Introduction to Mechanics (0J2)
Example Sheet 1

1. An airport has a straight, level runway which is 3000 m long. During take off, an aircraft starting from rest moves along the runway with constant acceleration, reaching its take off speed of 270 km per hour after 40 s. Find (i) the acceleration of the aircraft during take off in ms^{-2}; and (ii) the fraction of the runway used during take off.
[*Ans:* (i) 1.875ms^{-2}; (ii) $\frac{1}{2}$ of the runway is used.]

2. A particle moves on the straight line ABC. Starting from rest it moves from A to B with a constant acceleration 4 ms^{-2} and from B to C with a constant acceleration of 2 ms^{-2}. The times taken to travel from A to B and B to C are equal. Show that $AB = \frac{2}{5}BC$. If the speed on arrival at C is 30 ms^{-1}, find the time taken to travel from A to C and the distance AC.
[*Ans:* 10 s, 175 m.]

3. A car is timed between three successive points X, Y and Z where $XY = YZ = 2$ km. It takes 100 s to travel from X to Y and 150 s to travel from Y to Z. Given that the deceleration of the car is constant, calculate the magnitude of this deceleration. Find also how far the car travels beyond Z before it comes to rest.
[*Ans:* 4/75 ms^{-2}, $816\frac{2}{3}$ m.]

4. A particle moves along a straight line with constant deceleration, starting at O when $t = 0$ with speed u. Find the magnitude of this deceleration given that the particle is at the same point, P, which is 30 m from O, when $t = 2$ s and when $t = 6$ s. Find the time at which the particle arrives back at O.
[*Ans:* 5 ms^{-2}, 8 s.]

5. A stone is dropped down a well from rest, and the sound of the stone hitting the water at the bottom of the well is heard after time T. Show that the depth of the well, h, satisfies the quadratic equation

$$h^2 - \frac{2c^2}{g}\left(1 + \frac{gT}{c}\right)h + c^2T^2 = 0$$

where c is the speed of sound, and g is the acceleration due to gravity.

6. Masses of 3 kg and 4 kg are joined by a string placed over a light smooth pulley so that each mass hangs vertically. Find the acceleration with which the larger mass descends.
[*Ans:* $\frac{1}{7}g$ ms^{-2}.]

7. A mass of 10 kg is on a rough table 2 m from the edge. It is connected by a light inextensible string running at right angles to the edge of the table, to a 2 kg mass which hangs vertically (the string runs around a smooth pulley at the edge of the table). The system starts from rest and after 2 s the 10 kg mass reaches the edge of the table. Find the friction force exerted by the table on the mass, and the work done by this force. [*Take the acceleration due to gravity to be* 9.81 ms^{-2}.]
[*Ans:* 7.62 N, 15.24 J.]

8. Two empty buckets of mass 2 kg have their handles connected by a rope which passes over a smooth pulley. What mass of sand must be poured into one of the buckets for them to move with an acceleration of $\frac{1}{5}g$ ms^{-2}? Take $g = 9.81$ ms^{-2}.

The maximum tension that the rope can sustain is 40 N. What is the largest mass of sand which can be put into the other (empty) bucket without the rope snapping (and without changing the mass of sand in the first bucket). What is the acceleration of the buckets in this case?
[*Ans:* 1 kg, 4.36 kg, 3.52 ms^{-2}.]

9. Two particles of mass 2 kg and 3 kg are connected by a light inextensible string passing over a fixed pulley. Initially the system is at rest with the strings taut and vertical, and both particles 2 m above the ground. Taking $g = 9.81$ ms^{-2}, show that the time which elapses before the 3 kg mass hits the ground is 1.428 s.

10. This question is hard. Its solution will not be discussed in the tutorial but will be posted on the web along with the other solutions.

A light inextensible string passes over a light smooth pulley A which is fixed to a ceiling. The string has a mass of 5 kg fixed to one end and a light smooth pulley B at the other end. A light inextensible string passes over B and carries masses 2 kg and 3 kg at its ends The system is in motion with all the masses moving vertically. Find the tension in the moving strings.

Hint: If the 5 kg mass moves up by x then pulley B moves down by x.
Let the 3 kg mass move a distance y downwards relative to pulley B.
Clearly the 2 kg moves a distance y upwards relative to B.
Hence the 3 kg mass moves distance $x + y$ downwards and so
its acceleration is $\ddot{x} + \ddot{y}$ downwards.
Similarly the 2 kg mass moves distance $x - y$ downwards and
so its acceleration is $\ddot{x} - \ddot{y}$ downwards.

[*Ans:* $\frac{120}{49}g$ and $\frac{240}{49}g$.]