Introduction to Mechanics (0J2)

Example Sheet 2 – solutions

1. (i) On level ground at maximum speed, speed is constant so no net force on car.

$\therefore$ Force of engine F must equal restive force.

$\therefore$ $F = 1400\,N$. Power $P = Fv$ where v is the velocity.

$\therefore$ $v = P/F = \dfrac{44000}{1400} = 31.43\,m/s$.

(ii) On incline, force of engine must equal resistive force plus component of gravity down plane. $\therefore F = 1400 + mg\sin\theta$.

Since $\tan\theta = 1/20$, $\sin\theta = \dfrac{1}{\sqrt{401}} = 0.04994$

$\therefore$ $F = 1400 + 1100 * 9.81 * 0.04994 = 1939\,N$

$\therefore$ $v = P/F = \dfrac{44000}{1939} = 22.69\ \mathrm{ms}^{-1}$.

2. Kinetic energy at start $= \dfrac{1}{2}mv^2 = 0.5 * 0.1 * 8 * 8 = 3.2\,J$

Change in height to halfway up = 0.5 m.

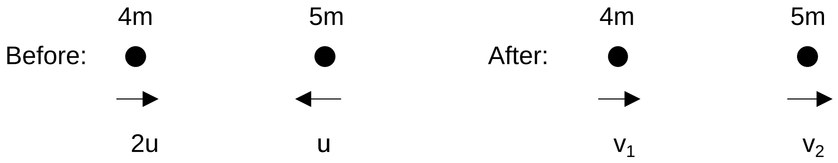
$\therefore$ Loss of potential energy $= mgh = 0.1 * 9.81 * 0.5 = 0.491$ J

$\therefore$ Kinetic energy = 2.710 J. $\Rightarrow v = \sqrt{2 * 2.71/0.1} = 7.361\ \mathrm{ms}^{-1}$.

Similarly at top, loss of PE = 0.982 J so KE = 2.218 J.

$\Rightarrow v = \sqrt{2 * 2.218/0.1} = 6.66\ \mathrm{ms}^{-1}$.

3.



Conservation of momentum: $8mu - 5mu = 4mv_1 + 5mv_2$

$$\Rightarrow \quad 4v_1 + 5v_2 = 3u \qquad (1)$$

Law of restitution: $v_2 - v_1 = \frac{1}{2}(3u) \quad \Rightarrow \quad 4v_2 - 4v_1 = 6u \qquad (2)$

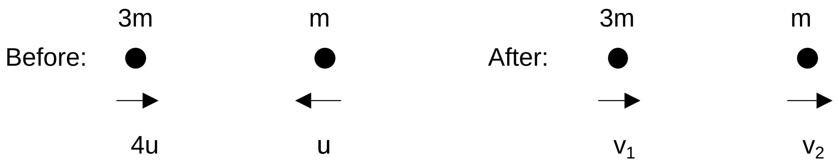
(1) + (2) gives: $9v_2 = 9u \quad \Rightarrow \quad v_2 = u,$ so $v_1 = -\frac{1}{2}u.$

Kinetic energy before: $2m(2u)^2 + \frac{5}{2}mu^2 = \frac{21}{2}mu^2$

Kinetic energy afterwards: $2m(\frac{1}{2}u)^2 + \frac{5}{2}mu^2 = \frac{6}{2}mu^2.$

Change in KE is thus $\frac{15}{2}mu^2.$

4.



Conservation of momentum: $3v_1 + v_2 = 11u \qquad (1)$

Law of restitution: $v_2 - v_1 = u \quad (2)$ so $4v_1 = 10u$

Hence $v_1 = \frac{5}{2}u$ and $v_2 = \frac{7}{2}u.$

Kinetic energy before: $\frac{1}{2}.\,3m(4u)^2 + \frac{1}{2}mu^2$

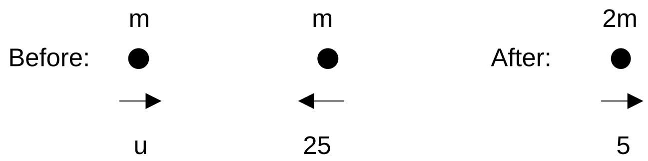
$$= \quad 24mu^2 + \frac{1}{2}mu^2 \quad = \quad \frac{49}{2}mu^2.$$

Kinetic energy afterwards: $\frac{1}{2}.\,3m(\frac{5}{2}u)^2 + \frac{1}{2}m(\frac{7}{2}u)^2$

$$= \quad \frac{1}{8}(75+49)mu^2 \quad = \quad \frac{1}{8}(124)mu^2 \quad = \quad \frac{31}{2}mu^2\ .$$

Change in KE is thus $\frac{1}{2}(49-31)mu^2 \quad = \quad \frac{1}{2}.\,18mu^2 = 9mu^2.$

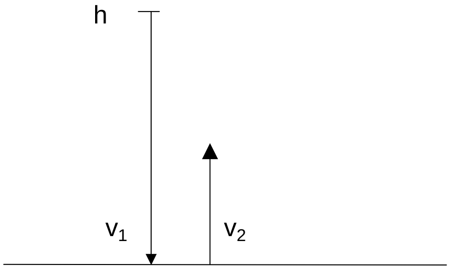
5.



Conservation of momentum: $mu - 25m = (2m)5$

Hence $u = 35 > 30$ so YES.

6.



Just before collision with the ground, by conservation of energy:

$$mgh = \tfrac{1}{2}mv_1^2 \quad \text{so} \quad v_1^2 = 2gh.$$

After hitting the ground the velocity is $v_2 = ev_1$ upwards.

It now rises to a height h_2 where, by conservation of energy again we have

$$mgh_2 = \tfrac{1}{2}mv_2^2.$$

But $h_2 = \frac{1}{2}h$, so $v_2^2 = hg$, and $v_2^2 = e^2v_1^2$.

Hence $hg = e^2v_1^2 = e^2(2gh)$ and thus $e = \frac{1}{\sqrt{2}}$.

Let time for the first bounce be T_1 and let height be y.

At start: $t = 0$, $y = 0$ and $u = +v_2$.

At end: $t = T_1$, $y = 0$ and $v = -v_2$.

Acceleration $a = -g$. Using $v = u + at$ we get:

$$-v_2 = v_2 - gT_1 \quad \text{so} \quad T_1 = \frac{-2v_2}{-g} = +2\sqrt{\frac{h}{g}}.$$

Immediately afterwards $v = v_3$ upwards

where $v_3 = ev_2 = \frac{1}{\sqrt{2}}v_2 = \sqrt{\frac{hg}{2}}$.

Let time for second bounce be T_2.

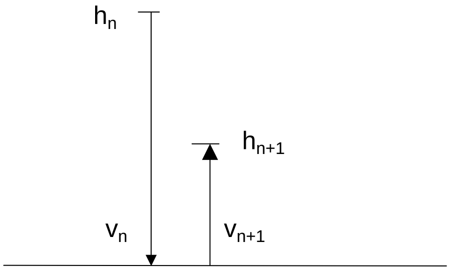
At start: $t = 0$, $y = 0$ and $u = +v_3$.

At end: $t = T_2$, $y = 0$ and $v = -v_3$.

Acceleration $a = -g$. Using $v = u + at$ we get:

$-v_3 = v_3 - gT_2$ so $T_2 = \frac{-2v_3}{-g} = +2\sqrt{\frac{h}{2g}}$.

7.



If at the top of the bounce the height is h_n the speed with which it strikes the ground next is v_n where $v_n^2 = gh_n$ by conservation of energy.

Immediately afterwards the speed is $v_{n+1} = ev_n$ upwards. It now rises to height h_{n+1} where, by conservation of energy again, $v_{n+1}^2 = gh_{n+1}$.

But $v_{n+1}^2 = r^2 v_n^2 = r^2 g h_n$ so $h_{n+1} = r^2 h_n$.

Hence $h_1 = r^2 h_0$, $h_2 = r^2 h_1$, $h_3 = r^2 h_2^2$ etc.

Distance from drop to first hitting floor is $h = h_0$. Distance travelled from first hit to second hit (first bounce) is $2h_1 = 2r^2 h_0 = 2r^2 h$. Distance travelled between second and third hits (second bounce) is $2h_2 = 2r^2 h_1 = 2r^4 h$. Distance travelled in third bounce is $2h_3 = 2r^2 h_2 = 2r^6 h$ etc.

Hence total distance travelled is:

$h + 2h_1 + 2h_2 + 2h_3 + \ldots = h + 2(h_2 + h_3 + h_4 + \ldots)$

$= h + 2hr^2(1 + r^2 + r^4 + r^6 + r^8 + \ldots)$

$= h + \frac{2hr^2}{1 - r^2}$ (Sum of geometric progression).

$= \frac{h(1 - r^2) + 2hr^2}{1 - r^2} = \left(\frac{1 + r^2}{1 - r^2}\right) h.$