiry, E and Ji, T., The effect of eccentricity on free vibration of composite floors, submitted to Computers and Structures (accepted subject to corrections)
5. Zhou, D and Ji, T, Dynamic characteristics of a generalized suspension system, submitted to the International Journal of Mechanical Sciences
6. El-Dardiry, E and Ji, T, Floor vibration induced by individual walking: loading, submitted to Engineering Structures.
7. El-Dardiry, E and Ji, T, Floor vibration induced by individual walking: response, submitted to Engineering Structures.
8. Ellis, B R, El-Dardiry, E and Ji, T, On determining the dynamic characteristics of multi-bay floors, to be submitted to The Structural Engineer

Special publication

9. Ellis, B R and Ji, T, (2004), BRE Digest 426, Response of Structures Subject to Dynamic Crowd Loads, October 1997. ISBN 1 86081 1744, 12 pages.

Conference paper

10. Ji, T and Ellis B R (2006), Sports stadia used for pop concerts, International Association for Shell and Spatial Structures Symposium: New Olympics, New Shell and Spatial Structures, Beijing, China, 16-19 October 2006. Invited Lecture

Consultancy Reports

11. Ji, T and Duarte, E (2006), Measurement of natural frequencies of the Goven stand at the Ibrox stadium before extension,
12. Ji, T and Duarte, E (2006), Measurement of natural frequencies of the Goven stand at the Ibrox stadium after extension,

Websites

13. Ji, T and Bell A J, (2006), Seeing and Touching Structural Concepts, www.structuralconcepts.org
14. Ji T, http://personalpages.manchester.ac.uk/staff/tianjian.ji/research/floorwalking/floormain.htm