MATH19852

TWO HOURS

To be Provided by Examinations Office: Mathematical Formula Tables

UNIVERSITY OF MANCHESTER

Mathematics 0J2

DATE: xx June 2012

TIME: 09.45 - 11.45

Foundation Year

Attempt SIX Questions

Calculators: University-approved calculators may be used.

P.T.O.

Take the acceleration due to gravity to be 9.81 ms^{-2} where needed

1. A lift rises from rest with an acceleration of 0.75 ms^{-2} for 4 s. It then travels at constant velocity for 3 s. Finally it decelerates with an acceleration of -1 ms^{-2} for 3 s, at which point it is at rest.

   (a) Draw a graph showing the velocity as a function of time.
   (The graph does not need to be on graph paper.) (4 marks)

   (b) Find the distance travelled by the lift. (3 marks)

   (c) The lift is raised by a single vertical cable. The mass of the lift is 400 kg. Find the tension in the cable during the first 4 seconds of the motion.
   (3 marks)

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2. Two particles A and B are connected by a light inextensible string. Particle A has mass 3 kg and rests on a rough horizontal table. The coefficient of friction is μ. The string is perpendicular to the edge of the table and passes over a smooth pulley at the edge. The other end is attached to particle B which has mass 2 kg and hangs vertically as shown.

The particles are released from rest and move with a constant acceleration of magnitude 0.9 ms^{-2}.

(a) By considering the motion of B, or otherwise, find the tension in the string. (3 marks)

(b) Find the magnitude of the friction force acting on A. (3 marks)

(c) Find the magnitude of the normal reaction force acting on A. (1 mark)

(d) Find μ. (1 mark)

(e) If A is initially 0.3 m from the edge of the table find how long it takes to reach the edge. (2 marks)

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3. Two particles, A and B, are travelling towards each other along a straight horizontal line.
   Particle A has velocity 2 ms^{-1} and mass m kg.
   Particle B has velocity -2 ms^{-1} and mass 3 kg.

   The particles collide.

   (a) If the particles move in opposite directions after the collision, each with speed 0.5 ms^{-1}

      i. Find the value of m. (3 marks)

      ii. Find the loss in kinetic energy in the collision. (2 marks)

   (b) If particle A has mass M and the particles coalesce during the collision, forming a single particle which moves with speed 0.5 ms^{-1}, find the two possible values of M. (5 marks)

4. A car with a mass of 800 kg moves up a hill inclined at an angle 15° to the horizontal with a constant acceleration of 2.5 ms^{-2}.

   (a) Find the force of the engine. (3 marks)

   It starts from rest and travels for 30 s.

   (b) What is the power the engine is producing at the start and at the end? (2 marks)

   (c) Find the velocity at the end and the work done in this time. (3 marks)

   At the end of this time the engine is switched off and the car moves without friction or braking.

   (d) What is the distance moved after the engine is switched off? (2 marks)

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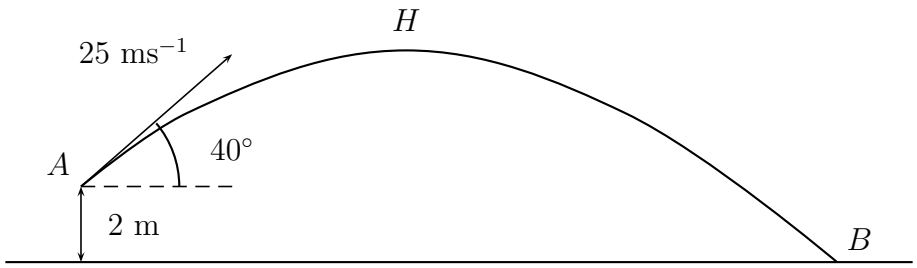
5. A lady, standing on level horizontal ground, throws a ball. When it leaves her hand it is 2 m above the ground, its velocity is 25 ms^{-1} and it is travelling at an angle of 40° above the horizontal. Assume that the ball is a particle and that its weight is the only force that acts on the ball after it has left her hand.



Let A be the point where it leaves her hand, H be the point of maximum height and B be the point where it lands on the ground, as shown.

(a) Calculate the horizontal and vertical components of velocity at A. (2 marks)

(b) Find the time of flight from A to B. (3 marks)

(c) Find the horizontal distance that it travels. (2 marks)

(d) Find the maximum height of the ball above the ground. (3 marks)

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6. A particle moves in 2D with a vector velocity which at time t is
$\mathbf{v} = (-0.4t\mathbf{i} + 5\mathbf{j})$ ms^{-1}.
At time $t = 0$ the particle is at (3, 2).

   (a) Find the acceleration vector at time t. (1 mark)

   (b) Find the position vector at time t. (3 marks)

   (c) Find the speed at time $t = 10$. (2 marks)

   (d) Find the time at which it crosses the Y-axis. (2 marks)

   (e) Show that the acceleration vector is perpendicular to the position vector at time $t = \sqrt{15}$ s. (2 marks)

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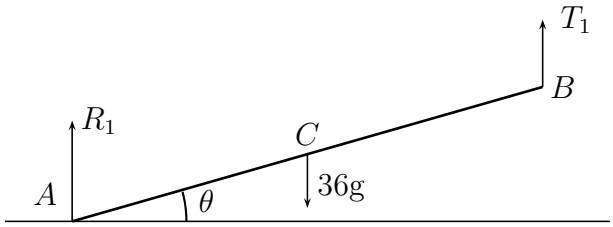
7. A uniform plank of wood of length 3 m and mass 36 kg has one end A resting on a rough horizontal surface with coefficient of friction $\mu = 0.5$. The other end B is attached to a light inextensible rope, which supports the end at a height of 1 m above the surface. C is the mid-point of the plank. The angle between the plank and the horizontal is θ.

   (a) If the the rope is vertical and the system is in equilibrium,

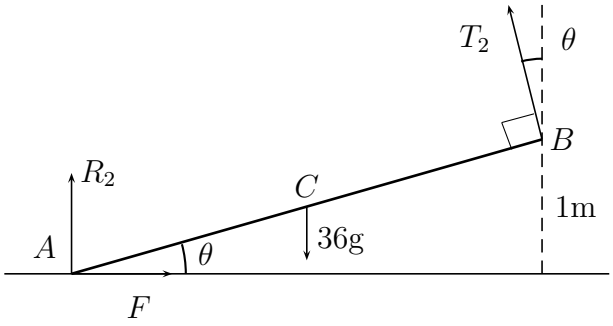


   i. Show that θ is 19.5° (1 mark)

   ii. Find the tension T_1 in the rope. (2 marks)

   iii. Find the normal reaction R_1 at A. (1 marks)

   (b) If the rope is at right angles to the plank



   i. Find the tension T_2 in the rope. (2 marks)

   ii. Find the normal reaction R_2 at A. (2 marks)

   iii. Find the friction force F at A. (1 mark)

   iv. Show that the plank will not slide. (1 mark)

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8. A uniform lamina is in the shape of a triangle with corners at $O(0,0)$, $A(10,0)$ and $B(10,6)$, with lengths measured in metres. The mass per unit area is ρ.

   (a) Show that the centre of mass is at $(6\frac{2}{3}, 2)$. (3 marks)

   A triangular piece with coordinates $D(7,0)$, $A(10,0)$ and $E(10,3)$ is now removed.

   (b) Find the centre of mass of the remaining piece. (5 marks)

   The remaining piece is freely suspended about corner O.

   (c) Find the angle that the line OD makes with the vertical. (2 marks)

9. Three forces act in 2D as follows:

   $\mathbf{F}_1 = 2\mathbf{i} + 3\mathbf{j}$ acts at $(5,-8)$

   $\mathbf{F}_2 = -\mathbf{i} + 5\mathbf{j}$ acts at $(-4,-6)$

   $\mathbf{F}_3 = 9\mathbf{i} - 3\mathbf{j}$ acts at $(2,-7)$

   Forces are measured in Newtons and distances in metres.

   (a) Find the resultant force $\mathbf{R}$ and the point where its line of action crosses the Y axis. (6 marks)

   A fourth force $\mathbf{F}_4$ is now applied at $(1,0)$ so that the four forces together form a couple.

   (b) Find the magnitude of the couple. (4 marks)

 END OF PAPER