1.3 The greens we eat

Problem 1.3 *The net primary productivity is the amount of elemental carbon that is converted from CO_2 to carbon containing molecules each year and is* $14.7 PgC$ *(Peta gramms of carbon) per year. If the energy content of dry biomass is* 1.6×10^4 $J\ g^{-1}$ *(note J≡joule, which is a unit of energy, see Appendix D) what fraction of the total annual plant growth on Earth was eaten by humans in 2008?*[1]

[1]The worlds population was 6.7×10^9 in 2008

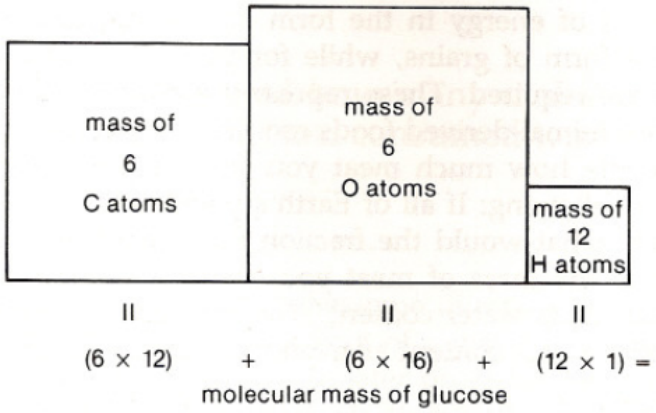

Figure 1.2: The molecular mass of glucose ($C_6H_{12}O_6$) is calculated by adding together the mass of 6 carbon atoms (6 ×12), 6 oxygen atoms (6 ×16), and 12 hydrogen atoms (12 ×1).

You will have to estimate how much energy a typical human eats (e.g. per day). Discuss this with and how you will use it with your group.

1.3.1 Notes

This is a good problem containing elements of moles, stoichiometry, fractions and unit conversions. You will also need to at least have some idea what energy is and that it is measured in joules or calories. We will cover it in more detail later though.

- The average daily food consumption of a human is approx. 2500 kcal. There are 4.2 J in 1 calorie, so this is $2500 \times 10^3\ \cancel{calories} \times 4.2\ \frac{J}{\cancel{calorie}} = 1.05 \times 10^7$ J.
- Annually (365 days) for the human population this is $1.05\times10^7\ J\ \cancel{day^{-1}}\ \cancel{person^{-1}} \times 6.7 \times 10^9\ \cancel{people} \times 365\ \cancel{days} year^{-1} \cong 2.6 \times 10^{19}$ J yr^{-1}.
- Net primary productivity is 14.7×10^{15}gC yr^{-1}. If we assume glucose is the biomass being made (a good approximation) then 14.7×10^{15}gC of carbon in glucose in being made, but glucose contains oxygen and hydrogen as well. So now the question turns to calculating what the mass of glucose would be if there were 14.7×10^{15}gC in the glucose. To answer this we need to calculation the fraction of glucose that is carbon by mass.

- Fraction of glucose that is carbon by mass is

$$\frac{Total\, mass\, of\ carbon\ in\, one\, mole}{Total\, mass\, of\ carbon\ +oxygen\ +hydrogen\, in\, one\, mole}$$

  or $\frac{6\times12\, g\,\cancel{mole^{-1}}}{6\times12+6\times16+12\times1\, g\,\cancel{mole^{-1}}} = \frac{72}{180} = 0.4\, g\,(carbon)\, g^{-1}\,(biomass)$.
- The total mass of glucose made is then the net primary productivity divided by the above fraction: $\frac{14.7\times10^{15}\,\cancel{g\,(carbon)}}{0.4\,\cancel{g\,(carbon)}\,g^{-1}\,(biomass)} \cong 3.67\times10^{16}$ g of biomass.
- Glucose contains 1.6×10^{4} J g^{-1} and so multiplying this by the number above: $1.6\times10^{4} J\,\cancel{g^{-1}} \times 3.67\times10^{16}\,\cancel{g} \cong 5.88\times10^{20}$ J.
- The annual global human food consumption calculated above divided by the energy in the glucose gives us an estimate of fraction of plants eaten by humans $\frac{2.6\times10^{19}}{5.88\times10^{20}} \cong 0.0425$; or about 4%.

1.3.2 Related problems

1. A formula representing the approximate chemical composition of typical dry freshly photo synthesized biomass is H_{2960} O_{1480}, C_{1480}, N_{160}, P_{18}, S_{10}, where each subscript denotes the relative number of atoms of that elemental type. If this more precise representation is used instead of $C_6H_{12}O_6$, recalculate the fraction, f, of the biomass that must be burned.
2. The production of animal-derived foods, such as beef, eggs, fish, and milk, requires the production of plants as fodder. To produce 1 J of energy in the form of beef requires about 8 J of energy in the form of grains, while for poultry about 3 J of energy from grains are required. These represent extremes. The production of 1 J of other animal-derived foods requires very roughly 5 J of plant matter. Lets say that people on average eat a diet of 1/4 poultry, 1/5 beef, 1/5 other and 7/20 veg what would the fraction f be? (Hint: first consider how much energy would be required to provide a diet of all poultry use the total energy consumed by humans calculated above).
3. About what fraction of Earth's current npp would we need to consume if we derived all the energy we now (2008) get from fossil fuel (roughly 5×10^{20} J) from biomass instead? What does your answer tell you about the wisdom of replacing all fossil fuels with biomass? What ecological problems would you anticipate this might cause?
4. If the human population continues to grow at about 2%/yr, in what year will humans be eating at Earth's current rate of npp?

1.3.3 Notes on related problems

1. This is done in exactly the same way as the first problem except that the fraction of biomass that is carbon by mass is different.
   - Instead use: $\frac{1480\times12\, g\, \cancel{mole^{-1}}}{(1\times2960)+(1480\times16)+(1480\times12)+(160\times14)+(18\times31)+(10\times32)\, g\, \cancel{mole^{-1}}} \cong 0.37\, g\, g^{-1}$.
   - So as before the total biomass is then the net primary productivity divided by the above fraction: $\frac{14.7\times10^{15}\, \cancel{g\, (carbon)}}{0.37\, \cancel{g\, (carbon)}\, g^{-1} (biomass)} \cong 3.93\times10^{16}$ g (biomass).
   - Say biomass contains 1.6×10^4 J g^{-1} and so multiplying this by the number above: $1.6 \times 10^4\, J\, \cancel{g^{-1}} \times 3.93 \times 10^{16}\, \cancel{g(biomass)} \cong 6.29 \times 10^{20}$ J.
   - The annual global human food consumption calculated above divided by the energy in the glucose gives us an estimate of fraction of plants eaten by humans $\frac{2.6\times10^{19}}{3.72\times10^{20}} \cong 0.0413$; or about 4%.
2. For this problem use the annual global food consumption: $\cong 2.6 \times 10^{19}$ J yr^{-1}.
   - If this was to be provided by poultry then we would require 3 times more energy (as it takes 3 J of plant food to produce 1 J of poultry). We can write down an expression that will give the total energy needed to fullfil the human food consumption requirement using fractions: total energy required = $3\times\frac{1}{4}\times2.6\times10^{19} + 8\times\frac{1}{5}\times2.6\times10^{19} + 5\frac{1}{5}\times2.6\times10^{19} + \frac{7}{20}\times2.6\times10^{19} \cong$ 9.62×10^{19}J yr^{-1}.

- Since the energy in biomass from the net primary productivity (calculated before) is 5.88×10^{20} J yr^{-1} the fraction is $\frac{9.62\times10^{19}}{5.88\times10^{20}} \cong 0.16$.

3. Just take the fraction of energy consumed globally to npp: $\frac{5\times10^{20}}{5.88\times10^{20}} \cong 0.85$.
4. You will need to have done logarithms to do this part. Leave if you are uncomfortable: we will come back to it later.

- The npp is 5.88×10^{20} J yr^{-1}. If every one consumes 1.05×10^{7} J per day (see above) then per year this is: $1.05 \times 10^7\ J\ \cancel{day^{-1}} \times 365\ \cancel{days}\ year^{-1} \cong$ 3.8×10^9 J year^{-1} person^{-1}.
- Divide the npp by the amount of energy that one person consumes per year to get the number of people it would take to consume the npp: $\frac{5.88\times10^{20}}{3.8\times10^{9}} \cong 1.53 \times 10^{11}$ people.
- If the population grows at 2% per year then after the first year it will be 1.02 times greater. After the second year it will be 1.02×1.02 or 1.02^2 times greater than the initial value. After x years it will be 1.02^x times greater than the initial value.
- We can write down an *indicial equation* (see Logarithms in the *Foundation Maths* support booklet) describing how long it would take the current population (growing at 2% per year) to equal this: $(6.7 \times 10^9) \times 1.02^x = 1.53 \times 10^{11}$.
- Solving for x we have $1.02^x = \frac{1.53\times10^{11}}{6.7\times10^{9}}$; then $\ln 1.02^x = \ln(\frac{1.53\times10^{11}}{6.7\times10^{9}})$; then $x \ln 1.02 = \ln(\frac{1.53\times10^{11}}{6.7\times10^{9}})$; then $x = \ln(\frac{1.53\times10^{11}}{6.7\times10^{9}}) / \ln 1.02 \cong 158$ years .

1.3.4 Homework and reading

Background reading:

- Review appendix C.
- Read appendix D in this booklet.
- Croft and Davison (2006, chapter on 'Transposing formulae') OR 2.7, 2.8. 2.9, 2.10, 2.11 and 2.12 of the *Foundation Maths Support Pack*

Do this weeks assessment on Blackboard:

- 'Transposing formulae'.
- 'The greens we eat'.
- If you are struggling complete the supplementary material and bring your answers to the next class to discuss with us.