1.4 A seam of gold

Problem 1.4 *A mineralised seam is visible at ground level but dips down into the surrounding rock. You are interested in mining this seam because it contains gold. However, you need to know whether it will be economic and therefore want to estimate the value of the gold it contains. To find out where it goes a hole has been drilled some distance away and the depth at which this seam reappears has been recorded.*
You can assume that the seam goes down at a constant angle and that you have followed it in the direction in which it dips most steeply.

Some information you might need:

- Horizontal distance from centre of seam to centre of borehole = 30m.
- Vertical distance from ground to centre of seam = 8m.
- Thickness of seam along ground = 1.8m
- Width perpendicular to dip = 15m.
- Density of ore 7.5 gcm^{-3}
- Concentration of gold in rock 3ppm by weight.
- Price of gold = £330 per troy ounce (NB. a troy ounce is 31.1g).

1.4.1 Notes

In order to solve this problem you should know about mass percent and how to calculate volumes and mass using density and also about Pythagoras' theorem and trigonometric ratios.

- Length along seam using Pythagoras: $\sqrt{30^2 + 8^2} \cong 31.05$m.
- Angle of dip using trig ratio: $\tan^{-1}\left(\frac{8}{30}\right) \cong 14.9°$.
- Thickness of seam (perpendicular to seam) using trig ratios: $\sin\theta = \frac{Opposite}{1.8}$ or $1.8\sin^{-1}(14.8) \cong 0.46$m.
- So the volume of the seam is $0.46 \times 31 \times 15 \cong 214\mathrm{m}^3$.
- The total mass of the ore is density×volume so in SI units $7500 \times 214 \cong 1.6 \times 10^6$ kg.
- Total mass of gold is this mass multiplied by the weight percent: $1.6{\times}10^6, \cancel{kg} \times 3 \times 10^{-6} kg\, \cancel{kg^{-1}}$ = 4.8kg (from the ppm above).
- Since the cost of gold is = £330 per 31.1g then to work out the total price of the gold we need to find how many 31.1 g are in 4.8kg: $\frac{4.8\times10^3\,\cancel{g}}{31.1\,\cancel{g}\,ounce^{-1}} \cong 154\,ounce$. Then multiply this number by £330 to get the total price: $154 \times$ £330 $\cong$ £51,000

1.4.2 Related problems

1. There is a seam with similar dimensions that contains gold. Without using trigonometry, calculate the volume of the seam if:
   - the horizontal distance from the centre of the seam (that is visible at the ground) to the bore hole is not given.
   - the vertical height between the seam and the horizontal plane is 7 m.
   - The width of the seam along strike is 15 m.
   - The thickness of the seam in the dip direction is 2 m.
2. In fact you can never be confident from one bore hole sample how much gold you will be able to mine. Instead, 5 bore hole samples are taken and give values of: 3, 5, 1, 5.5, 4 ppm of gold at different points. How would you use these bore hole samples to estimate how much gold could be mined?

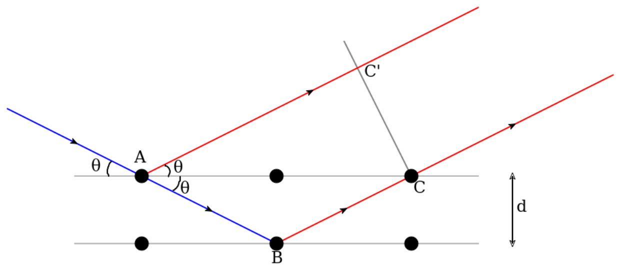
1.4.3 Answer to related problem

1. Tricky bit of Maths here:
   - Call the length of the seam h.
   - Using similar triangles, the ratio of the vertical depth of the seam below the ground, y, and the length of the seam, h, is equal to the ratio of the true thickness of the seam, d and the thickness of the seam in the dip direction, t. Therefore $\frac{y}{h} = \frac{d}{t}$.
   - Therefore, rearranging: $d = t \times \frac{y}{h}$.
   - To get the volume, multiply h by d by the length along strike, z: $volume = \cancel{h} \times t \times \frac{y}{\cancel{h}} \times z$.
   - We see that the h's cancel and can substitute numbers in to get: $volume = 2 \times 7 \times 15 = 210\ \mathrm{m}^3$.
   - Sometimes, if you don't know the value of something, try calling it a letter (like x) and you may be able to solve the problem.
2. Stats example. Calculate the average! then repeat.

1.4.4 Another related problem: Bragg's law

1.4.4.1 Theory

Atoms will scatter incident X-rays at different angles. In crystalography the atoms are tightly packed together, which produces interesting patterns of reflected X-rays. Bright and dark fringes are observed in the pattern of scattered X-rays.



The arrangement of atoms in the crystal above shows the principle behind Bragg's law. The incoming ray, at angle θ, is scattered at angle θ off the first layer of atoms. The same is true for the next layer. We consider light as a wave here (with troughs and crests like the sea). For some special angle, θ the two scattered rays will either completely cancel each other out or completely add together (constructive interference) in the strongest way possible. For constructive interference the distance: $\vec{AB} + \vec{BC} - \vec{AC'}$ must equal an integer number of wavelengths, λ.
The distance $\vec{AB} = \vec{BC}$ is equal to $\frac{d}{\sin\theta}$, whereas $\tan\theta = \frac{d}{\frac{1}{2}\vec{AC}}$.
The distance $\vec{AC}$ is therefore $\frac{2d}{\tan\theta}$. The distance $\vec{AC'}$ is $\vec{AC}$ multiplied by $\cos\theta$.
So we write down $n\lambda = \vec{AB} + \vec{BC} - \vec{AC'}$:

$$\begin{aligned} n\lambda &= \frac{2d}{\sin\theta} - \frac{2d}{\tan\theta}\cos\theta \\ &= \frac{2d}{\sin\theta} \times \sin^2\theta \\ &= 2d \times \sin\theta \end{aligned}$$

(where we have used $1 \equiv \sin^2\theta + \cos^2\theta$) which is known as Bragg's law.

Problem 1.5 *Example: for a crystal with layers separated by d = 50 Åwhat angles, measured from the face of the crystal, with X-rays of wavelength $\lambda = 1$ nm constructively interfere?*
Constructive interference is given when the path difference of two rays are multiples of λ. Rearrange Bragg's law for $\sin\theta$:

$$\sin\theta = \frac{n\lambda}{2d}$$

Substitute our values in to get:

$$\begin{aligned}\sin\theta &= \frac{n \times 1 \times 10^{-9}}{2 \times 50 \times 10^{-10}} \\ &= 0.1n\end{aligned}$$

with $n = 1, 2, 3, \ldots$
find the angles by taking inverse sin *to get $\theta = 5.74°, 11.54°, 17.46° \ \ldots.$*

1.4.4.2 Problem

1. Bragg's law: you study a crystal with X-rays of wavelength 2 nm and note the constructive interference maxima in intensity occur at angles of $\theta = 10°$, $20.3°$, ... Use Bragg's law to estimate the separation distance between layers in the crystal.

1.4.4.3 Answer to another related problem: Bragg's law

1. Rearrange Bragg's law for d:

$$d = \frac{n\lambda}{2\sin\theta}$$

Substitute $n = 1$, $\lambda = 2 \times 10^{-9}$ m and $\theta = 10°$ to get $d \cong 5.8 \times 10^{-9}$ m.

1.4.5 Another related problem: noise levels

1.4.5.1 Theory

Pressure waves of a certain frequency are heard as noise. The amplitude of the pressure wave is referred to as the *sound pressure level*, p and the intensity is proportional to the square of the sound pressure level ($I \propto p^2$). Sound levels, L, are often quoted using the *decibel scale*, which is ten multiplied by the log to base 10 of the ratio of the square of sound pressure level to some reference pressure level, p_{rel}:

$$L = 10 \times \log_{10}\left(\frac{p_1^2 + p_2^2 + \cdots + p_n^2}{p_{rel}^2}\right)$$

here, $p_{ref} = 20 \times 10^{-6}$ Pa.

Often, the sound level of sound sources are quoted at some distance from the source. For example you may be able to find that the sound level of a car is 60 dB(A) at 10 meters.

The intensity level of the sound source drops of as an inverse square law, which therefore implies (because $I \propto p^2$) that the pressure levels will drop off proportional to *inverse distance*, i.e. $p(x) = p_{lev} \times \frac{x_1}{x}$, where p_{lev} is the quoted sound pressure level at distance x_1 from the source.

Problem 1.6 *For example: the sound level of a car at 10 meters is 60 dB(A). What would be the sound level at 20 meters?*
We first calculate the sound pressure level

$$
\begin{aligned}
L &= 10 \times \log 10\left(\frac{p^2}{p_{rel}^2}\right) \\
\therefore &\quad \text{dividing by 10 and raising both sides} \\
&\quad \text{ to the power 10 to eliminate the logarithm} \\
10^{L/10} &= \frac{p^2}{p_{rel}^2} \\
\therefore &\quad \text{take the square root of both sides and rearrange for } p \\
p &= p_{rel}\sqrt{10^{L/10}} \\
\therefore &\quad \text{substitute numbers} \\
p &= 20 \times 10^{-6}\sqrt{10^{60/10}} \\
p &= 2 \times 10^{-2}\, Pa
\end{aligned}
$$

we then need to scale the sound pressure level by inverse distance. i.e.

$$
\begin{aligned}
p(x) &= p_{lev} \times \frac{x_1}{x} \\
\therefore &\quad \text{substitute the numbers from the problem} \\
p(20) &= 2 \times 10^{-2} \times \frac{10}{20} \\
p(20) &= 1 \times 10^{-2}\, Pa
\end{aligned}
$$

then to calculate the sound level that this pressure level corresponds to:

$$
\begin{aligned}
L &= 10 \times \log_{10}\left(\left[\frac{1 \times 10^{-2}}{20 \times 10^{-6}}\right]^2\right) \\
L &= \underline{53.98\, dB(A)}
\end{aligned}
$$

1.4.5.2 Problem

1. A house is situated 300m from the start of a straight road and 5m from the edge of the road. There are two cars on the road at 200m and 700m, from the start of the road, that each have a sound level of 60 dBA measured at 10 m. There is also an HGV at 305m from the start of the road. The HGV has a sound level of 84 dB(A) at 15 m distance. What will be the sound level at the location of the house?

1.4.5.3 Answer to another related problem: noise levels

1. You need to calculate the sound pressure level for a car and for an HGV.
   - The sound pressure level for the cars are the same as in the example shown: $p_{car} = 2 \times 10^{-2}$ Pa.
   - The sound pressure level for the HGV works out to be $p_{HGV} = 0.317$ Pa.
   - Now the distance between the houses and the vehicles needs to be calculated using Pythagoras' theorem.
     - Car 1: $\sqrt{(300-200)^2 + 5^2} = 100.12$ m
     - Car 2: $\sqrt{(300-700)^2 + 5^2} = 400.03$ m
     - HGV : $\sqrt{(300-305)^2 + 5^2} = 7.07$ m
   - Now scale the sound pressure levels by the inverse distance.
     - Car 1: $p_{car1} = 2 \times 10^{-2} \times \frac{10}{100.12} = 2 \times 10^{-3}$ Pa
     - Car 2: $p_{car2} = 2 \times 10^{-2} \times \frac{10}{400.03} = 5 \times 10^{-4}$ Pa
     - HGV : $p_{HGV} = 0.317 \times \frac{15}{7.07} = 0.67$ Pa
   - Calculate the sound level at the house:

$$L = 10 \times \log_{10}\left(\left[\frac{(2 \times 10^{-3})^2 + (5 \times 10^{-4})^2 + (0.67)^2}{(20 \times 10^{-6})^2}\right]\right)$$
$$L = 90.5\, dB(A)$$

1.4.6 Homework and reading

Background reading:

- Croft and Davison (2006, chapters on 'Angles', 'Solution of triangles' and 'Trig') OR 4.1, 4.2 and 4.5 of the *Foundation Maths Support Pack*.

Do this weeks assessment on Blackboard:

- 'Angles, Triangles and Trigonometry'.
- 'A seam of gold'.
- If you are struggling with trig., attempt the supplementary work and bring to class for discussion.