Introduction to Mechanics (0J2)
Example Sheet 2

Take the acceleration due to gravity to be $9.81\ ms^{-2}$.

1. A car of mass $1100\,kg$ has a maximum power output of $44\,kW$. The resistive forces (friction and air resistance) are constant with total $1400\,N$. Find the maximum speed of the car
(i) On level ground.
(ii) Up an incline of gradient 1 in 20.
[*Ans:* (i) 31.43 ms^{-1} (ii) 22.58 ms^{-1}.]

2. A smooth wire is bent into a circle of radius $0.5\,m$ with the plane of the circle vertical. A particle of mass $100\,gm$ is attached to and moves along the wire. It is started moving with velocity 8 ms^{-1} at the lowest point. What is its speed
(i) When it is moving vertically upwards.
(ii) When it reaches the top.
[*Ans:* (i) 7.36 ms^{-1} (ii) 6.66 ms^{-1}.]

3. A ball of mass $4m$ is moving with speed $2u$ and collides with a second ball with mass $5m$ and speed u in the opposite direction. If the coefficient of restitution is $\frac{1}{2}$, find the velocities of the balls after impact and the loss in total energy of the system due to the collision.
[*Ans:* $-\frac{1}{2}u$, u, $\frac{15}{2}mu^2$.]

4. A small smooth sphere of mass $3m$, moving with speed $4u$, collides directly with another smooth sphere of mass m, moving in the opposite direction with speed u. The coefficient of restitution between the spheres is $\frac{1}{5}$. Find the speed of each sphere after the impact and show that the energy lost in the impact is $9mu^2$.
[*Ans:* $\frac{5}{2}u$, $\frac{7}{2}u$.]

5. Two cars of equal mass are in a head-on collision, and immediately after collision the two cars are linked together with speed 5 kmh^{-1} towards the right. The cars are on a road with a 30 kmh^{-1} speed limit, and the driver in the car which was initially travelling to the left can prove she was moving at a speed of 25 kmh^{-1}. Was the other driver exceeding the speed limit at the time of the crash?
[*Ans:* Yes!]

6. A ball of mass m is dropped from a height h onto a horizontal floor and it rebounds to a maximum height $\frac{1}{2}h$. Calculate the value of the coefficient of restitution, the time between the first and second bounces and the time between the second and third bounces.
[*Ans:* $\frac{1}{\sqrt{2}}$, $2\sqrt{\frac{h}{g}}$, $2\sqrt{\frac{h}{2g}}$.]

7. A ball is dropped vertically under gravity from a height h above the floor. The coefficient of restitution between the ball and the floor is r, $0 < r < 1$. If h_n is the maximum height of the n^{th} bounce (with $h_0 = h$) show that $h_{n+1} = r^2 h_n$. Using induction or otherwise, show that this implies that $h_n = r^{2n}h$. Hence deduce that the total distance travelled by the ball as it bounces up and down is

$$\left(\frac{1+r^2}{1-r^2}\right)h$$

[**Hint:** You may find it useful to recall that if $0 < k < 1$ then the sum of the geometric series $1 + k + k^2 + k^3 + \ldots$ is $1/(1-k)$.]