Introduction to Mechanics (0J2)

Example Sheet 1 – solutions

1. Given $u = 0$, $v = \dfrac{270 \times 1000}{60 \times 60}$ ms^{-1} and $t = 40$ we have

$$a = \frac{v-u}{t} = \frac{2700}{36 \times 40} = 1.875ms^{-2}$$

$$s = ut + \frac{1}{2}at^2 = \frac{1}{2} \times 1,875 \times 40^2 = 1500m$$

$\therefore \frac{1}{2}$ runway used.

2. AB: $a = 4ms^{-2}, \quad u = 0 \qquad \therefore s = \frac{1}{2}at^2 \qquad \therefore s_1 = 2t^2.$

Also $v = u + at = 4t$. This is u for BC.

BC: $a = 2, u = 4t$. Using $s = ut + \frac{1}{2}at^2$

$$s_2 = 4t^2 + t^2 = 5t^2$$

$$\therefore s_1 = \frac{2}{5}s_2.$$

Also $v = 30$ for BC $\quad \therefore 30 = 4t + 2t \qquad \therefore t = 5.$

$\therefore$ Time from A to C is 10 secs.

Distance AC is $s_1 + s_2 = 2t^2 + 5t^2 = 7t^2 = 175$ metres.

3. XY: $s = 2000, t = 100$. Let initial velocity be u_1 and final velocity be v_1.

$$s = u_1t + \tfrac{1}{2}at^2$$

$$\therefore 2000 = 100u_1 + 5000a \qquad (1)$$

Also $v_1 = u_1 + at = u_1 + 100a$.

YZ: $s = 2000, t = 150$, and initial velocity is $u_1 + 100a$. Using $s = ut + \frac{1}{2}at^2$:

$$2000 = 150(u_1 + 100a) + \frac{22500}{2}a$$

$$2000 = 150u_1 + 26250a \qquad (2)$$

$(2) - \frac{3}{2}(1)$ gives:

$$-1000 = 0 + 18750a$$

$$\therefore a = -\frac{4}{75} ms^{-2}.$$

From (1) $u_1 = 20 - 50a = \dfrac{68}{3} ms^{-1}$.

Starting from X until it comes to rest:

$$s = \frac{v^2 - u^2}{2a} = \frac{0 - \frac{68^2}{9}}{-\frac{8}{75}} = \frac{68^2 \times 75}{9 \times 8} = 4816\frac{2}{3} m.$$

Distance beyond Z is $4816\frac{2}{3} - 4000 = 816\frac{2}{3} m$.

4. $s = 30$ when $t = 2$ <u>and</u> when $t = 6$.

$$\therefore 30 = ut + \tfrac{1}{2}at^2 = 2u + 2a \qquad (1)$$

and also $\quad 30 = 6u + 18a \qquad (2)$

From (1) and (2) $\quad a = -5ms^{-2}, \quad u = 20ms^{-1}$.

Arrives back at O when $s = 0$

$$\therefore 0 = 20T - 5\tfrac{1}{2}T^2$$

$$\therefore T = 0 \text{ (start) or } T = 8\text{sec}.$$

5. On the way down $s = ut + \frac{1}{2}at^2$

$$\therefore -h = 0 - \frac{1}{2}gt_1^2 \quad \text{and so} \quad t_1 = \sqrt{\frac{2h}{g}}.$$

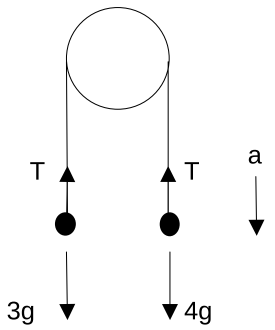
On the way up $t_2 = \dfrac{h}{c}$. The total time $T = t_1 + t_2$ so

$$T = \sqrt{\frac{2h}{g}} + \frac{h}{c} \qquad \therefore \sqrt{\frac{2h}{g}} = T - \frac{h}{c}$$

$$\frac{2h}{g} = (T - \frac{h}{c})^2 = T^2 - \frac{2h}{c}T + \frac{h^2}{c^2}$$

$$\therefore h^2 - 2hc(T + \frac{c}{g}) + c^2T^2 = 0.$$

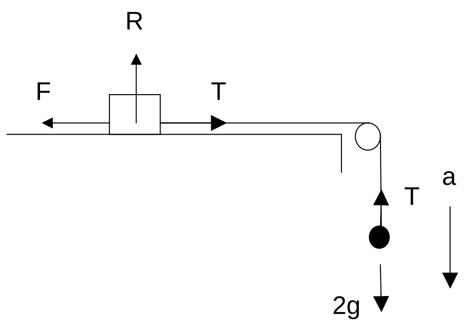
6. Apply Newton's second law to each mass:



$$\begin{aligned} \text{mass 3kg:} \quad 3a &= T - 3g \\ \text{mass 4kg:} \quad 4a &= 4g - T \end{aligned}$$

Adding gives $\quad 7a = g \quad$ so $\quad a = \frac{1}{7}g.$

7. Apply Newton's second law to each mass:



$$\begin{aligned} \text{mass 10kg:} \quad 10a &= T - F \\ \text{mass 2kg:} \quad 2a &= 2g - T \end{aligned}$$

Adding gives $\quad 12a = 2g - F \quad$ so $\quad a = \frac{1}{12}(2g - F).$

Now use $s = ut + \frac{1}{2}at^2$:

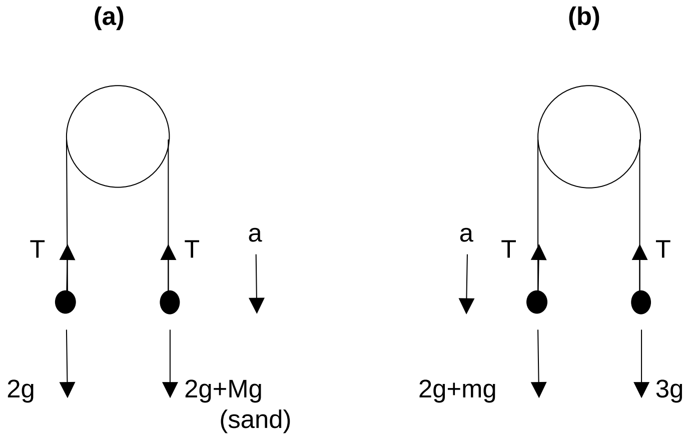
$2 = 0 + \frac{1}{2}\frac{1}{12}(19.62 - F)4$

(taking $g = 9.81ms^{-2}$).

$$\therefore F = 19.62 - 12 = 7.62 \text{ N.}$$

Work $= F\times$ distance $= 7.62 \times 2 = 15.24$ Joules.

8.



(a) Apply Newton's second law to each mass:

$$\text{mass 2kg:} \qquad 2a = T - 2g$$

$$\text{mass 2kg plus M kg of sand:} \qquad (2+M)a = (2+M)g - T$$

Adding gives $\quad (M+4)a = Mg \quad$ so $\quad a = \dfrac{M}{M+4}g.$

If $a = \dfrac{1}{5}g$ then $\dfrac{M}{M+4} = \dfrac{1}{5} \quad \Rightarrow M = 1$ kg.

(b) Similarly

$$\text{mass 2kg plus m kg of sand:} \qquad (2+m)a = (2+m)g - T$$

$$\text{mass 2kg plus 1 kg of sand:} \qquad 3a = T - 3g$$

Adding gives $\quad (m+5)a = (m-1)g \quad$ so $\quad a = \dfrac{m-1}{m+5}g.$

Also $T = 3(a+g)$ and when it snaps $T = 40$ so

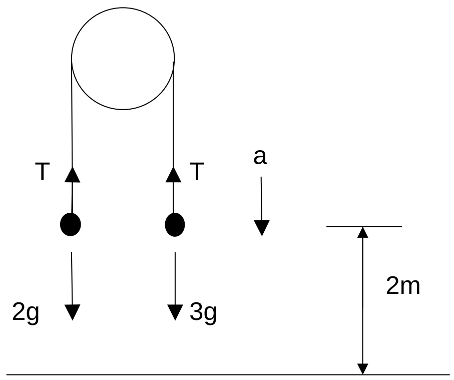
$$3\left(\frac{m-1}{m+5}+1\right)g = 40 \quad \Rightarrow \frac{3}{m+5}(2m+4)g = 40$$

$$\therefore \quad (6m+12) \times 9.81 = 40m + 200 \qquad (\text{Taking } g = 9.81ms^{-2}).$$

$$\therefore \quad 18.86m = 82.28 \quad \Rightarrow \quad m = 4.36\text{kg}.$$

and $a = \dfrac{3.36}{9.36} \times g = 3.52 \text{ ms}^{-2}$.

9. Apply Newton's second law to each mass:



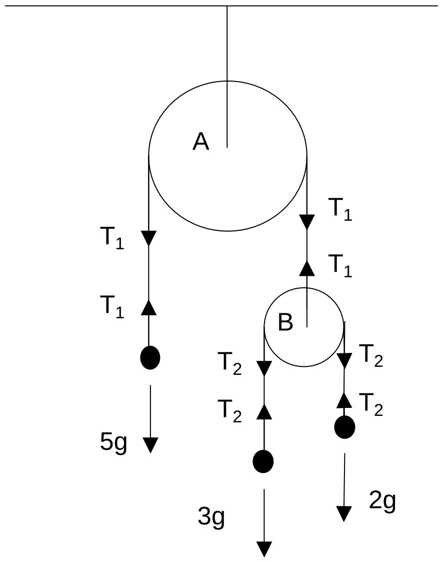
$$\begin{aligned} \text{mass 2kg:} \quad & 2a = T - 2g \\ \text{mass 3kg:} \quad & 3a = 3g - T \end{aligned}$$

Adding gives $5a = g$ so $a = \dfrac{1}{5}g = 1.962s^{-2}$.

Using $s = ut + \frac{1}{2}at^2$ gives $2 = 0 + \frac{1}{2} \times 1.962t^2$

$\Rightarrow \quad t^2 = 2.039 \quad \Rightarrow t = \sqrt{2.039} = 1.428$ s.

10.



Take tensions as shown in diagram. Pulley B has no mass so total force on it must be zero (F=ma).

$T_1 = 2T_2 \qquad (1)$

If 5kg mass moves upwards by an amount x then its acceleration is $\ddot{x}\left(=\dfrac{d^2x}{dx^2}\right)$ so by Newton's second law $5\ddot{x} = T_1 - 5g$ (as x is upwards) and so, using (1):

$$5\ddot{x} = 2T_2 - 5g \qquad (2)$$

If 5kg mass moves up by x then pulley B moves down by x. Let the 3kg mass move a distance y downwards <u>relative</u> to pulley B. Clearly the 2kg moves a distance y upwards <u>relative</u> to B. Hence

3kg mass moves distance $x+y$ downwards with acceleration $\ddot{x}+\ddot{y}$ downwards.

2kg mass moves distance $x-y$ downwards with acceleration $\ddot{x}-\ddot{y}$ downwards.

Applying Newton's second law for each mass:

$3(\ddot{x}+\ddot{y}) = 3g - T_2 \qquad (3)$

$2(\ddot{x}-\ddot{y}) = 2g - T_2 \qquad (4)$

$2\times(3) - 3\times(4)$ gives $\quad 12\ddot{x} = 12g - 5T_2. \qquad (5)$

Solve simultaneous equations (2) and (5) for T_2 by taking $5\times(5) - 12\times(2)$ to get:

$$0 = 120g - 49T_2 \quad \Rightarrow T_2 = \frac{120}{49}g.$$

Using (1) $\quad T_1 = \dfrac{240}{49}g.$