** Exercise 1

Consider the no claims discount model as in Example 3.11 in the notes. The insurance company is now considering to change the highest discount level from 40% to 50%. Research shows that as a result, the probability that a policyholder makes at least one claim in a year will drop from 1/4 to 1/6. Further, the average claim size per policyholder in a year, given that a claim is made, will go up from 250 to 320.

(a) Explain why it makes sense that the probability of making a claim goes down and that the average claim size goes up when the highest discount level is increased.

(b) Compute the expected profit per policyholder per year in the long run under the new regime and verify that it is lower than under the old regime.

(c) Despite the fact that the expected profit per policyholder drops, can you think of a reason why the insurance company should still go ahead with the new regime?

** Exercise 2

Draw the transition graphs of the two discrete time (and time homogeneous) Markov chains with state space $S = \{1, 2, 3, 4\}$ and transition matrices given by

(a) $P = \begin{pmatrix} 1/3 & 0 & 2/3 & 0 \\ 1/10 & 1/5 & 3/5 & 1/10 \\ 0 & 1/2 & 1/3 & 1/6 \\ 1/4 & 1/4 & 1/2 & 0 \end{pmatrix}$,  

(b) $P = \begin{pmatrix} 0 & 1/2 & 1/2 & 0 \\ 1/3 & 0 & 0 & 2/3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$.

Further determine for each MC whether it is irreducible or not and whether or not all states are aperiodic.

* Exercise 3

A firm rents cars and operates from 3 locations 1, 2, 3. Customers may return vehicles to any of the three locations. The company estimates that the probability of a car being returned to each location is as follows:

<table>
<thead>
<tr>
<th>car returned to</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>car hired from</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.50</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>2</td>
<td>0.25</td>
<td>0.75</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0.25</td>
<td>0.25</td>
<td>0.50</td>
</tr>
</tbody>
</table>

(a) Calculate the two step transition probabilities.

(b) Calculate the stationary distribution.
(c) Compute the probability that a car currently located at site 1 is subsequently at site 1, 2, 3.

** Exercise 4

Let $i, j, k$ be states of a Markov chain. Prove that if $i \leftrightarrow j$ and $j \leftrightarrow k$, then $i \leftrightarrow k$.

** Exercise 5  (**not so important**)

Below the one step transition matrices of two different discrete time Markov chains are given. For each case determine whether the Markov chain is irreducible and find its stationary distributions.

(a): $P = \begin{pmatrix} 1/2 & 1/4 & 1/4 & 0 \\ 2/3 & 0 & 1/3 & 0 \\ 0 & 0 & 2/3 & 1/3 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}$, (b): $P = \begin{pmatrix} 1/2 & 1/2 & 0 & 0 & 0 \\ 1/3 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2/3 & 1/3 \\ 0 & 0 & 0 & 1/2 & 1/2 \end{pmatrix}$.

*** Exercise 6  (**not so important**)

A time homogeneous Markov chain with state space $\{A, B, C\}$ has the following properties: (i) it is irreducible, (ii) it is not aperiodic and (iii) the probability of moving from A to B equals the probability of moving from A to C. Show that these properties uniquely define the transition matrix and sketch the transition graph/diagram of the process.