Introduction to Mechanics (0J2)
Example Sheet 6

1. A block with weight 40 N lies on a rough plane which is inclined at an angle of 30° to the horizontal. The coefficient of friction between the block and the plane is $\frac{3}{4}$. Show that in the absence of any external forces the block does not slip down the plane if it is initially at rest.

What is the smallest force parallel to the plane which can be applied (a) to make the block just begin to move down the plane; and (b) to make it just begin to move up the plane.

In an application, forces can only be applied horizontally. What is the smallest horizontal force which can be applied to make the block just begin to move down the plane.
[*Ans:* (a) $5(3\sqrt{3}-4)$; (b) $5(3\sqrt{3}+4)$; (c) $40(3\sqrt{3}-4)/(4\sqrt{3}+3)$.]

2. Two particles, A and B, with weights 30 N and 60 N respectively, lie on the line of greatest slope on a rough plane inclined at an angle of 30° to the horizontal with B above A. The coefficient of fiction between the particles and the plane is $\frac{1}{5}$. The particles are connected by a light inextensible string and a force of P N acting directly up the slope is applied to B. The system is in equilibrium with A and B on the point of moving up the plane. Find the value of P and the tension in the string.
[*Ans:* $P = 9(5+\sqrt{3})$, $T = 3(5+\sqrt{3})$.]

3. Forces represented by $5\mathbf{i}$, $4\mathbf{i}+2\mathbf{j}$, $-3\mathbf{j}$ and $\mathbf{i}-\mathbf{j}$ act through points with position vectors $\mathbf{i}+\mathbf{j}$, $\mathbf{i}-\mathbf{j}$, $3\mathbf{j}$ and $4\mathbf{i}$ respectively. Find their resultant moment (a) about the origin; and (b) about the point $-2\mathbf{i}-\mathbf{j}$.
[*Ans:* (a) -3; (b) -17.]

4. A light ladder leans against a smooth vertical wall at an angle of 30° to the horizontal. A load of 800 N is placed $\frac{3}{4}$ of the way up the ladder. If the ladder rests on a rough horizontal surface which prevents slipping, find the magnitude and direction of the (total) reaction between the ladder and the ground.
[*Ans:* $200\sqrt{43}$, $\tan^{-1}\left(\frac{4}{3\sqrt{3}}\right)$.]

5. A light rod AB of length 2ℓ has a weight W attached at its mid-point. The rod rests in equilibrium with the end A on rough horizontal ground and the end B in contact with a smooth vertical wall. If AB makes an angle α with the horizontal, where $\tan\alpha = \frac{4}{3}$, find the least possible value of μ (the coefficient of friction between the rod and the ground) for equilibrium to be preserved.

If $\mu = \frac{1}{2}$ find the distance from A to the highest point of the rod at which another load of weight W can be attached without equilibrium being disturbed.
[*Ans:* $\mu = \frac{3}{8}$; $h = \frac{5}{3}\ell$.]

6. One end of a light rod is attached to a vertical wall by a hinge, and a weight of 20 N attached is attached to the mid-point of the rod. A force acts at the unhinged end to keep the rod and weight in equilibrium with the rod at an angle of 30° to the (upward) vertical. By taking moments (or otherwise) find the magnitude of this force if (a) it is perpendicular to the rod; and (b) if it is horizontal.
[*Ans:* (a) $F = 5$ N; (b) $F = 10/\sqrt{3}$ N.]

7. A light beam $ABCD$ of length 3ℓ rests on supports at B and C with $AB = BC = CD = \ell$. A particle of weight W is placed at a point P on the beam. If a particle of weight 1 N is placed at the end D the beam just tilts. If this particle is removed and placed at the other end of the beam (i.e. at A) the reactions at the supports are equal. By taking moments about the supports, find W and the distance PC. [**Hint:** what does it mean to say that the beam *just tilts*].
[*Ans:* $W = 5$, $PC = \frac{1}{5}\ell$.]

8. A light beam AB of length $20m$ rests horizontally on two supports at C and D with $AC = DB = 4m$. A weight W is placed on the beam at P. The greatest mass that could be hung from A without upsetting the beam and weight is 8 kg, whilst the greatest mass that could be hung from B is 10 kg. Find the weight W and the distance AP.
[*Ans:* $W = 6g$ N, $AP = 9\frac{1}{3}$.]