0J2 June 2012 — Solutions

1. (a) First part $a = 0.75$, $u = 0$ $\quad \therefore v = u + at = 0.75t$.

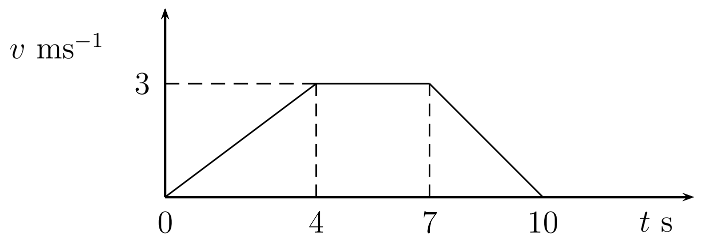
At $t = 4$, $v = 3$.

Second part $a = 0$, therefore v is constant at 3.

Third part $a = -1$, $u = 3$ $\quad \therefore v = u + at = 3 - t$

(Measuring t from the start of the third part).

Hence



(4 marks)

(b) Distance in first part is $s = (u+v)t/2 = (0+3) \times 4/2 = 6$ m.

Distance in second part is $s = vt = 3 \times 3 = 9$ m.

Distance in third part is $s = (u+v)t/2 = (3+0) \times 3/2 = 4.5$ m.

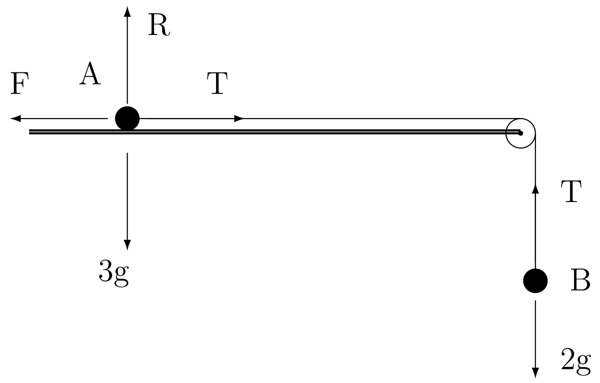
Total distance is $6 + 9 + 4.5 = 19.5$ m. (3 marks)

(c) Total force on the lift is $T - mg$, where T is the tension in the cable.

Also force $F = ma = 400 \times 0.75 = 300$ N.

$\therefore \quad T = 300 + mg = 300 + 400 \times 9.81 = 4224$ N. (3 marks)

2. Drawing in the forces with normal reaction R, friction F and tension T gives



(a) Total vertical force on B is $mg - T$ where T is the tension. Mass is 2 kg and acceleration is 0.9 ms^{-2} so using Newton's 2nd law gives

$$2g - T = 2 \times 0.9$$

$$\Rightarrow \quad T = 2g - 1.8 = 19.62 - 1.8 = 17.82 \text{ N} \qquad \text{(3 marks)}$$

(b) Horizontal force on A is $T - F$ where F if the friction force. Mass is 3 kg so using Newton's second law gives

$$T - F = ma \quad \Rightarrow \quad 17.82 - F = 3 \times 0.9$$

$$\Rightarrow \quad F = 17.82 - 2.7 = 15.12 \text{ N} \qquad \text{(3 marks)}$$

(c) No motion of A in the vertical direction, so total force is zero and resolving vertically for A: $R = 3g = 29.43$ N. (1 mark)

(d) $F = \mu R$ so

$$\mu = \frac{F}{R} = \frac{15.12}{29.43} = 0.514 \qquad \text{(1 mark)}$$

(e) Distance travelled is $s = 0.3$. $u = 0$, $a = 0.9$ so using $s = ut + \frac{1}{2}at^2$ gives

$$0.3 = 0 + \frac{1}{2} \times 0.9 \times t^2 \quad \Rightarrow \quad t^2 = \frac{0.3}{0.45}$$

$$\Rightarrow \quad t = \sqrt{0.6667} = 0.8165 \text{ s} \qquad \text{(2 marks)}$$

3.

(a) i. Conservation of momentum gives:

$$m \times 2 + 3 \times (-2) \;=\; m \times (-0.5) + 3 \times 0.5$$

since -0.5 is the velocity of A (to the left) after the collision, and 0.5 is the velocity of B (to the right) after the collision.

$$(m-3) \times 2 \;=\; (m-3) \times (-0.5)$$

$$\Rightarrow \quad (m-3) \times 2.5 \;=\; 0 \quad \Rightarrow \quad m \;=\; 3\text{kg} \qquad \text{(3 marks)}$$

ii. Kinetic energy before collision is $\frac{1}{2}m \times 2^{\,2} + \frac{1}{2}3 \times (-2)^2 \;=\; 12$ J.
Kinetic energy after collision is $\frac{1}{2}m \times (-0.5)^2 + \frac{1}{2}3 \times (0.5)^2 \;=\; 0.75$ J.
Thus loss in KE is 11.25 J. (2 marks)

(b) Conservation of momentum gives

$$M \times 2 + 3 \times (-2) \;=\; M \times v + 3 \times v$$

where v is the velocity after the collision.
If they are moving to the right then $v = 0.5$ so

$$(M-3) \times 2 \;=\; (M+3) \times (0.5)$$

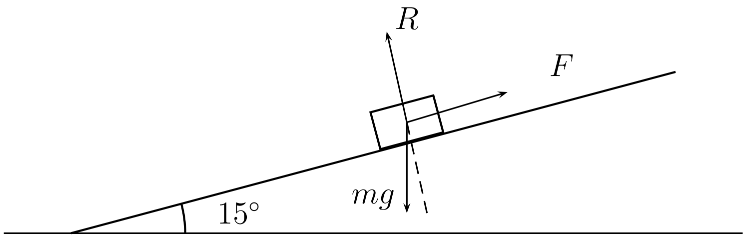
$$\Rightarrow \quad 1.5M \;=\; 3\times 2.5 \quad \Rightarrow \quad M \;=\; \frac{7.5}{1.5} \;=\; 5 \text{ kg} \qquad \text{(3 marks)}$$

If they are moving to the left then $v = -0.5$ so

$$(M-3) \times 2 \;=\; (M+3) \times (-0.5)$$

$$\Rightarrow \quad 2.5M \;=\; 3\times 1.5 \quad \Rightarrow \quad M \;=\; \frac{4.5}{2.5} \;=\; 1.8 \text{ kg} \qquad \text{(2 marks)}$$

4.



(a) Angle is 15°, acceleration $a = 2.5$ ms^{-2}, mass $m = 800$ kg.
Let force of engine be F.

Resolving parallel to the plane, using Newton's law (f=ma)

$$\begin{aligned} F - mg\sin 15^\circ &= ma \\ \text{so} \quad F &= 800 \times 9.81 \times 0.2598 + 800 \times 2.5 \\ &= 2031 + 2000 = 4031 \text{ N.} \end{aligned}$$ (3 marks)

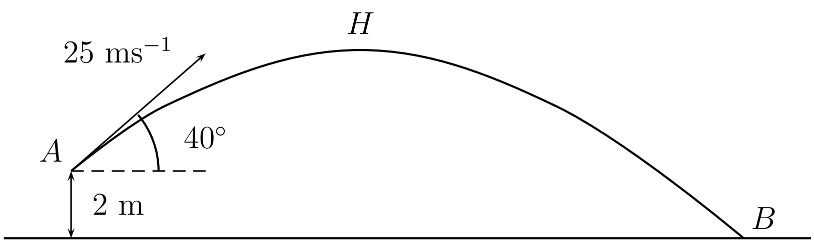
(b) Power is force × velocity.
At start velocity $u = 0$ so power is zero. (1 mark)
At end velocity is $v = u + at = 0 + 2.5 \times 30 = 75$ ms^{-1}.
so power is $4031 \times 75 = 302341$ watts. (1 mark)

(c) Work done is force × distance moved.
Distance moved is $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 2.5 \times 30^2 = 1125$ m.
so work $W = 4031 \times 1125 = 4,534,975$ J. (3 marks)
(Can also be done using energy)

(d) Resolving parallel to the plane the force is $mg\sin 15^\circ = 2031$ N, acting down the plane.
Therefore deceleration is $2031/800 = 2.53875$ ms^{-2}.
Distance s travelled after engine switched off is given by $v^2 = u^2 + 2as$

so $0 = 75^2 - 2 \times 2.53875s \quad \Rightarrow \quad s = \dfrac{75^2}{5.0775} = 1107.8$ m (2 marks)

5.



(a) Horizontal velocity is $25\cos 40° = 19.15$ ms^{-1}.
Vertical velocity is $25\sin 40° = 16.07$ ms^{-1}. (2 marks)

(b) Vertical motion measuring <u>downwards</u>
$u = -16.07$, $s = 2$, $a = g = 9.81$.
Using $s = ut + \frac{1}{2}at^2$ gives $2 = -16.07t + \frac{1}{2} \times 9.81 \times t^2$. This is a quadratic equation for t

$$4.905t^2 - 16.07t - 2 = 0$$

$$\Rightarrow \quad t = \frac{16.07 \pm \sqrt{258.25 + 39.24}}{9.81} = 3.396 \text{ or } -0.1201$$

Clearly the first of these is correct so $t = 3.396$ s. (3 marks)

(c) Horizontal distance d is the time of flight times the horizontal velocity since there is no acceleration horizontally.
Thus $d = 19.15 \times 3.396 = 65.03$ m. (2 marks)

(d) At H vertical velocity is 0. From A to H, considering the vertical motion, measuring <u>upwards</u> and using $v^2 = u^2 + 2as$ gives

$$0 = (16.07)^2 + 2 \times (-9.81) \times s \quad \Rightarrow \quad s = \frac{(16.07)^2}{19.62} = 13.16$$

Since A is 2 m above the ground, H is 15.16 m above the ground.
(3 marks)

6. (a) Acceleration vector $\mathbf{a}$ is given by

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = -0.4\mathbf{i}\ \mathrm{ms}^{-2} \qquad \text{(1 mark)}$$

(b) Position vector $\mathbf{r}$ is given by

$$\mathbf{r} = \int \mathbf{v}\, dt = (-0.4t^2/2 + r_1)\mathbf{i} + (5t + r_2)\mathbf{j}$$

where r_1 and r_2 are constants of integration.
Since $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j}$ at $t = 0$ we have $r_1 = 3$ and $r_2 = 2$.

$$\therefore \mathbf{r} = [(-0.2t^2 + 3)\mathbf{i} + (5t + 2)\mathbf{j}]\ \mathrm{m} \qquad \text{(3 marks)}$$

(c) Speed is magnitude of velocity. At $t = 10$ velocity is
$\mathbf{v} = -4\mathbf{i} + 5\mathbf{j}$. Magnitude is

$$v = \sqrt{(-4)^2 + 5^2} = 6.403\ \mathrm{ms}^{-1} \qquad \text{(2 marks)}$$

(d) Crosses the Y-axis when the X-coordinate is zero, i.e. when

$$-0.2t^2 + 3 = 0 \quad \Rightarrow \quad t = \sqrt{3/0.2} = \sqrt{15} = 3.873\ \mathrm{s} \qquad \text{(2 marks)}$$

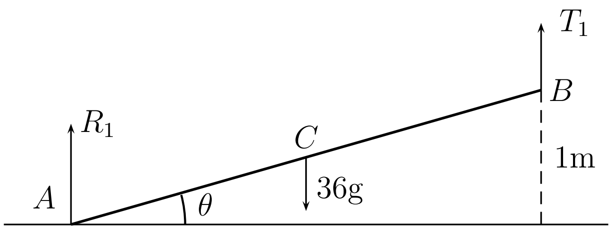
(e) At $t = \sqrt{15}$ the two vectors are

$$\mathbf{r} = (-0.2 \times 15 + 3)\mathbf{i} + (5\sqrt{15} + 2)\mathbf{j} = 0\mathbf{i} + (5\sqrt{15} + 2)\mathbf{j}$$

$$\text{and} \quad \mathbf{a} = -0.4\mathbf{i}$$

Scalar product of these is zero, so they are perpendicular. (2 marks)

7. (a)

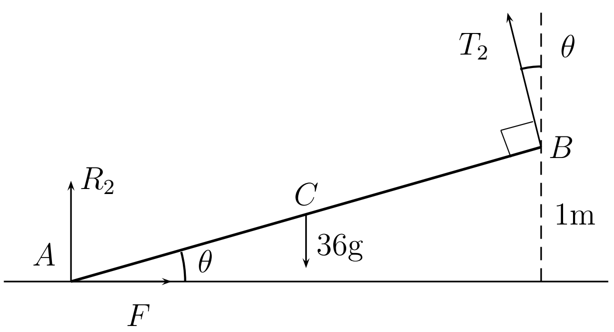


i. Clearly $\sin\theta = 1/3$ so $\theta = \sin^{-1} 1/3 = 19.5°$ (1 mark)

ii. Moments about A: $T_1 \times 3\cos\theta = 36g \times 1.5\cos\theta$
Therefore $T_1 = 18g = 176.6$ N. (2 marks)

iii. Resolve vertically: $T_1 + R_1 = 36g$
$\Rightarrow$ $R_1 = 36g - T_1 = 18g = 176.6$ N. (1 mark)

(b)



i. Moments about A: $T_2 \times 3 = 36g \times 1.5\cos\theta$
Therefore $T_2 = 18g\cos\theta = 166.5$ N. (2 marks)

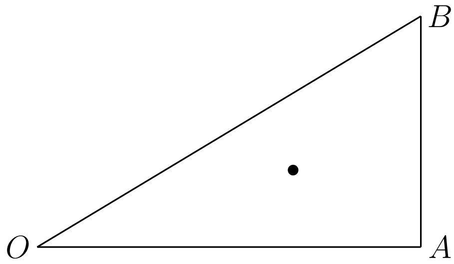
ii. Resolve vertically: $T_2\cos\theta + R_2 = 36g$
$\Rightarrow$ $R_2 = 36g - T_2\cos\theta = 353.2 - 157.0 = 196.2$ N. (2 marks)

iii. Resolve horizontally: $F = T_2\sin\theta = T_2/3 = 55.5$ N. (1 mark)

iv. Since $\dfrac{F}{R_2} = 0.283 < \mu$ the plank will not slide. (1 mark)

8. N.B the value of ρ is not actually needed. It is provided for assistance.

(a)



C of M of a triangle is 1/3 of the way from mid point of one side to the opposite corner (i.e. 1/3 of the way along the median). So it is 1/3 of the way from $(5,0)$ to $(10,6)$. Thus C of M is at (x_1, y_1) where

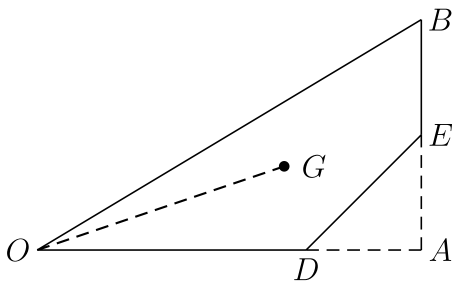
$$x_1 \;=\; 5+\frac{1}{3}5 \;=\; 6\frac{2}{3} \qquad \text{and} \qquad y_1 \;=\; 0+\frac{1}{3}6 \;=\; 2$$

i.e. at $(6\frac{2}{3}, 2)$. (2 marks)

Area is $\frac{1}{2}$ base $\times$ height $\;=\; \frac{1}{2}10\times 6 \;=\; 30$ m^2. (1 mark)

and mass is $m_1 \;=\; 30\times 0.28 \;=\; 8.4$ kg. (1 mark)

(b) N.B the value of ρ is not actually needed. It is provided for assistance.



C of M of removed triangle is at (x_2, y_2) where

$$x_2 \;=\; 7+\frac{1}{3}3 \;=\; 8 \qquad \text{and} \qquad y_2 \;=\; 0+\frac{1}{3}3 \;=\; 1$$

i.e. at $(8,1)$. (2 marks)

Area is $\;=\; \frac{1}{2}3\times 3 \;=\; 4.5$ m^2

and mass is $m_2 \;=\; 4.5\times 0.28 \;=\; 1.26$ kg. (1 mark)

Combining these we get C of M of remaining piece at $G(x_3, y_3)$ where

$$x_3 = \frac{m_1 x_1 - m_2 x_2}{M}; \quad y_3 = \frac{m_1 y_1 - m_2 y_2}{M}$$

where $M = m_1 - m_2 = 7.14$ kg, is the mass of the remaining piece. Thus

$$x_3 = \frac{8.4 \times 20/3 - 1.26 \times 8}{7.14} = 6.43$$

$$y_3 = \frac{8.4 \times 2 - 1.26 \times 1}{7.14} = 2.18 \qquad \text{(2 marks)}$$

(c) When freely suspended from O line OG will be vertical.

Angle between OG and OD is θ where $\tan\theta = \frac{2.18}{6.43} = 0.339$.

Hence $\theta = 0.3269$ rads $= 18.7°$. (2 marks)

9. (a) The resultant force $\mathbf{R}$ is the vector sum of the three forces i.e.:

$$\mathbf{R} \;=\; (2\mathbf{i}+3\mathbf{j})+(-\mathbf{i}+5\mathbf{j})+(9\mathbf{i}-3\mathbf{j}) \;=\; 10\mathbf{i}+5\mathbf{j} \text{ N.} \qquad \text{(1 mark)}$$

Let line of action cross the Y-axis at $(0, a)$. Moment of $\mathbf{R}$ about any point must be same as sum of the moment of the three forces about that point. Choose the origin as the point.

Moment of $F_X\mathbf{i}+F_Y\mathbf{j}$ applied at (x, y) about O is $-F_X y + F_Y x$ so:

Moment of $\mathbf{F}_1$ about O is $M_1 \;=\; -2\times(-8)+3\times 5 \;=\; 31$ Nm
Moment of $\mathbf{F}_2$ about O is $M_2 \;=\; -(-1)\times(-6)+5\times(-4) \;=\; -26$ Nm
Moment of $\mathbf{F}_3$ about O is $M_3 \;=\; -9\times(-7)+(-3)\times 2 \;=\; 57$ Nm
so total is 62 Nm.

Moment of $\mathbf{R}$ applied at $(0, a)$ is $-10\times a+0 \;=\; -10a$.
Hence $-10a \;=\; 62 \quad\Rightarrow\quad a \;=\; -6.2$ so the point is $(0, -6.2)$. (5 marks)

(b) The fourth force $\mathbf{F}_4$ must equal $-\mathbf{R}$ in order to form a couple. So $\mathbf{F}_4 \;=\; -10\mathbf{i}-5\mathbf{j}$. It is applied at $(1, 0)$ and thus has a moment about O given by

$$M_4 \;=\; -(-10)\times 0+(-5)\times 1 \;=\; -5 \qquad \text{(2 marks)}$$

The magnitude of the couple is the total moment of all four forces about O which equals $62-5 \;=\; 57$ Nm. (2 marks)