1.2 Measuring molecules

Problem 1.2 *Benjamin Franklin dropped oil on a lake's surface and noticed that a given amount of oil could not spread out beyond a certain area. If the number of drops of oil was doubled, then so was the maximum area to which it would spread. His measurements revealed that 0.1 cm^3 of oil spread to a maximum area of 40 m^2. How thick is such an oil layer?*

1.2.1 Notes

This problem tests your ability to use different units and convert between volumes and set up simple problems.

- Assume the layer is thickness, d.
- If d is in metres then the volume of the oil is $40 \times d\text{m}^3$.
- So since there are 1×10^6 cm^3 in 1 m^3: $0.1 \times 10^{-6} = 40d$
- Solving for d gives $d = \frac{0.1\times10^{-6}\cancel{m^3}}{40\cancel{m^2}} \cong 25 \times 10^{-10}\text{m}$ or 25Å.
- This is about the thickness of one molecule of oil and is why the layer does not spread any thinner.

1.2.2 Related problems

1. Franklin actually showed that 1 teaspoon of oil would spread to cover about 0.5 acre (Note: 10^4 m^2 = 2.47 acres). Determine how many cubic centimeters there are in a teaspoon.
2. Estimate the average spacing between H_2O molecules in liquid water by making use of two pieces of information: (a) liquid water has a density of 1 g/cm^3, and (b) every 18 g of water contains Avogadro's number (6.02×10^{23}) of H_2O molecules.
3. You hire a car that has 6/8 of a tank of fuel. The car hire agent asks for you to return the car with 6/8 of a tank of fuel otherwise it will charge for fuel at a premium rate. After using the car for a week the car has 1/4 tank of fuel. You look up that the capacity of the tank is 35 litres, how much should you spend on refilling the car so that you fill it just enough so that the car hire company doesn't charge you for extra fuel, but not so much that you are paying for more fuel than required? (assume the cost of fuel is £1.30 per litre.)
4. Sirs Andre Geim and Konstantin Novoselov were awarded the Nobel Prize in Physics (2010) for ground greaking experiments using graphene. Graphene is one of the crystalline forms of carbon in which carbon atoms are arranged in a regular hexagonal pattern. Given the information that each milli-gram of graphene has an area 1.299 m^2 and that carbon atoms have an atomic mass of 12 g mol^{-1}, what is the distance between carbon atoms in the 2-d material graphene? (hint: you will need to consider the area of an array of hexagons)
5. How many molecules of air are there in Earth's atmophere? (take the depth of the atmosphere to be 10 km; radius of Earth, $Re = 6.4 \times 10^6$ m; molecular weight of air $\sim$ 29 g mole^{-1}; density of air $\sim$ 1 kg m^{-3})

1.2.3 Notes on related problems

1. This problem tests unit conversion.

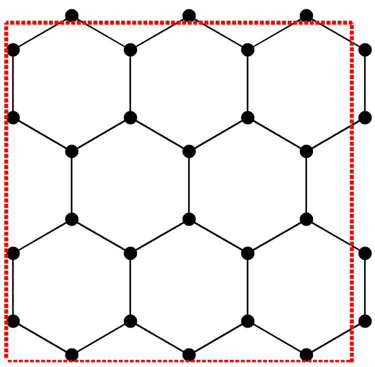
   - First need to determine how many metres squared are in 1 acre. This is calculated by taking the ratio of $\frac{10^4 m^2}{2.47 acre} \cong 4 \times 10^3\, m^2\, acre^{-1}$.
   - The oil spreads 0.5 acres or $0.5 \cancel{acre} \times 4 \times 10^3\, m^2 \cancel{acre^{-1}} = 2 \times 10^3$m^2.
   - If the oil is 25Å thick then the volume is the area multiplied by the thickness: $2 \times 10^3 \times 25 \times 10^{-10} \cong 5 \times 10^{-6}$m^3.
   - Since there are 1×10^6 cm^3 in 1 m^3 the volume is 5 cm^3.

2. This problem tests your understanding of what a mole is and your understanding of what density is. You will need to be able to use Indices and Fractional indices (see the *Foundation Maths* support booklet) and have read Appendix C.

   - If the density of water, ρ is 1 g cm^{-3} then this says that there is 1 gram of water for every cm^3 of water.

- Consult the Periodic Table of elements: The molecular weight of water is $2 \times H + 1 \times O = 18$ grams per mole.
- Using this we can work out the number of moles that occupy a cm^3 of water. It is just the density, ρ, divided by the molecular weight. Why did we do this? Look at the units. We have $\frac{\not{g} cm^{-3}}{\not{g} mol^{-1}}$. The grams cancel and we end up with cm^{-3} $\times \frac{1}{\text{mol}^{-1}}$. Using laws of indices this becomes mol cm^{-3}. So there $\frac{1}{18}$ mol cm^{-3} in liquid water. This says that every cm^{-3} contains $\frac{1}{18}$ moles of water.
- We can convert moles to number of molecules by multiplying by Avogadro's number: $\frac{1}{18} \times 6.02 \times 10^{23} \cong 3.3 \times 10^{22}$ molecules per cm^{-3}. So a cube of volume 1 cm^3 contains 3.3×10^{22} molecules.
- If the molecules are in a cubic arrangement then you can calculate the spacing by taking the cubed root of this number, giving the number of molecules per cm (length): $\sqrt[3]{3.3 \times 10^{22}\, cm^{-3}} \cong 3.2 \times 10^{7}\, cm^{-1}$.
- One over this number gives the distance between molecules: $\frac{1}{3.2 \times 10^{7}} \cong 3.1 \times 10^{-8}$ cm or 3.1×10^{-10}m.

3. Figure out what fraction must be refilled. You have $\frac{1}{4}$ tank and you started with $\frac{6}{8}$ so you must add: $\frac{6}{8} - \frac{1}{4} = \frac{1}{2}$. This means you need to put an additional half a tank of fuel in. To work out how much this costs, calculate how many litres it is: $\frac{1}{2} \times 35 = 17.5$ litres and then how much this costs: $17.5 \times £1.30 = £22.75$
4. Graphene example:
   - Consider the area per unit mass of 2-D graphene: 1.299 m^2 mg^{-1}.
   - If we multiply the above number by the atomic weight of six carbon atoms we get the area per mol: $1.299\, m^2\, mg^{-1} \times 12\, g\, mol^{-1} \cong 1.56 \times 10^4\, m^2\, mol^{-1}$
   - Divide the above number by Avogadros' number to get the area per single atom: $\frac{1.56 \times 10^4\, m^2\, mol^{-1}}{6.02 \times 10^23} \cong 2.59 \times 10^{-20}\, m^2\, atom^{-1}$.
   - Consider a 'square' of graphene with a row of 3 hexagons, connected to a row of two hexagons which are then connected to a row of 3 hexagons:



such a square contains 21 atoms

of carbon and has *length* $= 6 \times \frac{\sqrt{3}}{2} t$ and *height* $= 5t$

- So the area of the square is (*length* $\times$ *height*) $15\sqrt{3}t^2$ and the area per atom is $\frac{15\sqrt{3}t^2}{21}$.
- Set this equal to the area per atom for graphene $\frac{15\sqrt{3}t^2}{21} = 2.59 \times 10^{-20}$.
- Solving for t gives: $t = \sqrt{\frac{21 \times 2.59 \times 10^{-20}}{15\sqrt{3}}} \cong 1.45 \times 10^{-10}$ m.

5. Calculate the volume of the atmosphere as a thin shell, area of a sphere multiplied by the depth, h, taking care to convert h to metres: $4\pi Re^2 h \cong 5.15 \times 10^{18}$ m^3. Then calculate the mass by multiplying by the density of air: $\sim 5.15 \times 10^{18}$ kg. Then divide by the molecular weight to get number of moles (taking care to convert molecular weight to kg mole^{-1}: $\sim 1.78 \times 10^{20}$ moles. Then multiply by Avogadro's number to get number of molecules: $\sim 1.78 \times 10^{20} \times 6.02 \times 10^{23} \cong 1 \times 10^{44}$. Alternatively know that at STP 1 mole of gas occupies 22.4 litres so divide the volume of the atmosphere by this (taking care of units) and multiply by Avogadro's number (answer is close, 1.38×10^{44}, but not same, why?)

1.2.4 Homework and reading

Background reading:

- Review the periodic table handout.
- Optional: Hewitt et al. (2009, Chapters 9.1 and 11.7);
- Croft and Davison (2006, chapter on ’Indices’) OR 2.1, 2.2, 2.3, 2.4 and 2.5 of the *Foundation Maths Support Pack*.

Do this weeks assessment on Blackboard:

- ‘Indices and removing brackets’.
- ‘Measuring molecules’.
- If you are struggling with unit conversion and geometry, do the supplementary material and bring to next class to discuss with us.