* Exercise 1

Recall that for a Weibull distributed lifetime with parameters \( \alpha, \lambda > 0 \), the hazard function is given by \[ \mu(t) = \alpha \lambda t^{\alpha - 1}, \quad t \geq 0. \] Find the cumulative hazard function and the survival function of this lifetime.

* Exercise 2

The log-logistic survival time with parameters \( \alpha, \lambda > 0 \), has a survival function given by \[ S(t) = \frac{1}{1 + (\lambda t)^\alpha}, \quad t \geq 0. \]

(a) Find the cumulative hazard function and the hazard function of this survival time.

(b) Find the survival function, the cumulative hazard function and the hazard function of the residual lifetime of an individual aged \( x \) corresponding to the log-logistic survival time.

** Exercise 3

Let \( T \) be a survival time.

(a) Find a necessary and sufficient condition in terms of the hazard (or cumulative hazard) function such that the survival time is finite with probability one, i.e. \( \Pr(T < \infty) = 1 \).

(b) Find a necessary and sufficient condition in terms of the hazard (or cumulative hazard) function such that the survival time is unbounded, by which we mean that there exists no constant \( c > 0 \) such that \( \Pr(T \leq c) = 1 \).

** Exercise 4

In a certain population the hazard function of each lifetime is given by

\[
\mu(t) = \begin{cases} 
0.010 & 60 < t \leq 70 \\
0.015 & 70 < t \leq 80 \\
0.025 & t > 80.
\end{cases}
\]

Consider a man from this population aged exactly 65.

(a) Calculate the probability that he will live at least four more years.

(b) Calculate the probability that he will die before he turns 75.

(c) Calculate the probability that he will die between the ages 80 and 83.

(d) Calculate the expected residual lifetime of the man.