Introduction to Mechanics (0J2)
Example Sheet 5 – sketch answers

1. In each case let V denote the component of the force in the upwards (vertical) direction.
(i) A force of magnitude $10N$ applied at an angle of $30°$ to the vertical has $V = 10\cos 30° = 5\sqrt{3}$ with $V^2 = 75$.
(ii) A force of magnitude $14N$ applied at an angle of $45°$ to the vertical has $V = 14\cos 45° = 7\sqrt{2}$ with $V^2 = 98$.
(iii) A force of magnitude $9N$ at an angle of $0°$ to the vertical (i.e. directly upwards) has $V = 9$ i.e. $V^2 = 81$.
Comparing V^2 in each case we see that (ii) gives the largest lifting force.

2. A particle at the origin is subjected to a force $\mathbf{i} + 3\mathbf{j}$. The engineer can apply a horizontal force, $F\mathbf{i}$ say and a force along the line $x + y = 0$, i.e. a force of the form $G(\mathbf{i} - \mathbf{j})$. We want to choose F and G so that the forces are in equilibrium, i.e.

$$\mathbf{i} + 3\mathbf{j} + F\mathbf{i} + G(\mathbf{i} - \mathbf{j}) = \mathbf{0}$$

Looking at the $\mathbf{i}$ and $\mathbf{j}$ components separately gives

$$1 + F + G = 0 \qquad 3 - G = 0$$

respectively. Thus $G = 3$ and $F = -4$.
The magnitude of the horizontal force, $-4\mathbf{i}$ is 4, whilst the magnitude of $G(\mathbf{i} - \mathbf{j})$ is $\sqrt{9 + 9} = 3\sqrt{2}$.

3. (i) $\mathbf{F}_1 = 10\cos 30\mathbf{i} + 10\sin 30\mathbf{j} = 5\sqrt{3}\mathbf{i} + 5\mathbf{j}$.
(ii) $\mathbf{F}_2 = -5\mathbf{i}$.
(iii) $\mathbf{F}_3 = 8\cos 120\mathbf{i} + 8\cos 30\mathbf{j} = -4\mathbf{i} + 4\sqrt{3}\mathbf{j}$.
Their sum, $\mathbf{F} = (-9 + 5\sqrt{3})\mathbf{i} + (5 + 4\sqrt{3})\mathbf{j}$ and so

$$|\mathbf{F}|^2 = (-9 + 5\sqrt{3})^2 + (5 + 4\sqrt{3})^2 = 229 - 50\sqrt{3}$$

and so $|\mathbf{F}| = (229 - 50\sqrt{3})^{\frac{1}{2}}$ and the angle to the $\mathbf{i}-$axis is θ where

$$\tan\theta = -\frac{5 + 4\sqrt{3}}{9 - 5\sqrt{3}}$$

We leave the approximate values to your calculators.

4. To form a closed triangle of forces (the polygon of forces is closed for forces in equilibrium) a triangle with sides 5, 12 and 13 is a right-angled triangle (note that $5^2 + 12^2 = 13^2$) so the angle between the first two forces must be $90°$.

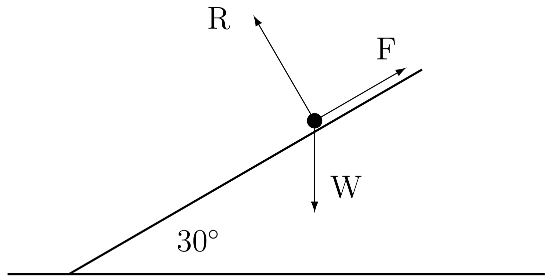
5. There are many ways to do this. For example, choose the force of 5 N to be in the $\mathbf{i}$ direction, with the 10 N force having an upwards vertical component and the 13 N force a downwards vertical component (we are only interested in the angles between

forces, so the orientation does not matter). Then the 5 N force is $5\mathbf{i}$, the 10 N force is $-5\mathbf{i} + 5\sqrt{3}\mathbf{j}$ and the 13 N force is $-\frac{13}{2}(\mathbf{i} + \sqrt{3}\mathbf{j})$. Adding to give the resultant $\mathbf{R}$ gives $\mathbf{R} = -\frac{13}{2}\mathbf{i} - \frac{3}{2}\sqrt{3}\mathbf{j}$ and so

$$|\mathbf{R}| = (\sqrt{169 + 27})/2 = 7$$

and the angle to the 5 N force is α where $\tan\alpha = \frac{3\sqrt{3}}{13}$ with $\alpha \in (\pi, 2\pi)$ for this choice of orientation, i.e. $\alpha \approx 201.8°$.

6.



$W = 20$ acts vertically down and the reaction R is at an angle of 30° to the vertical, perpendicular to the plane. Minimize F by choosing it parallel up the plane. Resolving along the plane:

$$F = W \sin 30 = 10$$

and resolve perpendicular to the plane

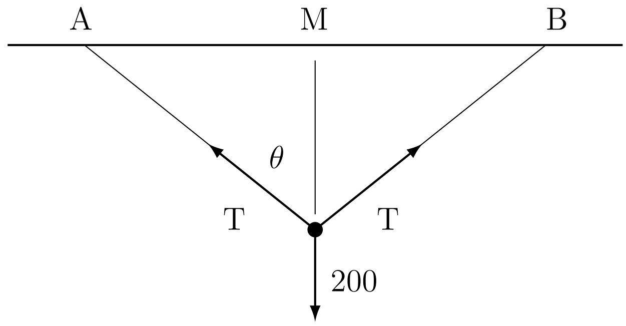
$$R = W \cos 30 = 10\sqrt{3}$$

7. There are many ways of doing this. Here is one. Note that $\vec{DA} + \vec{AB} = \vec{DB}$ and $\vec{DA} + \vec{AC} = \vec{DC}$ so

$$\begin{aligned} \vec{AB} + \vec{AC} + 2\vec{DA} + \vec{AE} + \vec{AF} &= (\vec{DA} + \vec{AB}) + (\vec{DA} + \vec{AC}) + \vec{AE} + \vec{AF} \\ &= (\vec{DB} + \vec{AE}) + (\vec{DC} + \vec{AF}) \end{aligned}$$

and each term in brackets is clearly zero (sum of anti-parallel sides).

8.



In this special case the tensions in the string are equal by symmetry (or resolve horizontally).
Also the triangle made by each string with the vertical and the ceiling is right-angled. If θ is the angle each string makes with the vertical, then $\cos\theta = 3/5$ (MB is 4, the hypoteneuse is 5, so the vertical distance is 3). Hence, resolving vertically:

$$200 \;=\; 2T\cos\theta = 6T/5$$

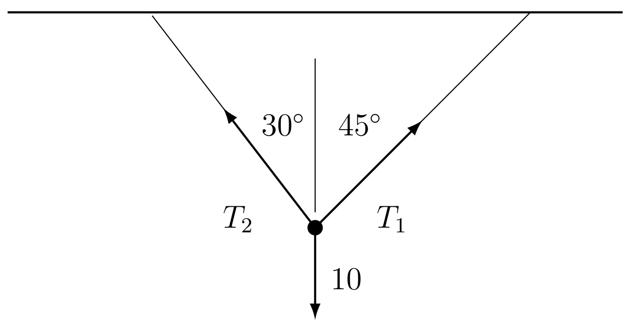
so $T = 500/3 = 166\frac{2}{3}\ N$.
In the general case, $\cos\theta = a/5$, where a is the vertical distance between the weight and the ceiling. Resolving vertically again:

$$200 = 2T\cos\theta \quad \text{i.e.} \quad T = 500/a$$

So $T \leq 125$ if and only if $500 \leq 125a$ or $a \geq 4$. But if $a \geq 4$ then $|MB| \leq 3$ (think of the 3,4,5 right-angled triangle again) and so

$$|AB| = 2|MB| \leq 6$$

9.



Let the tensions of the string at 45° to the vertical be T_1 and and that at 30° be T_2. Resolving horizontally and vertically respectively:

$$T_1\sin 45 = T_2\sin 30 \qquad T_1\cos 45 + T_2\cos 30 = 10$$

or

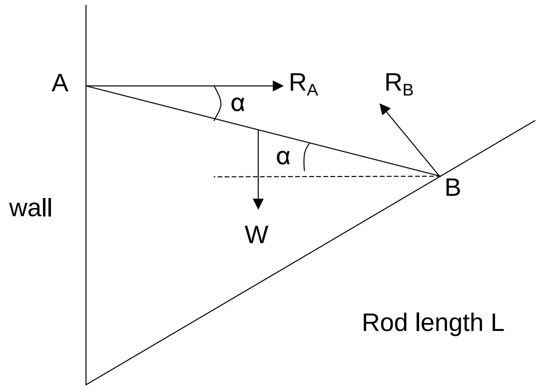
$$T_1/\sqrt{2} = T_2/2 \qquad T_1/\sqrt{2} + \sqrt{3}T_2/2 = 10$$

From the first of these, $T_2 = \sqrt{2}T_1$, and substituting into the second, $(1+\sqrt{3})T_1 = 10\sqrt{2}$. Hence

$$T_1 = 10\sqrt{2}/(1+\sqrt{3}) \quad \text{and} \quad T_2 = 20/(1+\sqrt{3})$$

The answers correct to 2 d.p. can be found using a calculator!

10.



R_A is horizontal, R_B is at 45° to the horizontal, *i.e.* ⊥ to the plane.

Forces acting on the rod:
<u>Horizontally</u>

$$\begin{aligned} R_A - R_B \cos 45^\circ &= 0 \\ \therefore R_A &= \frac{1}{\sqrt{2}} R_B \end{aligned}$$

<u>Vertically</u>

$$\begin{aligned} -W + R_B \cos 45^\circ &= 0 \\ \therefore R_B &= \sqrt{2} W \\ \therefore R_A &= W \end{aligned}$$

<u>Moments about B</u>

$$\begin{aligned} W \frac{L}{2} \cos\alpha - R_A L \sin\alpha &= 0 \\ \therefore \frac{1}{2}\cos\alpha - \sin\alpha &= 0 \\ \therefore \tan\alpha &= \frac{1}{2} \end{aligned}$$