Problem solving 2

Chemical reactions and equilibria

2.1 Altering the Atmosphere by burning fossil fuels

Problem 2.1 In a recent year how much O_2 is removed from the atmosphere due to the combustion of fossil fuel on Earth, and how much CO_2 and H_2O is produced in the combustion process?

The primary elemental constituents of the three major types of fossil fuel (natural gas, petroleum, and coal) are carbon and hydrogen. When fossil fuel is burned, oxygen from the atmosphere combines with the carbon to make CO₂ and with the hydrogen to make H₂O. In addition, coal contains some water (typically 10-15% by weight), which is released to the atmosphere upon combustion of the coal. All three types of fossil fuel contain various other substances such as ash, sulfur, and trace metals in even lower concentrations.

We will need to know the chemical composition of the fuel to calculate CO_2 and H_2O emission and the oxygen consumption, we need to balance the chemical combustion reactions for each fuel. Consider petroleum, first, with the approximate chemical composition $CH_{1.5}$. The combustion reaction for this fuel is

$$CH_{1.5} + xO_2 \rightarrow yCO_2 + zH_2O$$
 (2.1)

where x, y, and z are called stoichiometric constants. Their values, to be determined below, provide the answer to our problem: For each mole of petroleum consumed, x moles of oxygen are consumed, and y and z moles, respectively, of CO_2 and H_2O are produced.

Let's first determine the stoichiometric constants. Equating moles of C, H, and O on each side of the reaction gives us equations for x, y, and z. One mole of carbon in $CH_{1.5}$, for example, produces y moles of C in the form of CO_2 , and so

$$y = 1. (2.2)$$

Similarly, one and a half moles of H in $CH_{1.5}$ produces z moles of H_2 or 2z moles of H, and so

$$2z = 1.5 (2.3)$$

or

$$z = 0.75.$$
 (2.4)

Finally, 2x moles of O produces 2y moles of O in CO₂ and z moles of O in H₂O, or

$$2x = 2y + z \tag{2.5}$$

Combining Eqs. 2.2, 2.4 and 2.5, we get

$$x = 1.375 \tag{2.6}$$

The number of moles of $CH_{1.5}$ burned worldwide in a year now has to be determined. Say the energy content of the combusted petroleum in 1 year is 1.35×10^{20} J; and has a heat content of 4.3×10^{10} J/tonne. By weight, petroleum is 98% $CH_{1.5}$; thus, the amount of $CH_{1.5}$ consumed in the year is

$$0.98 \times \frac{1.35 \times 10^{20} \text{J}}{4.3 \times 10^{10} \text{J/tonne}} = 3.08 \times 10^{9} \text{tonnes}(\text{CH}_{1.5})$$
 (2.7)

$$= 3.08 \times 10^{15} g(CH_{1.5})$$
 (2.8)

Since one mole of $CH_{1.5}$ has a mass of 12 + 1.5 or 13.5 g, the number of moles of $CH_{1.5}$ consumed in the year is

$$\frac{3.08 \times 10^{15}}{13.5} = 2.28 \times 10^{14} \text{moles(CH}_{1.5})$$
 (2.9)

Hence $n(O_2)$, the number of moles of consumed O_2 , is x times this value, or

$$n(O_2) = 1.375 \times 2.28 \times 10^{14}$$
 (2.10)

$$= 3.14 \times 10^{14} \text{moles}(0_2) \tag{2.11}$$

The values of $n(CO_2)$ and $n(H_2O)$ can be obtained by multiplying 2.28 x 10¹⁴ moles(CH_{1.5}) by y and z, respectively. Hence,

$$n(CO_2) = 2.28 \times 10^{14} \text{moles}(CO_2)$$
 (2.12)

and

$$n(H_2O) = 1.71 \times 10^{14} \text{moles}(H_2O)$$
 (2.13)

A similar procedure is used for calculating $n(O_2)$, $n(CO_2)$, and n(H2O) from coal and natural gas combustion. However, now the more diverse chemical nature of the fuel must be taken into account. Say that the composition of natural gas, expressed as mole fractions, is CH_4 (75%), C_2H_6 (6%), C_3H_8 (4%), $C_4H_{10}(2\%)$, and $C_5H_{12}(1\%)$ and the remaining 12% is noncombustible. Using these mole fractions of each hydrocarbon in natural gas, first calculate an average mole fraction for each constituent and balance an equation similar to Eq 2.1.

For coal the average formula is $CH_{0.8}$ and 75% is combustible; however, there is a subtlety in that 13% of coal is H_2O and is liberated upon combustion.

Other information you will need is that $6.0 \times 10^{19} \text{J}$ were derived from natural gas and that natural gas contains $3.9 \times 10^7 \text{ J/m}^3$. For coal the numbers are $9.0 \times 10^{19} \text{J}$ and $2.93 \times 10^7 \text{ J/kg}$.

	$n(CO_2) \times 10^{14} \text{ moles}$	$n(H_2O) \times 10^{14} \text{ moles}$	$n(O_2) \times 10^{14}$ moles
Petroleum	2.28	1.71	3.14
Natural gas	?	?	?
Coal	?	?	?
Total	4.85	4.02	6.76

2.1.1 Notes

Follow the notes in the example.

2.1.2 Problems

- 1. Derive the missing results for natural gas and coal shown in the table above.
- 2. If all the CO_2 released to the atmosphere in a typical year from fossil-fuel burning remained there, by what percentage would that year's CO_2 emission increase the atmospheric concentration (radius of Earth= 6.4×10^6 m and the thickness of atmosphere ~ 10km)?
- 3. By what percentage does the O_2 consumed in a year by fossil-fuel burning deplete the atmosphere's stock of O_2 ?
- 4. There is great concern that the CO₂ we add to the atmosphere from fossil-fuel burning and the so-called "greenhouse" effect because this gas traps outgoing infrared radiation and thereby warms Earth's surface. Actually H₂O is a more effective absorber of infrared radiation than is CO₂. Given that emissions of H₂O were comparable to those of CO₂, why don't we worry about the effect of our H₂O emissions on the radiation balance in the atmosphere? (Hint: answering this requires only stock-flow-residence time considerations.)

2.1.3 Notes on problems

2.1.4 Homework and reading for next time

Background reading:

• Croft and Davison (2006, chapters on 'Simplifying algerbraic expressions', 'Factorisation' and 'Algerbraic fractions') OR 2.5 to 2.9 of the *Foundation Maths Support Pack*.

Do this weeks assessment on Blackboard:

- 'Factorising and simplifying fractions'.
- 'Altering the atmosphere by burning fossil fuels'.