Preservation of volumes in nonholonomic mechanics

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Abstract

In mechanics, constraints that restrict the possible configurations of the system are termed holonomic. A simple example is the fixed length of the rod of a pendulum. Mechanical systems with constraints on the velocities that do not arise as constraints on positions are called nonholonomic. These often arise in rolling systems, like a sphere rotating without slipping on a table.

The study of nonholonomic mechanical systems is challenging because the equations of motion are not Hamiltonian. The dynamics of the system can however be described in terms of a bracket of functions that fails to satisfy the Jacobi identity. One now speaks of an "almost Poisson bracket". The failure of the Jacobi identity leads to phenomena that are not shared by usual Hamiltonian systems. Open questions in nonholonomic mechanics that have received attention in recent years include determining general conditions for measure preservation, existence of asymptotic equilibria, relationship between symmetries and conservation laws, reduction, and integrability.

In the first part of this talk I will present a basic introduction to nonholonomic mechanics. I will then present my recent work with Y. Fedorov and J. C. Marrero in which we study the problem of measure preservation for nonholonomic systems possessing symmetries in a systematic manner. Our method allows us to identify specific parameter values for which there exists a preserved measure for concrete mechanical examples.