Data analysis 1

Equations needed for this module

1.1 Basic statistics and histograms

1.1.1 Central tendency

Revise calculating the mode, median and mean. Formula for the mean:

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \tag{1.1}$$

If you have data tabulated in a frequency table you can calculate the mean by multiplying the frequencies by the mid-points of the bins, adding them up and dividing by the sum of all frequencies. For example:

Example 1.1 *The UK Met.Office (Met.Office) publish climatologies for the number of days of sunshine in the UK. Their data state that, for the month of January, the 30 year average hours of sunshine between 1971 and 2000, had a distribution over the UK and was binned into a histogram with the mid points:*

Mid-points (hrs)	*Frequency*
20.0	*9610*
22.5	*27343*
27.5	*14380*
32.5	*11265*
37.5	*31694*
42.5	*4704*
47.5	*3793*
52.5	*3623*
57.5	*2525*

What was the mean hours of sunshine for January over this period?
Calculate the sum of the product of bin mid points and frequencies and divide this by the sum of frequencies:

$$\begin{aligned} Sum\,of\,bin\,mid\,points \times freqs. &= \\ &(20 \times 9610 + \ldots + 57.5 \times 2525) = 3.47 \times 10^6 \\ Sum\,of\,frequencies &= (9610 + 27343 + \ldots + 2525) = 108937 \\ \therefore Mean &= \frac{347 \times 10^6}{108937} = 31.88\,hours. \end{aligned}$$

1.1.2 Variability

Revise range, interquartile range and standard deviation. Calculating the range:

$$range = x_{largest} - x_{lowest} \tag{1.2}$$

Calculating the standard deviation for a sample of data:

$$s = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}} \tag{1.3}$$

Calculating the standard deviation for the whole population of data:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N}} \tag{1.4}$$

Example 1.2 *Calculate the standard deviation of the numbers 1,2 and 5.*

- *in Excel enter in a cell* $= STDEV(1,2,5)$
- *in MATLAB enter* $std([1,2,5])$

The variance is just the standard deviation squared.

$$s^2 = \frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1} \tag{1.5}$$

Example 1.3 *Calculate the variance of the numbers 1,2 and 5.*

- *in Excel enter* $= VAR(1,2,5)$ *or* $= STDEV(1,2,5)\hat{}2$
- *in MATLAB enter* $var([1,2,5])$ *or* $std([1,2,5])\hat{}2$

1.1.3 z-score

It is often useful to know how many standard deviations a value is away from the mean. The formula that gives you this is:

$$z = \frac{x - \mu}{s} \tag{1.6}$$

where x is the value or measurement, μ is the mean and s is the standard deviation.

1.1.4 Symmetry

The symmetry of a dataset can be described using skewness

$$g_1 = \frac{\sqrt{N}\sum_{i=1}^{N}(x_i - \bar{x})^3}{\left(\sum_{i=1}^{N}(x_i - \bar{x})^2\right)^{3/2}} \tag{1.7}$$

If more data is less than the mean than greater than the mean then we say the data is *positively skewed* (the value g_1 is positive) and vice-versa.

Example 1.4 *Calculate the skewness of the numbers 1,2 and 5.*

- *in Excel enter* $= SKEW(1, 2, 5)$
- *in MATLAB enter* $skewness([1, 2, 5], 0)$

Another measure of symmetry is kurtosis:

$$g_2 = \frac{N\sum_{i=1}^{N}(x_i - \bar{x})^4}{\left(\sum_{i=1}^{N}(x_i - \bar{x})^2\right)^2} - 3 \tag{1.8}$$

Example 1.5 *Calculate the kurtosis of the numbers 1,2, 5 and 6.*

- *in Excel enter* $= KURT(1, 2, 5, 6) - 3$
- *in MATLAB enter* $kurtosis([1, 2, 5, 6], 0)$

1.1.5 Accuracy and precision

The *accuracy* of an instrument is related to *systematic error*. It is given by the magnitude of the difference between the actual value of the property being measured and the mean of measured values.

$$accuracy = |\bar{x} - x_{true}| \tag{1.9}$$

The *precision* of an instrument is related to *random error*. In other words you if take a measurement of the same thing with an inprecise instrument you will get a different answer every time. It can be quantified by the standard error of the mean (also called the standard deviation of the sample means), SE. In other words we can improve the precision of a measured property by taking lots of measurements and calculating the mean of those measurements. The standard error of the mean tells us how confident we can be in our measurement.

$$SE_{\bar{x}} = \frac{s}{\sqrt{N}} \tag{1.10}$$

here N is the number of measurements taken and s is the standard deviation. So this means that precision of your measurements can be improved by making more measurements and averaging them together (i.e. increasing the sample size N).

1.1.6 Central Limit Theorem

The standard error of the mean finds application in the *Central Limit Theorem*. The Central Limit Theorem states that if you take a sample of data and calculate the mean and then repeat this lots of time, to create a list of sample means, then those sample means will be normally distributed with a standard deviation equal to the standard error of the mean, $SE_{\bar{x}}$ (which is defined above).

Example 1.6 *The freezing temperatures of a population of cloud drops are normally distributed with a mean of* $\mu = -21.33°C$ *and standard deviation of* $\sigma = 0.93°C$. *If you select 1 of those drops what is the probability of the freezing temperature being less than* $-22°C$?. *What is the probability of a sample of 10 drops having a mean freezing temperture less than* $-22°C$?

1. *Sample from the normal distribution. For the single drop:* $normcdf(-22, -21.33, 0.93)$ *in MATLAB or* $= NORMDIST(-22, -21.33, 0.93, 1)$ *in Excel. The answer is* 0.24 *so there is a 24% chance.*
2. *For the sample of 10 drops, the standard deviation is equal to* $0.93/\sqrt{10} = 0.29$. *Therefore there will be less chance of the sample of 10 drops having a mean freezing temperature less than* $-22°C$: $normcdf(-22, -21.33, 0.29)$ *in MATLAB or* $= NORMDIST(-22, -21.33, 0.29, 1)$ *in Excel. The answer is* 0.01 *so a there is only 1% chance.*

Also, because of this concept any error bars that you put on your sample data should have width equal to, $SE_{\bar{x}}$.

1.2 Confidence intervals

Confidence interval on population parameters from sample statistics.

$$\hat{p} - E < p < \hat{p} + E \tag{1.11}$$

where $\hat{p}$ is the sample proportion of sucesses (e.g. the proportion of people who say yes in a survey) and p is the population proportion (i.e. the proportion of the population who would say yes if they took the survey). E is the margin of error on the estimate.
It can also be written:

$$\hat{p} \pm E \tag{1.12}$$

where

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{N}} \tag{1.13}$$

Note that $\hat{q} = 1 - \hat{p}$.
People doing surveys as part of a third year project take note... of this important fact...
You can estimate the size of your sample to have a certain level of confidence in your answer by calculating:

$$N = \frac{[z_{\alpha/2}]^2 \times 0.25}{E^2} \tag{1.14}$$

where, $z_{\alpha/2}$ is the confidence interval at a significance level of $\alpha/2$ and E is the desired margin of error on your answer. This is very important as it tells you how many people you need to ask to be able to infer something about a population.

Example 1.7 *How many people should you ask in a survey whether they believe human induced climate change is true, if you want to be 90% confident that you can state the confidence interval on the whole population's opinion to within a margin of error of 20%.*

- *calculate* $z_{\alpha/2}$ *for a two tailed distribution at 90% confidence (e.g.* $norminv(0.1/2, 0, 1)$*).*
- *calculate N using Equation 1.14 with* $E = 0.20$
- *You should have* $N \cong 17$*, so you need to ask at least 17 people.*

Also, if you know the confidence interval and want to get the best estimate of the population proportion and the margin or error you can do this by using the fact that the best estimate is in the middle of the confidence interval and the confidence interval has a width of twice the margin of error.

$$\hat{p} = \frac{\text{upper confidence interval limit+lower confidence interval limit}}{2} \tag{1.15}$$

$$E = \frac{\text{upper confidence interval limit}-\text{lower confidence interval limit}}{2} \tag{1.16}$$

The confidence interval on population mean from sample mean is calculated in a similar way to that for proportions:

$$\bar{x} - E < \mu < \bar{x} + E \tag{1.17}$$

where $\bar{x}$ is the sample mean and μ is the population mean. E is the margin of error. This is equivalent to:

$$\bar{x} \pm E \tag{1.18}$$

where

$$E = z_{\alpha/2}\frac{\sigma}{\sqrt{N}} \tag{1.19}$$

You can estimate the size of the sample you need to have a certain level of confidence in your answer:

$$N = \left[\frac{z_{\alpha/2} \times \sigma}{E}\right]^2 \tag{1.20}$$

so for instance, say you had a requirement to estimate a population mean to a certain level of accuracy. You would have take N measurements to be able to strictly state this. Note that if you do not know what the population standard deviation, σ, is (and you often wont), then use the sample standard deviation, s, in place of it, and instead of using the normal distribution to calculate $z_{\alpha/2}$, use the t-distribution.
If you know the confidence interval and want to get the best estimate of the population mean and the margin or error you can do this by using the fact that the best estimate is in the middle of the confidence interval and the confidence interval has a width of twice the margin of error.

$$\mu = \frac{\text{upper confidence interval limit+lower confidence interval limit}}{2} \tag{1.21}$$

$$E = \frac{\text{upper confidence interval limit}-\text{lower confidence interval limit}}{2} \tag{1.22}$$

1.3 Hypothesis testing

Test statistic for proportion:

$$z = \frac{\hat{p} - p}{\sqrt{pq/N}} \tag{1.23}$$

Example 1.8 *A sample of 20 people surveyed are asked whether they think it rains all the time in Manchester. The proportion who said yes was* 0.7*. Test the hypothesis that in the population, the proportion of people who think it rains all the time is larger than* 0.5 *i.e. more people think it rains more than those who dont.*

- *the null hypothesis is that the sample proportion is close enough to* 0.5 *to be considered the same.*
- *Calculate the z statistic above* $\frac{0.7-0.5}{\sqrt{0.5\times0.5/20}} = 1.79$.
- *Assign a confidence level (how confident you want to be), e.g. 90%, so significance level is* $100 - 90 = 10\%$.
- *Calculate the* z_α *for a one-tailed test (e.g.* $norminv(0.1, 0, 1)$*), which should equal* -1.28*—ignore the sign.*
- *The z-statistic is greater than the critical value, so we can reject the null hypothesis. Hence, we accept the alternate hypothesis which is more people think it rains all the time than do not.*

Test statistic for mean:

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{N}} \tag{1.24}$$

or if you do not know the population standard deviation (often the case):

$$t = \frac{\bar{x} - \mu}{s/\sqrt{N}} \tag{1.25}$$

with $N - 1$ degrees of freedom.

Example 1.9 *An EU directive states that PM10 loadings (particulate matter with size less than 10 μm) must not exceed an mean over a 24 hour period of 0.05 mg* m^{-3} *(milli grams of PM10 per cubic metre of air). 20 spot measurements are made over a 24 hour period which have a mean of* 0.055 *and a sample standard deviation of* 0.04. *Has the PM10 exceeded the directive?*

- *We can choose the null hypothesis to be that the sample mean is close enough to be considered the same. The alternate hypothesis is that they are different.*
- *Calculate the t statistic above* $\frac{0.055-0.05}{0.04/\sqrt{20}} = 0.56$.
- *Assign a confidence level (how confident you want to be), e.g. 90%, so significance level is* $100 - 90 = 10\%$.
- *Calculate the* t_α *for a one tailed test (e.g.* $tinv(0.1, 19)$ *in MATLAB or* $= tinv(0.1 \times 2, 19)$ *in Excel), which should equal* -1.33*—ignore the sign.*
- *The t-statistic is less than the critical value, so we can accept the null hypothesis, which is the sample mean is not significantly different to* 0.05. *Hence we cannot say that there has been an exceedance.*
- *To the untrained scientist, the fact that the sample mean exceeds* 0.05 *might seem like evidence there has been an exceedance, but the standard deviation is quite high, so there is not enough evidence to be conclusive.*

1.4 Inferences from two samples

Note, those doing surveys might want to compare different groups—e.g. Men vs Women, both asked the same yes/no question.
Usually you will state the null hypothesis: e.g. H_0: men have different opinions than women.

$$z = \frac{(\hat{p}_1 - \hat{p}_2)}{\sqrt{\frac{\bar{p}\bar{q}}{n_1} + \frac{\bar{p}\bar{q}}{n_2}}} \tag{1.26}$$

$$\hat{p}_1 = \frac{x_1}{n_1} \tag{1.27}$$

$$\hat{p}_2 = \frac{x_2}{n_2} \tag{1.28}$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} \tag{1.29}$$

$$\bar{q} = 1 - \bar{p} \tag{1.30}$$

where x_1 is the number of successes in the 1st sample (i.e. the number of people saying yes to a question) and n_1 is the size of the 1st sample group and x_2 and n_2 are the same for the 2nd sample group.

Example 1.10 *A survey is conducted in which a question is "yes/no would you buy an electric car?". The data can be stratified into both male and female responses. 10 females were asked and and 5 males. The proportion of females that said "yes" was 60% (i.e. 6 out of the 10 asked), while the proportion of males who said yes was 20% (i.e. 1 out of the 5 asked). Is there a difference of opinion between males and females?*

- *The null hypothesis is that there is no difference of opinions between groups.*
- $\bar{p} = \frac{6+1}{15} = 0.4667$ *and* $\bar{q} = \frac{4+4}{15} = 0.5333$
- *Calculate the z-value above:* $z = \frac{0.6-0.2}{\sqrt{\frac{0.4667\times0.5333}{10}+\frac{0.4667\times0.5333}{5}}} = 1.4638$
- *Assign a significance level, say* 0.05 *(i.e. we want to be 95% confident in our claim), and calculate* $z_{\alpha/2}$ *(i.e.* $norminv(0.05/2, 0, 1)$*), which should equal* -1.96 *(ignore the sign).*
- *Since the z-statistic is less than the critical value of z we accept the null hypothesis. There is not enough data to say the two groups have different opinions.*
- *If the question we asked was is the proportion of women who would buy an electric car larger than men, we would have to conduct a one-tailed test. Make sure you know what the differences would be.*

If you want to test if collected data sets have the same population mean:

$$t = \frac{(\bar{x_1}-\bar{x_2})}{\sqrt{\frac{s_p^2}{n_1}+\frac{s_p^2}{n_2}}} \tag{1.31}$$

$$s_p^2 = \frac{(n_1-1)s_1^2+(n_2-1)s_2^2}{(n_1-1)+(n_2-1)} \tag{1.32}$$

with $n_1 + n_2 - 2$ degrees of freedom.

These statistics are then either compared to a normal or t-distribution at some significance level, depending, and if the statistic is large compared to the distribution we reject the null hypothesis. If is it less than the value from the distribution we accept the null hypothesis.

Example 1.11 *You observe the freezing of two samples of 100 cloud drops. One consists of pure water only, and the other contains* 10 $g\ L^{-1}$ *of volcanic ash. The sample with pure water freezes with a mean freezing temperature of* $\bar{x}_1 = -21.33°C$ *and a standard deviation of* $s_1 = 0.93$*, whereas the sample with volcanic ash freezes with a mean freezing temperature of* $\bar{x}_2 = -18.20°C$ *and standard deviation of* $s_2 = 0.88$*. Does volcanic ash affect the freezing temperature of cloud drops?*

- *The null hypothesis is that there is no volcanic ash does not affect the freezing temperature.*
- *Calculate the t-value above. You will first have to calculate* S_p^2 *and then t, the answer should be* $S_p^2 = 0.82$ *and* $t = -24.45$*.*
- *Assign a significance level, say* 0.01 *(i.e. we want to be 99% confident in our claim), and calculate* $t_{\alpha/2}$ *(i.e. tinv*$(0.01/2, 198)$ *in MATLAB or tinv*$(0.01, 198)$ *in Excel), which should equal* -2.60 *(ignore the sign).*
- *Note because we are interested if the two are different it is a two-tailed test.*
- *Since the t-statistic much larger (in magnitude) than the critical value of t we reject the null hypothesis. Volcanic ash clearly affects the freezing of cloud drops.*
- *If we asked the question is the mean freezing temperature of the pure drops lower than the volcanic ash laden drops we would have to conduct a one-tailed test. Make sure you understand what the difference in the procedure would be.*

1.5 Correlation and regression

In order to test whether the population correlation coefficient is equal to zero (no correlation).

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}} \tag{1.33}$$

with $n - 2$ degrees of freedom. Here r is the correlation coefficient.
In order to fit a straight line of the form $y = b_0 + b_1 \times x$:

$$b_1 = r\frac{s_y}{s_x} \tag{1.34}$$

$$b_0 = \bar{y} - b_1\bar{x} \tag{1.35}$$

Note that not all equations you fit will be linear, you should think about what form of equation should best fit the data.
For instance, if you were looking at the freezing of cloud drops to form ice crystals, then the theory suggests that $N_{drop}(t) = N\exp(-Jt)$, so if you were to fit a curve you should fit a linear-exponential curve. The same method can be used if you fit $N_{drop}(t)$ vs $\exp(t)$.

Also, some data might fit a power law, $y = ax^b$. For example, the terminal velocity of water drops in air vs their size or similarly sediments in free-fall in a fluid. Again, linear regression can be used in the following way:

1. Take logs of both sides:

$$\begin{aligned} \ln y &= \ln\left(ax^b\right) \\ \ln y &= \ln a + b \ln x \end{aligned}$$

2. This is just the same as fitting a straight line if we fit the natural logarithm of y against the natural logarithm of x.
3. The gradient will be b and the intercept will be $\ln a$. Therefore a can be found by raising e to the power of the intercept.
4. This seems hard but it is quite easy if you get your head round it.

1.6 Propagating errors

This is not assessed. If you make two measurements x and y, which have independent errors σ_x and σ_y, and use those measurements to calculate a new number, z, the corresponding error in z, σ_z is shown in the table below.

Relationship	Error propagation
$z = x + y$	$\sigma_z^2 = \sigma_x^2 + \sigma_y^2$
$z = x - y$	$\sigma_z^2 = \sigma_x^2 + \sigma_y^2$
$z = x \times y$	$\left(\frac{\sigma_z}{z}\right)^2 = \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2$
$z = \frac{x}{y}$	$\left(\frac{\sigma_z}{z}\right)^2 = \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2$
$z = k \times x$	$\sigma_z = k \times \sigma_x$
$z = x^n$	$\frac{\sigma_z}{z} = n\frac{\sigma_x}{x}$
$z = \ln(x)$	$\sigma_z = \frac{\sigma_x}{x}$
$z = \exp(x)$	$\frac{\sigma_z}{z} = \sigma_x$

Example 1.12 *You make a measurement of the length of along strike of a mineralised seam. Your tape measure is 10 m long, which is just shorter than the seam so you measure the length in two steps. The two readings are* 1000 ± 1 *cm and* 403 ± 1 *cm. As the precision of the tape measure is* $\sigma \cong \pm 1$*cm, propagate the error in your measurement.*

- *Use the first formula in the table—i.e. for* $z = x + y$; $\sigma_z^2 = \sigma_x^2 + \sigma_y^2$.
- *This means that the error in* z *can be calculated as:* $\sigma_z = \sqrt{1^2 + 1^2} \cong 1.41$ *cm.*

Example 1.13 *You want to estimate the mass loading of aerosol between* 2.5 *and* 10 μm. *You have two separate instruments that measure either PM2.5 or PM10 (i.e. the total number less than* 2.5μm *and the total number less than* 10μm*). Both instruments have quoted precisions of* $\sigma = 0.01$ *mg* m^{-3}. *Your measurements are PM2.5*=0.02 ± 0.01 *mg* m^{-3} *and PM10*=0.05 ± 0.01 *mg* m^{-3}. *What is the mass loading between* 2.5 *and* 10μm*?*

- *Use the second formula in the table—i.e. for* $z = x - y$; $\sigma_z^2 = \sigma_x^2 + \sigma_y^2$.
- *This means that the error in* z *can be calculated as:* $\sigma_z = \sqrt{0.01^2 + 0.01^2} \cong 0.0141$.
- *So the value you quote should be* 0.03 ± 0.014 *mg* m^{-3}.

Example 1.14 *You want to estimate the density of a rock. You do this by independently measuring the mass of the rock and also its volume. In order to measure the rock's mass you place on the scales and get* $M = 30000 \pm 10$ *g (*10 *is the quoted precision). In order the measure the rock's volume, you place the rock in a rectangular tub of water, that has an area of* 2500 ± 10 cm^2, *and note the increase in height level of the water. Before the rock is placed in the water the height of the water in the tub is* 30 ± 1 *cm and after it is* 33 ± 1 *cm.*

- *This calculation has several points where the errors need to be considered and propagated.*
- *Firstly, consider the change in height level of the water* $l = 33 \pm 1 - (30 \pm 1)$. *To propagate this error use the second formula in the table:* $\sigma_l = \sqrt{1^2 + 1^2} = 1.41$ *cm.*
- *Secondly consider the volume of the rock* $v = l \times A = 7500$ cm^3. *To propagate this error use the 3rd formula in the table:* $\left(\frac{\sigma_z}{z}\right)^2 = \left(\frac{\sigma_x}{x}\right)^2 + \left(\frac{\sigma_y}{y}\right)^2$, *so* $\sigma_v = v\sqrt{\left(\frac{\sigma_l}{l}\right)^2 + \left(\frac{\sigma_A}{A}\right)^2}$. *So* $\sigma_v = 7500\sqrt{\left(\frac{1.41}{3}\right)^2 + \left(\frac{10}{2500}\right)^2} \cong 3.5 \times 10^3$ cm^3.
- *Finally, the density of the rock is calculated by dividing the mass by the volume,* $\rho = \frac{M}{v}$. *To propagate the error use the fourth formula in the table, which is the same form as above* $\sigma_\rho = \rho\sqrt{\left(\frac{\sigma_M}{M}\right)^2 + \left(\frac{\sigma_v}{v}\right)^2}$. *So* $\sigma_\rho = 4\sqrt{\left(\frac{10}{30000}\right)^2 + \left(\frac{3.5\times10^3}{7500}\right)^2} \cong 1.87$ *g* cm^{-3}.
- *So the answer you should quote is* $\rho = \frac{30000}{7500} \cong 4 \pm 1.87$ *g* cm^{-3}.

Example 1.15 *You wish to estimate the volume of a cuboid with side* $l = 10 \pm 1$ *cm.*

- *Use the sixth formula in the table: i.e. for* $z = x^n$ *use* $\frac{\sigma_z}{z} = n\frac{\sigma_x}{x}$.
- *Therefore* $z = 10^3 = 1000$ *and* $\sigma_z = z \times n\frac{\sigma x}{x} \cong 300$ cm^3.
- *Quote your answer as* $z = 1000 \pm 300$ cm^3.

Data analysis 2
Cumulative probability tables

2.1 The standard normal distribution

The tables below show the cumulative probability for a given number of standard deviations away from the mean, z. Read the tables as follows: if you want to know the cumulative probability for a value of z (to up to 2 dps), find the nearest value of z in the column for z, then count across the columns for the second decimal place. The value in the table that corresponds to this row and column is the cumulative probability.

For example: what is the cumulative probability for $z = -2.05$? *Answer: go down the 1st column until you get to a z-value of* -2.0 *then go along the rows until the top column reads* 0.05*: this means* $z = -2.05$. *Then read of the probability, which is* 0.0202.

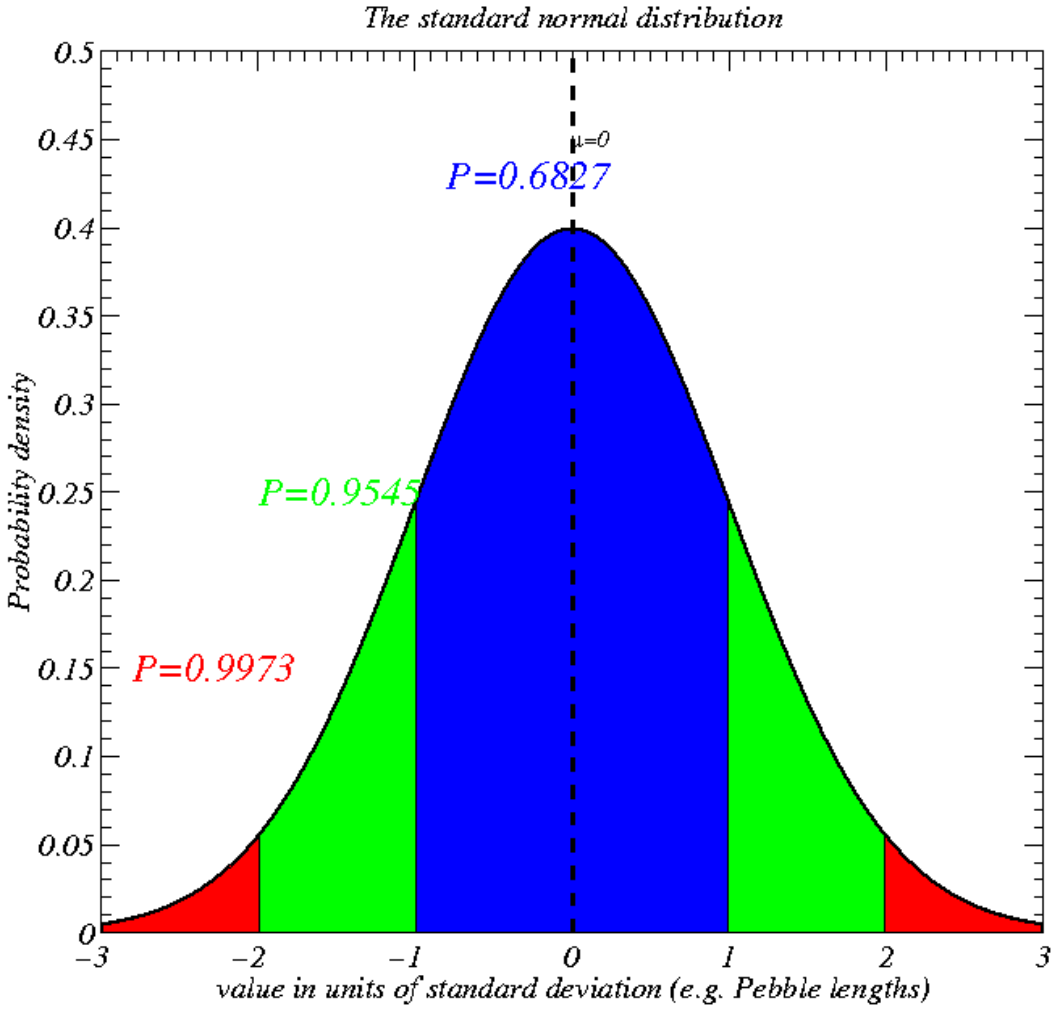
z	0.00	0.01	0.02	0.03	0.04	**0.05**	0.06	0.07	0.08	0.09
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	**0.0202**	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

The same concept applies for the right side of the distribution. For example: what is the cumulative probability for a value of $z = 2.05$? *Answer: go down the 1st column until you get to a z-value of* 2.0 *then go along the rows until the top column reads* 0.05*: this means* $z = 2.05$. *Then read of the probability, which is* 0.0202.

z	0.00	0.01	0.02	0.03	0.04	**0.05**	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	**0.9798**	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

IMPORTANT: you can use these tables for other course, reports, etc; however, for this course I'd like you to either use Excel, or Matlab to calculate the cumulative probabilities. The functions are:

- Excel: =`normdist(`z`,`μ`,`σ`,1)`—calculates the cumulative probability of the normal distribution for a z-score, z.
- MATLAB: `normcdf(`z`,`μ`,`σ`)`—calculates the cumulative probability of the normal distribution for a z-score, z.
- Excel: =`norminv(`P`,`μ`,`σ`)`—calculates the z-score for a cumulative probability, P, for the normal distribution.
- MATLAB: `norminv(`P`,`μ`,`σ`)`—calculates the z-score for a cumulative probability, P, for the normal distribution.

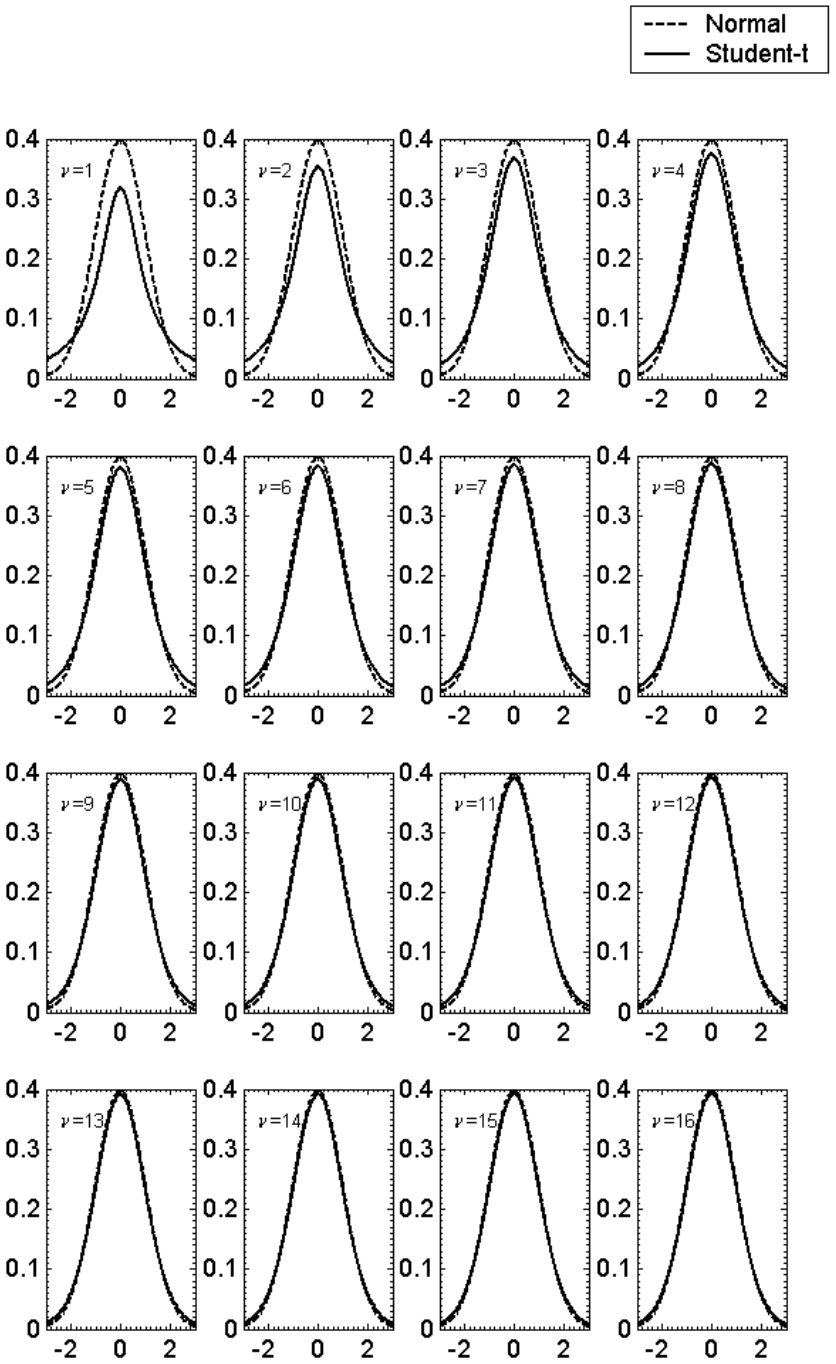

The standard normal distribution
μ=0
P=0.6827
P=0.9545
P=0.9973
Probability density
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
−3
−2
−1
0
1
2
3
value in units of standard deviation (e.g. Pebble lengths)

2.2 t-distribution

The table of the t-distribution is presented in a slightly different way to the normal distribution. For a given probability and degrees of freedom it gives the t-score (i.e. distance from the mean). For example: if degrees of freedom is = 10, what is the t-score where the cumulative probability (i.e. area to the left of t-score) is 0.025? *This is a one tailed example, so find the column where the "area in one tail" is* 0.025 *and the row for* $df = 10$. *This gives* $t = 2.228$.

Degrees of freedom	Area in one tail				
	0.005	0.01	**0.025**	0.05	0.10
	Area in two tails				
	0.01	0.02	0.05	0.10	0.20
1	63.657	31.821	12.706	6.314	3.078
2	9.925	6.965	4.303	2.920	1.886
3	5.841	4.541	3.182	2.353	1.638
4	4.604	3.747	2.776	2.132	1.533
5	4.032	3.365	2.571	2.015	1.476
6	3.707	3.143	2.447	1.943	1.440
7	3.499	2.998	2.365	1.895	1.415
8	3.355	2.896	2.306	1.860	1.397
9	3.250	2.821	2.262	1.833	1.383
10	3.169	2.764	**2.228**	1.812	1.372
11	3.106	2.718	2.201	1.796	1.363
12	3.055	2.681	2.179	1.782	1.356
13	3.012	2.650	2.160	1.771	1.350
14	2.977	2.624	2.145	1.761	1.345
15	2.947	2.602	2.131	1.753	1.341
16	2.921	2.583	2.120	1.746	1.337
17	2.898	2.567	2.110	1.740	1.333
18	2.878	2.552	2.101	1.734	1.330
19	2.861	2.539	2.093	1.729	1.328
20	2.845	2.528	2.086	1.725	1.325
21	2.831	2.518	2.080	1.721	1.323
22	2.819	2.508	2.074	1.717	1.321
23	2.807	2.500	2.069	1.714	1.319
24	2.797	2.492	2.064	1.711	1.318
25	2.787	2.485	2.060	1.708	1.316
26	2.779	2.479	2.056	1.706	1.315
27	2.771	2.473	2.052	1.703	1.314
28	2.763	2.467	2.048	1.701	1.313
29	2.756	2.462	2.045	1.699	1.311
30	2.750	2.457	2.042	1.697	1.310
31	2.744	2.453	2.040	1.696	1.309
32	2.738	2.449	2.037	1.694	1.309
33	2.733	2.445	2.035	1.692	1.308
34	2.728	2.441	2.032	1.691	1.307
35	2.724	2.438	2.030	1.690	1.306
36	2.719	2.434	2.028	1.688	1.306
37	2.715	2.431	2.026	1.687	1.305
38	2.712	2.429	2.024	1.686	1.304
39	2.708	2.426	2.023	1.685	1.304
40	2.704	2.423	2.021	1.684	1.303
45	2.690	2.412	2.014	1.679	1.301
50	2.678	2.403	2.009	1.676	1.299
60	2.660	2.390	2.000	1.671	1.296
70	2.648	2.381	1.994	1.667	1.294
80	2.639	2.374	1.990	1.664	1.292
90	2.632	2.368	1.987	1.662	1.291
100	2.626	2.364	1.984	1.660	1.290
200	2.601	2.345	1.972	1.653	1.286
300	2.592	2.339	1.968	1.650	1.284
400	2.588	2.336	1.966	1.649	1.284
500	2.586	2.334	1.965	1.648	1.283
1000	2.581	2.330	1.962	1.646	1.282
2000	2.578	2.328	1.961	1.646	1.282
∞	2.576	2.326	1.960	1.645	1.282

Note that students t-distribution is practically the same as a normal distribution for large degrees of freedom, ν.



IMPORTANT: you can use this table for other course, reports, etc; however, for this course I'd like you to either use Excel, or Matlab to calculate the t-scores. The

functions are:

- Excel: `=tdist(t,df,1)`—calculates the cumulative probability of the t distribution for a t-score, *t*.
- MATLAB: `tcdf(t,df)`—calculates the cumulative probability of the t distribution for a t-score, *t*.
- Excel: `=tinv(P*2,df)`—calculates the t-score for a cumulative probability, *P*, for the t distribution. To calculate the t-score for two-tails do `=tinv(P,df)`
- MATLAB: `tinv(P,df)`—calculates the t-score for a cumulative probability, *P*, for the t distribution. To calculate the t-score for two-tails do `tinv(P/2,df)`

Data analysis 3

Introduction to Excel

Microsoft Excel is a widely used program commonly found throughout business and industry. Job applicants with a knowledge of Excel enjoy an advantage over those without that knowledge. Excel is a spreadsheet program that includes procedures for calculations and graphs; a *spreadsheet* is a popular way to organize data.

> A spreadsheet is a collection of data organized in an array of cells arranged in rows and columns, and it is used to summarize, analyze, and perform calculations with the data.

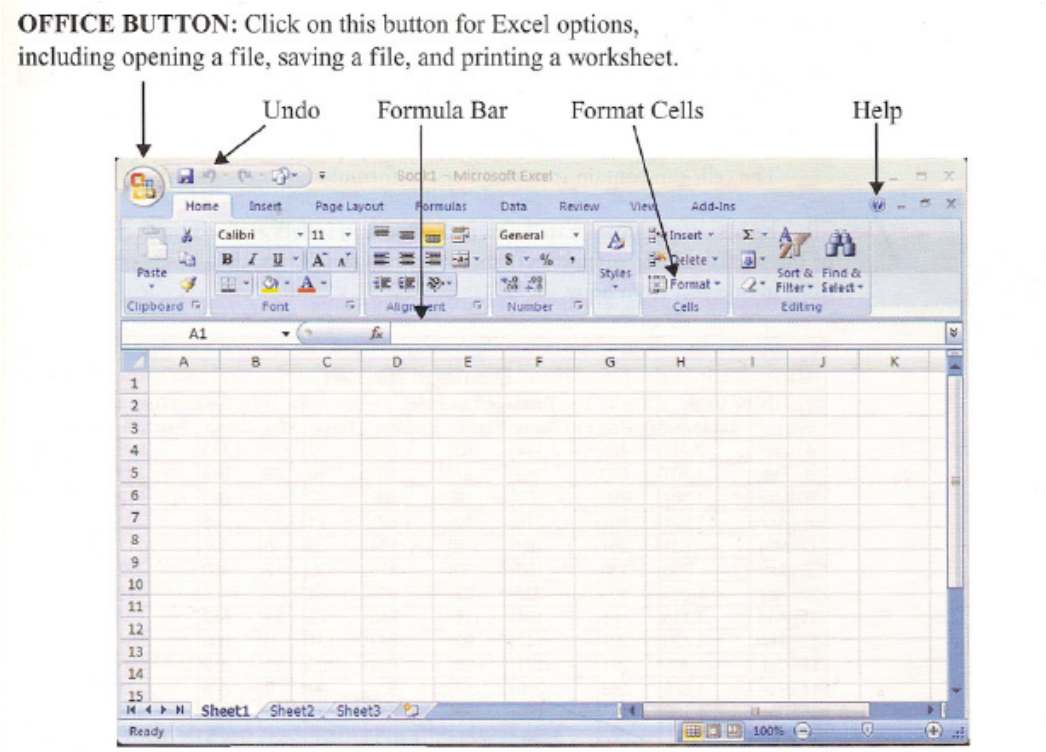


Figure 3.1: Spreadsheet in Excel 2007

Figures 3.1 and 3.2 show the basic format of an Excel spreadsheet. The body of the spreadsheet consists of cells in columns labeled A, B, C, ... , and rows labeled 1, 2, 3, ... An individual spreadsheet is part of a larger Excel structure, described as follows.

```
Excel file = Workbook = 3 worksheets
```

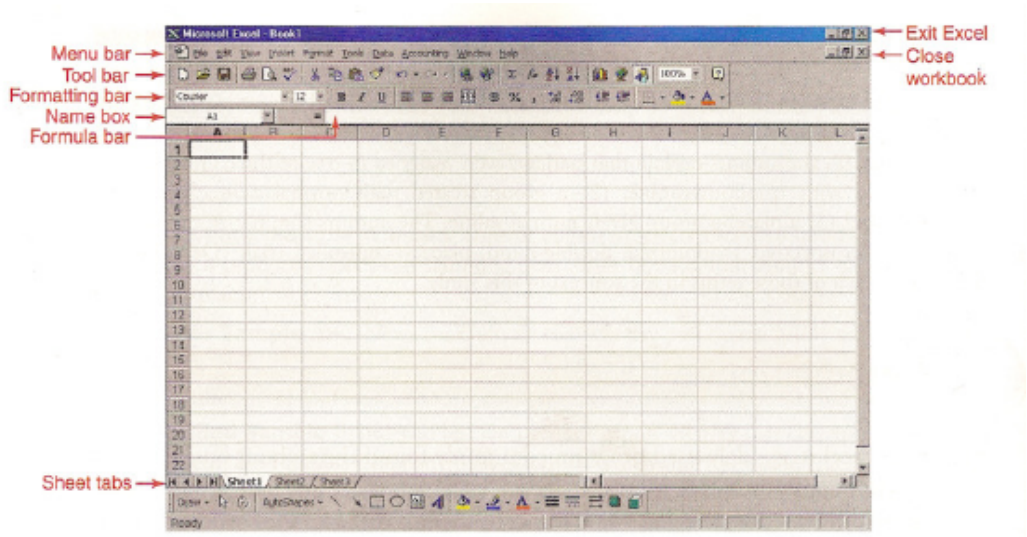


Figure 3.2: Spreadsheet in Excel 2003

(see Figure 3.1 or Figure 3.2)

Although Excel is actually a spreadsheet program, we typically refer to worksheets instead of spreadsheets. (The number of worksheets is 3 by default, but you can insert additional worksheets or delete existing worksheets.)

> An Excel worksheet is one page or sheet (or spreadsheet), consisting of cells arranged in an array of rows and columns (as in Figure 3.1 or Figure 3.2). The cells can contain text, numbers, or formulas.

The best way to learn Excel is to actually use it, so let's walk through the creation of the worksheet (or spreadsheet) shown in Figure 3.3, which is based on damaged units in one year from Sony.

3.1 Creating a an Excel Worksheet

1. Launch Excel to get a blank worksheet. This typically requires that you double-click on the Excel icon. Shown below are the Windows desktop icons for Excel 2007 and Excel 2003.

(Excel should be installed on your computer. If it is not installed, follow the directions provided with the software.)

	A	B	C	D	E	F	G	H	I	J	K
1	**Damage Report**										
2		**Units**	**Damage Location**					**Type of Damage**			
3		**Damaged**	**Rcving**	**Put Away**	**Static**	**Picking**	**Dock**	**Wet**	**Crush**	**Puncture**	**Scuffing**
4	Jan	2,217	135	1,124	183	698	77	136	1,142	903	36
5	Feb	1,701	560	352	87	607	95	39	943	442	277
6	Mar	1,387	295	257	36	707	92	2	863	512	10
7	Apr	1,227	307	216	12	621	71	12	768	428	19
8	May	831	281	109	8	394	39	8	597	211	15
9	June	798	130	158	0	475	35	7	460	322	9
10	July	1,506	405	327	22	668	84	5	758	743	0
11	Aug	1,118	96	184	118	648	72	23	539	487	69
12	Sept	1,420	207	147	93	842	131	0	806	567	47
13	Oct	1,495	291	191	69	835	109	21	852	590	32
14	Nov	1,676	365	278	57	877	99	0	866	774	36
15	Dec	1,186	263	155	38	643	87	0	481	673	32
16	Total	16,562	3,335	3,498	723	8,015	991	253	9,075	6,662	582

Figure 3.3: Excel Spreadsheet of a Damage Report

2. Click on the cell in the upper left corner and type ”Damage Report” and , then press the Enter key. To modify the title so that it appears as shown in Figure 3.3, follow these steps.
   (a) Click on the inside of the cell and, while holding the mouse button down, drag the mouse to the right so that columns A through K in row 1 are all highlighted. Release the mouse button.
   (b) Click on the menu item of Format, select Format Cells (or Cells), select Alignment, then proceed to select the Horizontal option of Center. Also, click on the box to the left of Merge cells. Click OK.
   (c) Click on the menu item of Format Cells (or Format), select Format Cells (or Cells), select Font and proceed to select the Font style of Bold and the Size of 14. Click OK.
3. Click on the fourth cell in the first column and enter ”Jan.” Proceed to enter the other data shown in Figure 3.3. The tides of ”Damage Location” and ”Type of Damage” can be modified by using the same procedures described in parts a, b, and c of Step 2. The bold borders can be created as follows:
   (a) Click and drag the mouse to highlight the desired cells.
   (b) Click on Format, then Format Cells (or Cells), then Border.
   (c) Click on the desired line Style.
   (d) Click on the desired border pattern from the available choices, then click OK.
4. Columns can be made narrower or wider by clicking on a cell in the desired column, selecting Format, then Column Width (or Column, then Width). Enter a value for the desired width, such as 15. You may have to experiment with

different values to find your desired column width. After creating the Excel worksheet shown in Figure 3.3, we could save the worksheet, print it, edit it, perform calculations, do statistical analyses, and construct graphs. Some of these important functions will be discussed in the chapters that follow. For now, we consider a few of the more important general functions of Excel.

3.2 Getting help

To get help while you are running Excel: In Excel 2007, click on the question mark icon located on the top menu bar. In Excel 2003, click on Help located to the right of the menu bar at the top, then click on the question mark icon.

3.3 Undoing your work

When using Excel, it often happens that you do something terribly wrong and you wish that you hadn't done it. Great news: You can undo your last action. Simply click on the counterclockwise arrow located on the menu bar. It looks like this:

3.4 Saving an Excel File

1. In Excel 2007, click on the large round Office Button in the upper left corner, as shown in Figure 3.1; in Excel 2003, click on File. Select the option of Save As.
2. You should now see a dialog box labeled "Save As.
   (a) In the "Save in" box at the top, select the directory and folder in which you want your file saved. To change the drive that -is shown, click on the dropdown button and then click on the desired location, such as (C:). (If you select C, you will get a list of folders available in directory C, and you should click on the desired folder, such as "My Documents.")
   (b) Enter a name (such as "Damages") in the File name box.
   (c) In the "Save as type" box, select Excel Workbook (or Microsoft Excel Workbook) .
   (d) Click on Save.

3.5 Doing calculations in Excel

By entering an equal sign followed by an arithmetic expression in the formula bar, you can perform calculations. Because the standard keyboard does not have special keys for operations such as division, exponents, and square roots, we must use the keys or commands designated for those purposes. Here are a few of the more common operations, with their Excel expressions:

Operation	Excel Expression	Excel Example	Result
Addition	+	=2+3	5
Subtraction	−	=6−4	2
Multiplication	*	=3*4	12
Division	/	=12/3	4
Exponentiation	^	=2^3	8
Square root	SQRT( )	=SQRT(49)	7
Absolute value	ABS( )	=ABS(−5)	5

Using operations such as those listed, we can use Excel to evaluate arithmetic expressions as follows.

1. Click on an empty cell or the formula bar.
2. Enter an equal sign (=) followed by the desired arithmetic expression.
3. Press the Enter key.

Whether using calculators or computer software, it often happens that the result of an arithmetic expression is given in a form of scientific notation, and we should be able to express such results in a standard format. See the following two examples.

Math Expression	Excel Expression	Excel Result	Result in Standard Format
0.2^{10}	=0.2^10	1.024E−07	0.0000001024
3^{25}	=3^25	8.47289E+11	847,289,000,000

3.6 Printing an Excel Worksheet

To print all of the cells in the current worksheet: In Excel 2007, click on the Office Button, then select Print; in Excel 2003, click on File, then select Print.

3.7 Closing files and closing Excel

It is the end of the day, your work is done, you have saved your Excel file, and you now want to close your Excel file so that you can proceed to shut down the whole Excel program, shut down the computer, turn out the lights, and go home to feed your pet iguana.

- Closing an Excelfile: In Excel 2007, click on the Office Button in the upper left corner, then select Close; in Excel 2003, click on File, then click on Close .
- Closing the entire Excel program: Simply click on the X box located in the upper right corner of the screen.

3.8 Installing Add-Ins

In the following lectures, we will describe the use of Excel for a variety of different statistical functions. Those functions typically come from two sources:

1. Click on fx found on the tool bar.You will have access to many built-in functions.
2. Use the Data Analysis set of statistical procedures. In Excel 2007, click on Data, then click on Data Analysis; in Excel 2003, click on Tools, then click on Data Analysis.

Instructions for installing Excel add-ins are available through Excel's Help feature. In Excel 2007, go to the Help feature by clicking on the question mark icon (?) located near the upper-right corner; in Excel 2003, click on Help. In the Search box, enter "add-in" and press the Enter key. Follow Excel's instructions for installing add-in programs. Select **Analysis ToolPak**.

3.9 Cell referencing

You can reference the values or text within a Excel cell by using what is referred to as either *absolute*, *relative or mixed cell references*, these can save a lot of time and are best illustrated with examples.

3.9.1 Relative cell references

As a first example enter the number 0 in cell `A1` of an Excel spreadsheet. Now in cell `A2` enter `=A1+1`, and you should find that the number displayed in cell `A2` is `1`. Click on cell `A2` and then copy it's contents (`ctrl-c`), then go down to cell `A10` and click on that cell with the mouse; hold the keys `ctrl + shift` and press the up arrow key; you should find that cells `A2 to A10` are now highlighted. Now paste the contents of what you copied by pressing `ctrl-v`. You should find that the numbers 0-9 are in cells `A1` to `A10`. The reason being is that you told each cell below `A1` to be equal to the number in the cell above plus 1.

Excel 'knew' to add 1 to the cell above it because we used relative cell references; that is we referenced the cells using only the letter corresponding to the column and the number corresponding to the row.

Relative cell references are useful when you want to multiply (or divide or add or subtract) two lists of numbers to make a third list.

3.9.2 Absolute cell references

The other main type of cell reference is an absolute cell reference, again best illustrated with an example.

As an example enter the number 0 in cell `A1` of an Excel spreadsheet. Now in cell `A2` enter `=$A$1+1`, and you should find that the number displayed in cell `A2` is `1`. Click on cell `A2` and then copy it's contents (`ctrl-c`), then go down to cell `A10` and click on that cell with the mouse; hold the keys `ctrl + shift` and press the up arrow key; you should find that cells `A2 to A10` are now highlighted. Now paste the contents of what you copied by pressing `ctrl=v`. You should find that the number 0 is in cell `A1` and the number 1 is in cells `A2` to `A10`. The reason being is that you told each cell below `A1` to be equal to the number in the cell `A1` plus 1, which is always 1.

Excel 'knew' to add 1 to always reference the cell `A1` because we used absolute cell references; that is we referenced the cells a dollar symbol before the letter corresponding to the column and a dollar symbol before the number corresponding to the row.

Absolute cell references are useful when you want to multiply (or perform some mathematical operation) a list of numbers by the same number each time.

3.9.3 Mixed cell references

Mixed cell references can be useful in certain circumstances, such as computing 2-dimensional tables of quantities and costs and 2-dimensional surface functions (plus countless other things that are hard to explain in words).

Consequently I've put a video on YouTube showing examples of where you might find relative, absolute and mixed cell references useful—see `http://www.youtube.com/watch?v=y3xbluhWY6k`

Data analysis 4
Using Excel for different graphs

Excel sucks for histogram / frequency distribution generation. Instead of reading the pages that follow take a look at my YouTube vid `http://www.youtube.com/watch?v=mdixXI5hAW0`.

4.1 Using Excel to construct a frequency distribution

You can use Excel to construct a frequency distribution from a list of sample data. In Excel, the process of constructing a frequency distribution is called binning the data, because each category acts like a separate bin into which we can pour some of the individual data values. Correct interpretation of an Excel frequency distribution requires that you know this important principle:

Excel's bins (classes) are based on upper class limits.

The following Excel procedure for constructing a frequency distribution uses the menu item of "Histogram," which is a type of graph. However, the same dialog box is used to generate a frequency distribution or a histogram graph.

1. Enter your sample data in a column of the Excel worksheet, or open an existing data set. To manually enter data, type the first value, then press the Enter key. Type the second value, then press the Enter key, and so on.
2. If using Excel 2007, click on Data, then click on Data Analysis; if using Excel 2003, click on Tools, then click on Data Analysis.
3. Click on Histogram, then click OK.
4. You should now see a Histogram dialog box
   (a) First enter the Input Range. For example, if the data are listed in cells 1 through 40 of column A, enter the input range of Al:A40.
   (b) For the Bin Range, you have two options: (1) Leave the bin range blank and let Excel decide how to construct the frequency table; (2) if you want specific class limits, specify a range of cells that you have previously filled in with the values of the upper class limits.
   (c) Select Output Range, then enter a cell location for the frequency table, such as E2. Finally, click OK.
5. You can also use Excel to generate a frequency table with cumulative percentages. Follow the same five steps listed above, but in part c of Step 4, click on the box labeled "Cumulative Percentage" before clicking OK.

4.2 Using Excel for Histograms

CAUTION: Be very careful when reading and interpreting an Excel-generated histogram, because the values shown on the horizontal scale might appear to be class midpoints, but they are actually upper class limits.

The following procedure is quite complicated, but uses techniques that will also apply to many other Excel graphs, so the effort and time spent now will help later.

1. To generate a histogram, follow the procedure described in Section 4.1 for constructing a frequency distribution, but make the following addition. In part c of Step 4, click on the box labeled Chart output before clicking OK. Excel's default histogram requires considerable effort to make it suitable, so proceed patiently with the following steps.
2. Enlarge the Graph The original histogram will be quite small, so enlarge it. Click any area just inside of the border. Click and drag the black boxes on the outer edges so that the graph is enlarged.
3. Delete the Frequency Box To delete the "Frequency" box located to the right of the histogram, right-click on that box and then select either Delete or Clear.
4. Delete the Spaces Between Bars If using Excel 2007, right click on one of the bars, select Format Data Point, set the "Gap Width" to 0, then click on Close. If using Excel 2003, double click on one of the bars, click on the Options tab, change the "Gap width" to 0, then click OK.
5. Delete the Bin Label Remove the label of "Bin" for the horizontal axis by right-clicking on it, then selecting either Delete or Clear.
6. Delete the Bin Values on the Horizontal Scale The default values on the horizontal scale are bin values, so delete them as follows: Use the mouse to right-click on one of those values, then click on the option of Delete or Clear.
7. Delete the Background Color If using Excel 2003, delete the gray background color by right-clicking on any portion of that background, selecting Format Plot Area, clicking on the box colored white, then clicking OK.
8. Insert Class Boundary Values: Using Excel 2007 to insert class boundaries: Click on the Insert tab, then click on Text Box and releasethe mouse button. Move the cursor to the upper left portion of the area that will contain the text for the class boundary values (or class midpoint values), then hold the mouse button and slide the mouse to get a box large enough to contain the text for all of the class boundaries (or class midpoints). (Hint: You can make more room for the text box by clicking on an empty area just above a bar of the histogram, then click and hold the mouse on the lower right corner of the box that contains the bars, then slide the cursor upward.) Enter the class boundaries (or class midpoints) in the text box and use spacing to position them correctly.To change the size or style of the text, use the formatting options available by clicking on Home and selecting Font.
   Using Excel 2003 to insert class boundaries: Double-click on the Text Box icon at the bottom. (The Text Box icon looks like a page of a book with a large "A" in the upper left corner.) Now move the mouse cursor to the area just below the graph. While holding the mouse button down, slide the mouse to enclose a region that will be suitable for entering the class boundaries. (If

your text box isn't suitable, change it by clicking and dragging one of the small squares on its border.) Type the values of the class boundaries and use spacing to position them correctly. Hint 1:To change the size or style of the text, use the mouse to click and drag over all of the text so that it is highlighted, then select Format to change the text as desired. Hint 2: If there is not enough room for the text box at the bottom, click on the inner border that is attached to the histogram, then use the mouse to click and drag (upward) the black box in the middle of the bottom portion of that border. Hint 3: To delete a border around the text box itself, right-click on that border, select Format Text Box, choose Colors and Lines, and in the "Line" section of options, click on the down arrow to the right of "Color" and click on No Line.

9. Insert a Label for the Horizontal Scale The histogram should have a label for the values on the horizontal scale, so enter it by using the procedure in Step 8. That is, create a text box below the values of the class boundaries and enter the label within that box. Remember, the size and style of the font can be edited as desired.

Excel's "Chart Wizard" is very flexible and allows you to generate a wide variety of different graphs. You can usually edit graphs by right-clicking or double-clicking on the element that you want to edit.

4.3 Using Excel for frequency polygons

The following Excel procedure for generating a frequency polygon is much easier than the procedure for generating a histogram.

1. Start with a clear worksheet. In column A, enter 0 in the first cell, followed by the class frequenciet, followed by a 0 at the end. For example, if you want to generate a frequency polygon based on a table (with frequencies of 12, 14, 11,1,1,0, 1), enter these frequencies in column A: 0, 12, 14, 11, 1, 1,0, 1,0.
2. Click on Insert, then select Chart and, among the various charts, select Line.
3. Select the line graph in the upper left corner.
4. The frequency polygon will be displayed without the correct class midpoint values listed along the horizontal axis, so that must be fixed along with any other modifications that would improve the graph. The displayed values along the horizontal axis can be deleted, and the correct values can be inserted as a text box by using the procedure given in Step 8 for creating a histogram.

Data analysis 5
Introduction to Matlab

Topics discussed here include the Command Window, numbers and arithmetic operations, saving and reloading a work, using help, MATLAB demos, interrupting a running program, long command lines, and MATLAB resources on the Internet.

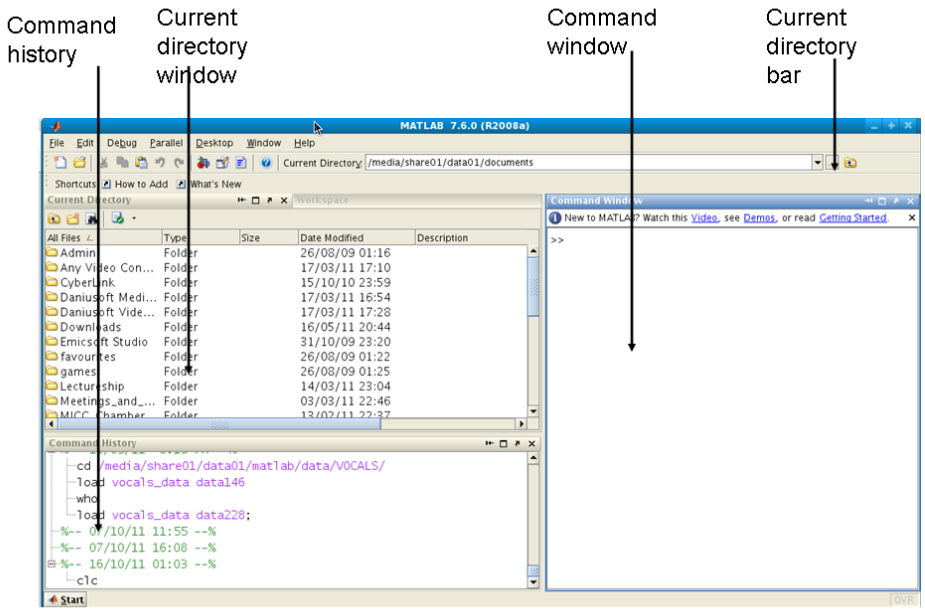
There are some statistics functions that do not ship with the standard version of MATLAB. You may need to download these for the course (unless you have the statistics toolbox). Go to Blackboard, 'Course content→ Semester→ Matlab resources' and download the *stats_toolbox.zip* file to your hard drive and unzip it. It will need to be on your MATLAB path so that you can use it.

5.1 Overview

You can start MATLAB by double clicking on the MATLAB icon that should be on the desktop of your computer. This brings up the MATLAB program and contained within the whole program (amongst other things) are:

- the Command Window.
- the Command History window
- the Current Directory Window
- the Current Directory bar.

as shown below.



The Command History prints a list of anything entered into the Command Window. The Current Directory shows where you are within the System File Structure.

The Command Window is perhaps the most important window. It is where commands can be entered and is now described.

5.2 The command window

This window allows a user to enter simple commands. To clear the Command Window type clc and next press the Enter or Return key. To perform a simple computations type a command and next press the Enter or Return key. For instance,

```
s = 1 + 2
s =
     3
fun = sin(pi/4)
fun =
    0.7071
```

In the second example the trigonometric function sine and the constant π are used. In MATLAB they are named sin and pi, respectively.

Note that the results of these computations are saved in variables whose names are chosen by the user. If they will be needed during your current MATLAB session, then you can obtain their values typing their names and pressing the Enter or Return key. For instance,

```
s
s =
     3
```

Variable name begins with a letter, followed by letters, numbers or underscores. MATLAB recognizes only the first 31 characters of a variable name.

To change a format of numbers displayed in the Command Window you can use one of the several formats that are available in MATLAB. The default format is called short (four digits after the decimal point.) In order to display more digits click on File, select Preferences..., and next select a format you wish to use. They are listed below the Numeric Format. Next click on Apply and OK and close the current window. You can also select a new format from within the Command Window. For instance, the following command

```
format long
```

changes a current format to the format long. To display more digits of the variable fun type

```
fun
fun =
    0.70710678118655

% To change a current format to the default one type
format short
fun
fun =
    0.7071
```

To close MATLAB type exit in the Command Window and next press Enter or Return key. A second way to close your current MATLAB session is to select File in the MATLAB's toolbar and next click on Exit MATLAB option. All unsaved information residing in the MATLAB Workspace will be lost.

5.3 Numbers and arithmetic operations in MATLAB

There are three kinds of numbers used in MATLAB:

1. integers
2. real numbers
3. complex numbers

Integers are entered without the decimal point

```
xi = 10
xi =
      10
```

However, the following number

```
xr = 10.01
xr =
    10.0100
```

is saved as the real number.

Complex numbers in MATLAB are represented in rectangular form. The imaginary unit $\sqrt{-1}$ is denoted either by i or j

```
i
ans =
             0 + 1.0000i
```

However, we will not make use of complex numbers in this course.

In addition to classes of numbers mentioned above, MATLAB has three variables representing the nonnumbers:

- -Inf
- Inf
- NaN

The Inf and Inf are the IEEE representations for the negative and positive infinity, respectively. Infinity is generated by overflow or by the operation of dividing by zero. The NaN stands for the not-a-number and is obtained as a result of the mathematically undefined operations such as $0.0/0.0$ or ∞ or $-\infty$.

The list of basic arithmetic operations in MATLAB include six operations

Operation	Symbol
addition	+
subtraction	-
multiplication	*
division	/ or \
exponentiation (to the power)	^

MATLAB has two division operators /—the right division and \—the left division. They do not produce the same results

```
rd = 47/3
rd =
    15.6667
ld = 47\3
ld =
     0.0638
```

as you can see, the first returns the value of $47 \div 3$ and the second $3 \div 47$.

5.4 Vectors, Matrices / arrays

MATLAB is excellent for dealing with vectors and matrices (it stands for MATrix LABoratory after all). Let us call a vector a 1-d list of numbers, you can type some in using the commandline as follows:

```
vector=[1; 2; 1; 3; 7 ; 10]

% and matlab prints the following
vector =

     1
     2
     1
     3
     7
    10
```

The ';' tells MATLAB to start a new row. You can use ',' instead to start a new column:

```
rowvector=[1, 2, 1, 3, 7, 10]

% and matlab prints the following
rowvector =

     1     2     1     3     7    10
```

You can also append two column vectors or row vectors together:

```
columnvector1=[1; 2; 1; 3; 7; 10];

columnvector2=[2; 3; 7; 10];


new=[columnvector1;columnvector2]

% and MATLAB will print
new =

     1
     2
     1
     3
     7
    10
     2
     3
     7
    10
```

If you want to append two row vectors together the syntax is slightly different:

```
rowvector1=[1, 2, 1, 3, 7, 10];

rowvector2=[2, 3, 7, 10];


new=[rowvector1,rowvector2]

% and MATLAB will print
new =

     1     2     1     3     7    10     2     3     7    10
```

If two vectors have the same dimensions you can add / subtract them:

```
columnvector1=[1; 2; 1; 3];

columnvector2=[2; 3; 7; 10];


new=columnvector1+columnvector2

% and MATLAB will print
new =

     3
     5
     8
    13
```

or you can multiply or divide each column (or row). When multiplying or dividing vectors in this way you must use the `.*` or `./` syntax (the '.' is important it tells MATLAB to multiply or divide each column (or row):

```
columnvector1=[1; 2; 1; 3];

columnvector2=[2; 3; 7; 10];


new=columnvector1.*columnvector2

% and MATLAB will print
new =

     2
     6
     7
    30
```

Finally here we append two matrices together:

```
matrix1=[1, 2, 4;  5, 7, 10 ]; % 2 by 3 matrix

matrix2=[2, 3, 7; 6, 4, 1; 0, 1, 4; 3, 4, 3]; % 4 by 3 matrix


new=[matrix1;matrix2]

% and MATLAB will print
new =

     1     2     4
     5     7    10
     2     3     7
```

```
     6     4     1
     0     1     4
     3     4     3
```

There are ways you can multiply and divide matrices and vectors that are different to element wise multiplication above. These will not be covered in this course, but they are very useful.

There is an interesting point about arrays in computer languages, which allows us to gain insight into how computer memory works. Try the two following pieces of code, which both do the same thing:

```
a=zeros(10000,10000);

tic
for i=1:10000
    for j=1:10000
        a(i,j)=a(i,j)+1;
    end
end
toc
```

on my computer the output is: `Elapsed time is 1.024994 seconds.`

```
a=zeros(10000,10000);

tic
for j=1:10000
    for i=1:10000
        a(i,j)=a(i,j)+1;
    end
end
toc
```

on my computer the output is: `Elapsed time is 0.498409 seconds.`

So we see that the way in which you access data (memory) can make a difference to the speed of the computations. Very important!

Matlab treats its arrays so that the first dimension (rows) are arranged sequentially in memory. If you access columns sequentially the computer has to work over time to go forward an backward to access the data.

5.5 Saving and reloading your work

All variables used in the current MATLAB session are saved in the Workspace. You can view the content of the Workspace by clicking on File in the toolbar and next selecting Show Workspace from the pull-down menu. You can also check contents of the Workspace typing whos in the Command Window. For instance,

```
whos
  Name          Size                Bytes  Class
  ans           1x1                    16  double  array (complex)
  fun           1x1                     8  double  array
  ld            1x1                     8  double  array
  rd            1x1                     8  double  array
  s             1x1                     8  double  array
  xi            1x1                     8  double  array
  xr            1x1                     8  double  array
Grand total is 7 elements using 64 bytes
```

shows all variables used in current session. You can also use command who to generate a list of variables used in current session

```
who

Your variables are:
ans         ld          s
fun         rd          xi
```

To save your current workspace select Save Workspace as... from the File menu. Chose a name for your file, e.g. filename.mat and next click on Save. Remember that the file you just created must be located in MATLAB's search path. Another way of saving your workspace is to type

```
save <filename>
```

where <filename> is the name of the file you want to save. in the Command Window. The following command

```
save <filename> s
```

where <filename> is the name of the file, saves only the variable s.

Another way to save your workspace is to type the command:

```
 diary <filename>
```

where <filename> is the name of the file, in the Command Window. All commands and variables created from now will be saved in your file. The following command:

```
diary off
```

will close the file and save it as the text file. You can open this file in a text editor, by double clicking on the name of your file, and modify its contents if you wish to do so.

To load contents of the file named filename into MATLAB's workspace type

```
load <filename>
```

in the Command Window.

More advanced computations often require execution of several lines of computer code. Rather than typing those commands in the Command Window you should create an m-file, which is just a text file with commands to be executed one after another. The m-file is saved with a '.m' file extension. Each time you will need to repeat computations just invoke your m-file by typing the name in the MATLAB command window (without the '.m' extension). Another advantage of using m-files is the ease to modify its contents.

5.6 Help

One of the nice features of MATLAB is its help system. To learn more about a function you are to use, say normpdf, type in the Command Window

```
help normpdf

 NORMPDF Normal probability density function (pdf).
    Y = NORMPDF(X,MU,SIGMA) returns the pdf of the normal distribution with
    mean MU and standard deviation SIGMA, evaluated at the values in X.
    The size of Y is the common size of the input arguments.  A scalar
    input functions as a constant matrix of the same size as the other
    inputs.

    Default values for MU and SIGMA are 0 and 1 respectively.

    See also NORMCDF, NORMFIT, NORMINV, NORMLIKE, NORMRND, NORMSTAT.
```

The helpwin command, invoked without arguments, opens a new window on the screen. To find an information you need double click on the name of the sub-directory and next double click on a function to see the help text for that function. You can go directly to the help text of your function invoking helpwin command followed by an argument. For instance, executing the following command

```
helpwin zeros
ZEROS Zeros array.
      ZEROS(N) is an N-by-N matrix of zeros.
      ZEROS(M,N) or ZEROS([M,N]) is an M-by-N matrix of zeros.
      ZEROS(M,N,P,...) or ZEROS([M N P ...]) is an M-by-N-by-P-by-...
      array of zeros.
      ZEROS(SIZE(A)) is the same size as A and all zeros.
      See also ONES.
generates an information about MATLAB's function zeros.
```

5.7 Demos

To learn more about MATLAB capabilities you can execute the demo command in the Command Window:

```
demo
```

or click on Help and next select Demos from the pull-down menu. You can then use the explorer bar on the left to learn about different aspects of MATLAB and its toolboxes. Some of the MATLAB demos use both the Command and the Figure windows.

To learn about plots and graphics in MATLAB open the demo window using one of the methods described above. In the left pane select Graphics and in the right pane select 2-D Plots. The information will be displayed in the help pane. You can run a demo by clicking the link "Run in the Command Window", you may have to return to the command window to press return—and follow instructions.

I recommend trying some of these demos and looking at what MATLAB commands are needed to do them. There is enough information to 'learn by doing' for those interested (and motivated to do so).

5.8 Interupting a running program

To interrupt a running program press simultaneously the Ctrl-c keys. Sometimes you have to repeat pressing these keys a couple of times to halt execution of your

program. This is not a recommended way to exit a program, however, in certain circumstances it is a necessity. For instance, a poorly written computer code can put MATLAB in the infinite loop and this would be the only option you will have left.

5.9 Long command lines

To enter a statement that is too long to be typed in one line, use three periods, ... , followed by Enter or Return. For instance,

```
x = sin(1) - sin(2) + sin(3) - sin(4) + sin(5) -...
        sin(6) + sin(7) - sin(8) + sin(9) - sin(10)
x =
     0.7744
```

You can suppress output to the screen by adding a semicolon after the statement

```
u = 2 + 3;
```

5.10 Plotting and histograms

To plot a graph of $f(x) = x^2$ on the interval $0 \leq x \leq 10$ you may type:

```
X=0:0.1:10 % create a vector x ranging from zero to 10 in steps of 0.1
plot(X,X.^2) % plot out the function f(x)=x^2
xlabel('x variable');
ylabel('f(x)=x^2')
```

To plot a histogram of some random data:

```
X=rand(1000,1); % create a vector of 1000 random numbers ranging from 0 to 1
hist(X,[0:0.2:1]) % create a histogram of those values binned into intervals of 0.2
xlabel('x')
ylabel('Frequency')
```

There are many other ways of visualising data in MATLAB. Take a look at the Graphics and 3-D visualisation demos for some examples.

5.11 Reading data into MATLAB

For this course you can use csvread to read in a csv (comma separated variable) file:

```
dat=csvread('filename') % 'filename' is the name of the file to be read into the variable 'dat'
```

You may also be able to xlsread to read in an Excel spreadsheet (try 'help xlsread') to learn about this function:

```
[dat,text]=xlsread('filename') % 'filename' is the name of the file to be read into the variables 'da
```

5.12 MATLAB resources on the Internet

- The MathWorks Web site: `http://www.mathworks.com/` The MathWorks, the makers of MATLAB, maintains an important Web site. Here you can find information about new products, MATLAB related books, user supplied files and much more.

- Especially this link: `http://www.mathworks.co.uk/academia/student_center/tutorials/launchpad.html`, which gives plenty of resources and interactive tutorials.
- The Mastering Matlab Web site: `http://www.eece.maine.edu/mm` Link to the books: Mastering Matlab. A Comprehensive Tutorial and Reference, which are also available from the John Rylands library.

Data analysis 6

Table of equations and functions

Here I give a quick reference guide to common things you may want to do and the corresponding functions to look up in either Microsoft Excel or Matlab—see table opposite.

What you want to do	Equation	Excel functions	Matlab functions
Import data	N/A	N/A	`dat=csvread;` `[dat,text]=xlsread`
Create a frequency distribution of your data	N/A	frequency	`hist`
Create a cumulative frequency distribution of your data	N/A	use `frequency` and then in another column calculate the cumulative sum using absolute and relative cell references.	use `[N,X]=hist(dat,bins)` and then `N2=cumsum(N)` to calculate the cumulative frequency
Create a relative frequency distribution of your data	N/A	use `frequency` then in another column reference the frequency distribution and divide by `sum()` of all values in the frequency distribution (using absolute cell references)	use `[N,X]=hist(dat,bins)` and then divide by `sum(N)`
Create a bar chart of your data	N/A	Insert chart	`bar`
Create a pie chart of your data	N/A	Insert chart	`pie`
Create an x-y scatter plot of your data	N/A	Insert chart	`plot(x,y)`
Calculate the arithmetic mean	$\bar{x} = \frac{\sum x}{N}$	`average`	`mean(x)`
Calculate the median	N/A	`median`	`median(x)`
Calculate the mode	N/A	`mode`	`mode(x)`

Continued on next page

What you want to do	Equation	Excel functions	Matlab functions
Calculate the range	N/A	`max()-min()`, and pass cell references	`range(x)`
Calculate the sample standard deviation	$s = \sqrt{\frac{\sum(x-\bar{x})^2}{N-1}}$	`stdev`	`std(x,0)`
Calculate the population standard deviation	$\sigma = \sqrt{\frac{\sum(x-\bar{x})^2}{N}}$	`stdevp`	`std(x,1)`
Calculate a percentile	N/A	`percentile(arr,k)`	`prcentile`
Calculate an interquartile range	$Q_3 - Q_1$	`percentile(arr,75)-percentile(arr,25)`	`iqr`
Calculate a z-score	$z = \frac{x-\bar{x}}{s}$		`[Z,MU,SIGMA] = zscore(X)`
Calculate probability density for the normal distribution	$y = \frac{\exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)}{\sigma\sqrt{2\pi}}$	`normdist(x,mean,s,0)`	`normpdf(x,mean,s)`
Calculate cumulative probability for the normal distribution	$\frac{1}{2}\left[1 + \mathrm{erf}\left(\frac{x-\mu}{\sqrt{2\sigma^2}}\right)\right]$	`normdist(x,mean,s,1)`	`normcdf(x,mean,s)`
Sample from the normal distribution	N/A	`norminv(P,mean,s)`	`norminv(P,mean,s)`
Calculate cumulative probability for the student t-distribution	nasty	`tdist(P,degF,tails)` note that `tails` can be either 1 or 2 and refers to whether the area, P, is on both sides or one	`tcdf` this is the one sided cumulative dist.

Continued on next page

What you want to do	Equation	Excel functions	Matlab functions
Sample from the t-distribution	N/A	`tinv(P,degF)` note this returns the two-tailed distribution. Multiply P by two to return the one-tailed	`tinv(P,degF)` note this returns the one-tailed distribution. Divide P by two to return the two-tailed.
Calculate the correlation coefficient	see sheet of formulas	`CORREL(x,y)`	`corrcoef(x,y)`
Perform linear regression	N/A	Use regression in Analysis toolpak	`P=polyfit(x,y,1)` P will contain the gradient and the intercept
Calculate linear regression line	N/A	N/A	`y=polyval(P,x)` Where `x` is a list of numbers spanning the lower and upper values (e.g. `x=-10:0.01:10`) and P contains the output of `polyfit`

References

Hanselman, D., and B. Littlefield, 2001: *Mastering Matlab 6: A comprehensive tutorial and reference*. Pearson.

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Triola, M. F., 2009: *Elementary statistics using Excel*. 4th ed., Pearson education.