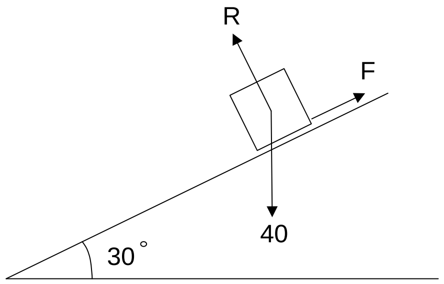
Introduction to Mechanics (0J2)

Example Sheet 6 – solutions

1.



For equilibrium:

Resolve down plane: $40 \cos 60 = F$ (friction) $\therefore F = 20$

Resolve perpendicular to plane: $R = 40 \cos 30 = 20\sqrt{3}$

For consistency $20 \leq \mu R = \frac{3}{4}.20\sqrt{3} = 15\sqrt{3} \approx 25.98$ so the friction force required is less than the maximum possible force.

a) Let the applied force be P, friction force F, acting up the plane. Resolve down the plane with $F = \mu R$ (limiting case), looking for equilibrium, so just a slightly greater force cannot be in equilibrium and the block starts sliding.

Resolve down the plane: $P + 40 \cos 60 = \mu R$

Perpendicular to plane: $R = 40 \cos 30 = 20\sqrt{3}$

so $P + 20 = 20\sqrt{3} \times \frac{3}{4}$, *i.e.* $P = 20(3\sqrt{3}/4 - 1) = 5(3\sqrt{3} - 4)$

b) Now P is up the plane, so $F = \mu R$ down the plane and $R = 20\sqrt{3}$ as before. So resolving down the plane $40 \cos 60 + \mu R = P \quad \Rightarrow P = 20 + \frac{3}{4}20\sqrt{3} = 5(3\sqrt{3} + 4)$.

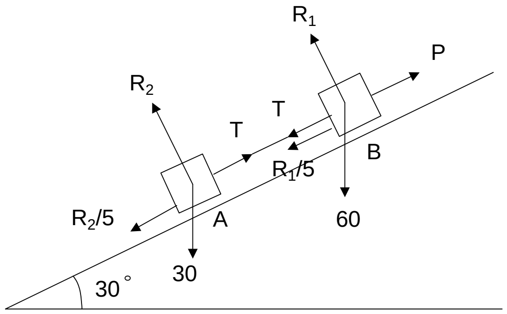
Finally: P applied horizontally with a component down the plane, so again the limiting friction is $F = \mu R$ *up* the plane.

Resolving perpendicular to the plane: $R + P \cos 60 = 20\sqrt{3} \quad \Rightarrow R = 20\sqrt{3} - \frac{1}{2}P$.

Down the plane: $P \cos 30 + 40 \cos 60 = \mu R = \frac{3}{4}R \Rightarrow \frac{\sqrt{3}}{2}P + 20 = \frac{3}{4}R = \frac{3}{4}(20\sqrt{3} - \frac{1}{2}P)$.

Hence $(\frac{\sqrt{3}}{2} + \frac{3}{8})P = 15\sqrt{3} - 20$ and so $P = 40(3\sqrt{3} - 4)/(4\sqrt{3} + 3)$.

2.



At B: Parallel to the plane: $P = T + \frac{1}{5}R_1 + 30$

Perpendicular to the plane: $R_1 = 30\sqrt{3}$

so $P = T + 6\sqrt{3} + 30$. (1)

At A: Parallel to the plane: $T = \frac{1}{5}R_2 + 15$

Perpendicular to the plane: $R_2 = 15\sqrt{3}$

Hence $T = 3\sqrt{3} + 15$. (2)

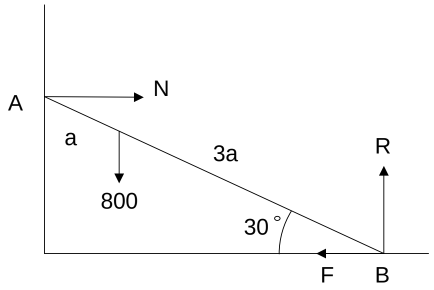
Substitute (2) into (1): $P = 9\sqrt{3} + 45 = 9(\sqrt{3} + 5)$

and $T = 3(\sqrt{3} + 5)$.

3. If a force $F_1\mathbf{i} + F_2\mathbf{j}$ is acting at the point (x, y) then the moment about point (X, Y) is $(x - X)F_2 - (y - Y)F_1$. Hence

Force	at	Moment about O	$(x+2)\underline{i} + (y+1)\underline{j}$	Moment about $(-2, -1)$
$5\mathbf{i}$	$(1, 1)$	-5	$3\mathbf{i} + 2\mathbf{j}$	-10
$4\mathbf{i} + 2\mathbf{j}$	$(1, -1)$	6	$3\mathbf{i}$	6
$-3\mathbf{i}$	$(0, 3)$	0	$2\mathbf{i} + 4\mathbf{j}$	-6
$\mathbf{i} - \mathbf{j}$	$(4, 0)$	-4	$6\mathbf{i} + \mathbf{j}$	-7
total		-3		-17

4.



Vertically: $R = 800$

Horizontally: $N = F$

Moments about B:

$800 \times 3a \cos 30 - N \times 4a \sin 30 = 0$

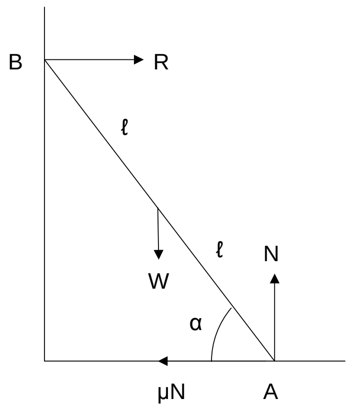
$\therefore N = 800 \times \frac{3}{4} \times \cot 30 = 600\sqrt{3} = F$

Total reaction magnitude: $= \sqrt{R^2 + F^2} = \sqrt{800^2 + 600^2 \times 3} = 200\sqrt{43}N$

Angle of reaction: $\tan \alpha = \frac{R}{F} = \frac{4}{3\sqrt{3}}$

$\therefore \alpha = \tan^{-1}(\frac{4}{3\sqrt{3}})$

5.



Vertically: $N = W$

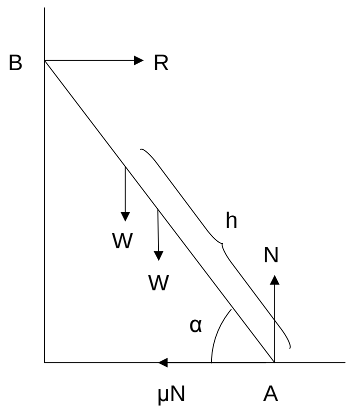
Horizontally: $\mu N = R$

Moments about A: $W\ell \cos \alpha = R \times 2\ell \sin \alpha$

$\therefore \cos \alpha = 2\mu \sin \alpha$

$\therefore$ (Minimum) $\mu = \frac{1}{2} \cot \alpha = \frac{3}{8}$

Now let $\mu = \frac{1}{2}$ and attach weight W at distance h along rod from A.



Vertically: $N = 2W$

Horizontally: $R = \mu N = \frac{1}{2}N \qquad \therefore R = W$

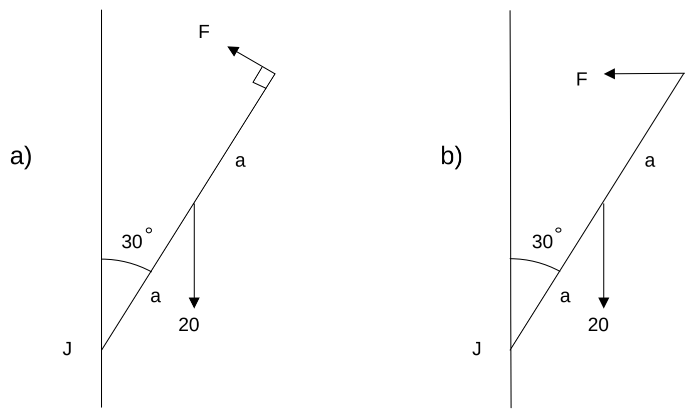
Moments about A: $W\ell\cos\alpha + Wh\cos\alpha = R \times 2\ell\sin\alpha$

$\therefore W(\ell + h)\cos\alpha = W \times 2\ell\sin\alpha$

$\therefore \ell + h = 2\ell\tan\alpha = \frac{8\ell}{3}$

$\therefore h = (\frac{8}{3} - 1)\ell = \frac{5}{3}\ell.$

6.



a) **F** perpendicular to the rod.

Moments about J: $F \times 2a = 20a\sin 30$

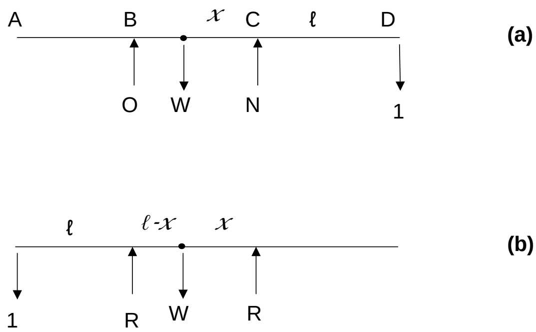
$\therefore F = 10\sin 30 = 5N.$

b) **F** horizontal.

Moments about J: $F \times 2a\cos 30 = 20a\sin 30$

$\therefore F = 10\tan 30 = \frac{10}{\sqrt{3}}N.$

7.



Case a) Moments about C: $Wx - 1\ell \; = \; 0$ (1)

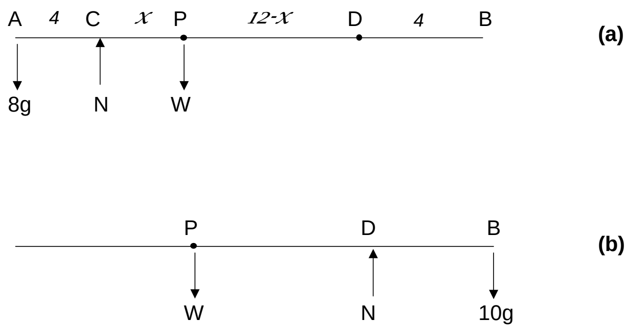
Case b) Resolve vertically: $2R = W + 1$ (2)

Moments about B: $1\ell - W(\ell - x) + R\ell = 0$

Therefore, using (2), $\frac{1}{2}(W+3)\ell = W(\ell - x)$ (3)

and using (1), $(W+3)\ell = 2W\ell - 2\ell \quad \therefore W = 5 \quad \Rightarrow x = \frac{1}{5}\ell$

8.



Case a) Moments about C: $Wx = 32g$ (1)

Case b) Moments about B: $W(12 - x) = 40g$ (2)

From (1) and (2) obtain $x = 5\frac{1}{3}, \quad AP = 9\frac{1}{3}. \quad \therefore W = 6g.$