Teleoperation with memoryless, monotone, and bounded environments:
A Zames-Falb multiplier approach

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Abstract—Absolute stability of bilateral teleoperation has been investigated under the assumption that both human and environment behave as passive systems. This robust design is required since human and environment must be considered as unknown systems at the design stage. If this approach is widely used in the literature, it leads to conservative conditions. Recently, teleoperation has been described in the integral quadratic constraint (IQC) framework, which provides powerful tools to develop less conservative description of both, human and environment. In this initial work, we consider that the environment can be described by a memoryless, monotone, and bounded nonlinearity. Then, Zames-Falb multipliers can be introduced to relax the conservatism of current state-of-the-art stability conditions. The usefulness of our result is shown in a 2-channel position-force teleoperation; where the well known lack of passivity of a PD-F controller is corrected by the use of Zames-Falb multipliers.

I. INTRODUCTION

Bilateral teleoperation enables humans to reach, manipulate, and also sense the objects located at a distance. In some applications such as space, telesurgery, under sea exploration, etc.: it plays a key role, therefore it has been extensively studied for decades [1], [2], [3]. Overall, it consists of a master device manipulated by an operator, slave device mimicking the master’s behaviours, and a communication medium enabling data transmission between two manipulators [4]. The system has two main performance criteria; transparency that defines how well the environmental contact force or task impedance is transmitted to the operator, and motion tracking. From a control point of view, designing a stable bilateral teleoperation system is a challenging task as it contains highly uncertain human-environment pairs, and time delay in the communication channel. Moreover, there exists a trade off between transparency and stability [5], [6].

Passivity is an energy-based phenomenon: A system is called passive if it is not possible to extract more energy from its ports than what is supplied to. Due to this external description the modelling of a passive system can be model-free [7]. That convenience makes passivity theory the main technique used by many researchers to analyse absolute stability of the bilateral teleoperation as it contains highly variable human and environment, see [8] and references there in. Consequently, each components master, slave, along with others were designed to be passive over and above this passivation was implemented for non-passive elements [9], [10]. Conservatism, however, is the main drawback of this method as passivity of the each element is required. Hereby, performance either never been mentioned or been considered. On the other hand, less conservative designs were obtained with passivity-based Llewellyn two-port network absolute stability conditions [11]. These criteria give exact information about design’s stability while interconnected system needs to be designed as a two port network that connected with passive linear time invariant (LTI) human and environment [12]. Consequently, the design is going to be robust against a wide range of uncertainties, hence performance needs to be sacrificed to obtain this level of robustness such that stability of the systems with simple controllers might not be guaranteed [13], [14].

Willaert et al. [14] have shown that conservativeness of the passivity based design can be reduced by being robust against an upper bound of pure stiff or mass environment, considered to be the worst case conditions in the passivity point of view [15], rather than all passive environments. By restricting the passivity bound in environment side performance of the design was increased under favour of the trade off between performance and robustness. That shows the importance of the assumption has been made to cover the uncertainties, but with Llewellyn’s criteria human and environment need to be defined as LTI passive operators. It would seem, therefore, that further investigations are needed in order to be able to define and analyse different uncertainty sets.

Recently, integral quadratic constraints (IQC) [16] have been used for analysing stability of the bilateral teleoperation by Polat and Scherer in [17]. They have shown that classical stability analyses in teleoperation can be unified into frequency domain inequalities transformed to a search for frequency dependent multipliers by Linear Matrix Inequalities (LMI), see [18] for more detailed and broader information. Also this method, as highlighted in [17], gives chance to extend definitions about uncertainties which play a key role to reduce the conservatism of the existing analyses. On the other hand, the class of Zames-Falb multipliers [19], [20] has recently attracted much attention [21], [22], [23], [24], [25], [26], [27]. This class of multipliers is used to describe memoryless, monotone, and bounded nonlinearities.

The purpose of this paper is to explore stability of the well known 2 channel position-force bilateral teleoperation architecture with Zames-Falb multipliers. Unlike the existing studies, the environment is assumed to be a memoryless, monotone, and bounded nonlinear operator. Therefore, there is no need to pre-design system’s and controller’s parameters.
Then, the positive feedback interconnection of $G$ and $\Delta$ is stable.

A nonlinearity $\phi : \mathcal{L}_2[0, \infty) \to \mathcal{L}_2[0, \infty)$ is said to be memoryless if there exists $N : \mathbb{R} \to \mathbb{R}$ such that $\phi(v)(t) = N(v(t))$ for all $t \in \mathbb{R}$. Henceforth we assume that $N(0) = 0$. A memoryless nonlinearity $\phi$ is said to be bounded if there exists a positive constant $C$ such that $|N(x)| \leq C|x|$ for all $x \in \mathbb{R}$. The nonlinearity $\phi$ is said to be monotone if for any two real numbers $x_1$ and $x_2$ we have

$$0 \leq \frac{N(x_1) - N(x_2)}{x_1 - x_2}.$$ 

**Definition 2 (Zames-Falb Multiplier [19], [20]):** Let $Z$ be a rational transfer function. Then $Z$ belongs to the multiplier class of Zames-Falb multipliers $\mathcal{Z}$, if the following three conditions are satisfied:

- $Z(j\omega) = z_0 - \int_{-\infty}^{\infty} z(t)e^{-j\omega t}dt$ \hspace{1cm} (1)
- $\int_{-\infty}^{\infty} |z(t)|dt < z_0$ \hspace{1cm} (2)
- $z(t) \geq 0$, \hspace{1cm} $\forall t \in \mathbb{R}$ \hspace{1cm} (3)

If a multiplier $Z$ belongs to the above class of multipliers, then it preserves the positivity of any memoryless, monotone, and bounded nonlinearity $\phi$. Loosely speaking, if this multiplier also achieve passivity of $G$, i.e. $GM$ is positive ($M \in \mathcal{Z}$), then the feedback interconnection between $G$ and $\phi$ is stable. In this paper, we will use only the first result:

**Lemma 2 ([20]):** Given a memoryless, monotone and bounded nonlinearity $\phi$ and any $Z \in \mathcal{Z}$, then

$$\int_{-\infty}^{\infty} \left[ \hat{\phi}(j\omega) \right]^* \left[ \begin{array}{c} Z(j\omega) \\ \phi(u)(j\omega) \end{array} \right] \right] dw \geq 0$$ 

(4)

A comparison between the rest of the classes for this type of nonlinearities and the class $\mathcal{Z}$ is given in [28]. In this work, we will parameterise the Zames-Falb multiplier using Szegö’s polynomials. Following [29], $Z \in \mathcal{Z}$ in Definition 2, it can be approximated by

$$Z(j\omega) = z_0 - \sum_{i=0}^{N} \left( \frac{a_i}{(j\omega + 1)^{i+1}} - \frac{b_i}{(j\omega - 1)^{i+1}} \right) i!$$

if $N$ is chosen sufficiently large. As a result, condition (1) in Definition 2 can be transformed to its $N$-th order approximation and the other conditions, (2) and (3), can be redefined as well

$$\sum_{i=0}^{N} (a_i + (-1)^ib_i)i! < z_0$$

(5)

and, the transfer functions

$$H_1(s) = \sum_{i=0}^{N} a_i(-1)^is^{2i} \geq 0,$$

$$H_2(s) = \sum_{i=0}^{N} b_is^{2i} \geq 0,$$

(6)
satisfy \(H_j(j\omega) > 0\) for \(j = 1, 2\) and all \(\omega \in \mathbb{R}\).
On the other hand, the human operator is assumed to be a bounded LTI passive system. An LTI system \(\Delta \in \mathbb{RH}_\infty\) is said to be passive if \(\Delta(j\omega) + \Delta(j\omega)^* \geq 0\) for all \(\omega \in \mathbb{R}\). The class of multipliers preserving the positivity of this class is defined in [30]:

**Definition 3:** Let \(\lambda\) be a transfer function, then \(\lambda\) belongs to the class of passive multipliers \(\mathcal{P}\) if \(\lambda(j\omega) = \lambda(j\omega)^*\) and \(\lambda(j\omega) > 0\).

**Lemma 3:** Given a bounded LTI passive system \(\Delta\) and \(\lambda \in \mathcal{P}\), then

\[
\int_{-\infty}^{\infty} \left[ \frac{\tilde{u}(j\omega)}{\Delta u(j\omega)} \right]^* \begin{bmatrix} 0 & \lambda(j\omega) \\ \lambda(j\omega) & 0 \end{bmatrix} \left[ \frac{\tilde{u}(j\omega)}{\Delta u(j\omega)} \right] d\omega \geq 0
\]

(7)

Finally, the frequency domain inequality in Theorem 1 is transformed in an LMI using the KYP lemma:

**Lemma 4 ([31]):** Given \(A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, M = M^T \in \mathbb{R}^{(n+m) \times (n+m)}\), with \(\text{det}(j\omega I - A) \neq 0\) for all \(\omega\), where \([A, B]\) are controllable. The following two statements are equivalent:

1) \[
\left[ (j\omega I - A)^{-1}B \right]^* M \left[ (j\omega I - A)^{-1}B \right] \leq 0
\]

2) There is a matrix \(P \in \mathbb{R}\) such that \(P = P^T\) and \[
M + \begin{bmatrix} A^T P + PA & PB \\ B^T P & 0 \end{bmatrix} \leq 0
\]

**III. METHODOLOGY**

Uncertainties in bilateral teleoperator are one of the main obstructions that prevent physically realizable stable devices. In practice, it is not possible to obtain a model that defines every operator’s or environmental task’s dynamics, therefore it is convenient to analyse stability against an uncertainty set.

![Fig. 1. Bilateral teleoperation as a classical nominal plant-uncertainty interconnection](image)

For comparison, definition about human uncertainty is not being changed: Human, \(\Delta_h\), is being assumed as an LTI passive operator. On the other hand, definition about environment linearity is being questioned and redefined because majority of the objects’ physical contact forces show non-linear phenomena such as surface friction, and so on. Therefore, in our analyses we considered the environment as memoryless, monotone, bounded, and nonlinear operator. In order to obtain an analytical advantage from this assumption, we need to search for Zames-Falb multipliers which are the widest available class of multipliers [28].

IQC is a powerful framework that enables us to analyse different types of uncertainties in one characterization. Combination of the multipliers preserving positivity of the uncertainties inserted into a single matrix; we combined frequency dependent passive, (7), and Zames-Falb multipliers, (4), such that

\[
\Pi(j\omega) = \begin{bmatrix} 0 & 0 & \lambda(\omega) & 0 \\ 0 & 0 & 0 & Z(j\omega)^* \\ \lambda(\omega) & 0 & 0 & 0 \\ 0 & Z(j\omega) & 0 & 0 \end{bmatrix}
\]

As a result, we will need to parametrise both multipliers, \(\lambda(\omega)\) and \(Z(j\omega)\) in order to develop a convex search of \(\Pi(j\omega)\) such that the condition

\[
\left[ -Y(j\omega) \right] \Pi(j\omega) \left[ -Y(j\omega)^* \right] \leq -\epsilon I, \quad \forall \omega \in \mathbb{R}
\]

is satisfied for some \(\epsilon > 0\), where \(Y(j\omega)\) is admittance transfer function matrix of the designed teleoperation system. The existence of \(\Pi(j\omega)\) will imply that the interconnection depicted in Fig. 1 is stable.

Firstly, in the spirit of [17], we propose the following parametrization for frequency dependent passive multiplier \(\lambda(\omega)\). Given \(N_1\), let define the transfer function vector \(\Lambda(j\omega)\) as

\[
\Lambda(a, j\omega) = \left[ 1 \quad \frac{1}{j\omega+a} \quad \ldots \quad \frac{1}{(j\omega+a)^{1+\epsilon}} \right]^T, \quad a > 0
\]

Then a subclass of the set of multipliers \(\mathcal{P}\) is given by \(\lambda(\omega) = \Lambda(j\omega)^* K_h \Lambda(j\omega)\), where \(K_h = K_h^T\) and \(\lambda(\omega) \in \mathbb{R}\) under the condition \(\lambda(\omega) \geq 0\). This second condition, positivity, will be tested using the KYP Lemma with state space representation of \(\Lambda(j\omega)\). Let \(\Lambda \sim \begin{bmatrix} A_\Lambda & B_\Lambda \\ C_\Lambda & D_\Lambda \end{bmatrix}\), then positivity of \(\lambda(\omega)\) is equivalent to

\[
(*)^* K_h \begin{bmatrix} C_\Lambda & D_\Lambda \end{bmatrix} \left[ (j\omega I - A_\Lambda)^{-1}B_\Lambda \right] \geq 0,
\]

for all \(\omega \in \mathbb{R}\). Therefore, (10) can be transformed into an equivalent LMI by using Lemma 4 (KYP) as follow

\[
\begin{bmatrix} A_\Lambda^T P_\Lambda + P_\Lambda A_\Lambda & P_\Lambda B_\Lambda \\ B_\Lambda^T P_\Lambda & 0 \end{bmatrix} + M_\Lambda \geq 0
\]

(11)

where \(M_\Lambda = \begin{bmatrix} C_\Lambda & D_\Lambda \end{bmatrix}^T K_h \begin{bmatrix} C_\Lambda & D_\Lambda \end{bmatrix}\) and if there exists \(P_\Lambda\) such that \(P_\Lambda = P_\Lambda^T\) the mentioned equivalence holds.

Secondly, and following [29], \(Z(j\omega)\) is factorized as \(Z(j\omega) = \Xi(j\omega)^* K_z \Xi(j\omega)\), where \(\Xi(j\omega)\) is a vector containing all the dynamics of the \(Z(j\omega)\) such that \(\Xi(j\omega) = \Lambda(1, j\omega)\) while \(N_1 = N + 2\) and \(K_z\) is a matrix contains constant parameters of \(Z\) function \((z_0, a_i, b_i)\) at its first column and row

\[
K_z = \begin{bmatrix} z_0 & -a_0 & -a_1 & \cdots & -a_N N! \\ -b_0 & 0 & 0 & \cdots & 0 \\ b_1 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ (-1)^{N+1} b_N N! & 0 & 0 & \cdots & 0 \end{bmatrix}
\]
We ensure that $Z(j\omega) \in \mathbb{Z}$ by using conditions given (5) and (6). Note that condition (6) requires to test the positivity of two transfer functions and they can be converted to equivalent LMIs by using Positive Real Lemma [32]. If $H_j(s) \sim \begin{bmatrix} A_{j} & B_{j} \\ C_{j} & D_{j} \end{bmatrix}$ then $H_j(j\omega) > 0$ for all $\omega \in \mathbb{R}$ if
\[
\begin{bmatrix} A_{j}^T X_j + X_j A_{j} & C_{j}^T - X_j B_{j} \\ C_{j} - B_{j}^T X_j & -(D_{j} + D_{j}^T) \end{bmatrix} \leq 0 \tag{12}
\]
where $X_j = X_j^T$ for $j = 1, 2$.

Once we make sure that the above inequalities hold, we conclude that there exists a Zames-Falb multiplier which gives exact characterization of the stability against slope restricted nonlinearities.

Now we are ready to analyse stability of the bilateral teleoperation illustrated in Fig. 2, redesigned as in Fig. 1, by using inequality (9) and aforementioned parametrizations. We can rewrite the IQC multiplier, (8), as follows:
\[
\Pi = (\ast)^* \begin{bmatrix} 0 & 0 & K_h & 0 \\ 0 & 0 & 0 & K^T_h \\ K_h & 0 & 0 & 0 \\ 0 & K_z & 0 & 0 \end{bmatrix} \begin{bmatrix} \Lambda & 0 & 0 & 0 \\ 0 & \Xi & 0 & 0 \\ 0 & 0 & \Lambda & 0 \\ 0 & 0 & 0 & \Xi \end{bmatrix} \begin{bmatrix} \Psi & 0 \\ 0 & \Psi \end{bmatrix} \begin{bmatrix} -Y \\ I \end{bmatrix} \leq -\epsilon I \tag{13}
\]
for all $\omega \in \mathbb{R}$, where $\Psi(j\omega) = \begin{bmatrix} \Lambda(j\omega) & 0 \\ 0 & \Xi(j\omega) \end{bmatrix}$. And $G_\psi$ can be defined with its state space representation, $G_\psi(j\omega) = C(j\omega I - A)^{-1} B + D$, as a result inequality is re-defined as
\[
\begin{bmatrix} (j\omega I - A)^{-1} B \\ I \end{bmatrix}^* M \begin{bmatrix} (j\omega I - A)^{-1} B \\ I \end{bmatrix} \leq 0 \quad \forall \omega \in \mathbb{R} \tag{14}
\]
where $M = \begin{bmatrix} C & D \end{bmatrix}^T \begin{bmatrix} K & C \end{bmatrix}$. Based on the KYP Lemma we conclude that (14) is equivalent to the existence of symmetric matrices $P$ and $K$ such that
\[
\begin{bmatrix} A^T P + PA & PB \\ B^T P & 0 \end{bmatrix} + M \leq 0. \tag{15}
\]

Remark 1: In [17] at Remark 7 it has been stated that nominal system, $Y$, needs to be perturbed because the inequalities cannot be satisfied strictly when $\omega \to \infty$, as the plant is strictly proper. Here we use the same approach as [17]: using $Y + \epsilon I$, with $\epsilon = 10^{-4}$, instead of $Y$. The meaning of this constant is that our set of uncertainties are within the sector $(0, \epsilon^{-1})$.

In the following section, we state the formal stability result, which is given as a convex optimization problem. We have used Yalmip with sdpt3 solver [33] to test the LMI conditions.

IV. MAIN RESULT

Assuming human operator and environment are modelled as LTI passive system and a monotone and bounded nonlinearity, respectively; absolute stability of the bilateral teleoperation system can be analysed as follows:

Corollary 5: Let $\phi_m$ and $\Delta_m$ be a memoryless, monotone and bounded nonlinearity and a bounded and passive LTI system. Let
\[
G_\psi \sim \begin{bmatrix} A & B \\ C & D \end{bmatrix}
\]
where it has been defined in (13). Then the feedback between $Y$ and diag($\Delta_m, \phi_m$) in Fig. 1 is stable if there exist symmetric matrices $P$ and $K$; where $K$ is given in (13), such that
\[
\begin{bmatrix} A^T P + PA & PB \\ B^T P & 0 \end{bmatrix} + \begin{bmatrix} C^T K C & C^T K D \\ D^T K C & D^T K D \end{bmatrix} \leq 0
\]
and symmetric matrices $P_A$ and $X_j$ for $j = 1, 2$ such that the LMI (11) and the set of LMI (12) are satisfied.

V. EXAMPLES

The analyses are based on the position-force control architecture where master and slave are 1 degrees-of-freedom (DOF) rigid robotic manipulators. The system’s equation of motion
\[
v_m = Y_m(F_h + \tau_m), \quad v_s = Y_s(\tau_s - F_e)
\]
where, $Y_m$ and $Y_s$ are admittance of the manipulators, $\tau_m$ and $\tau_s$ are forces generated by controllers, $F_h$ and $F_e$ are applied human and environmental contact forces, respectively. Due to the rigid body, manipulator’s speed are equal to operator’s and environment’s speed such that $v_m = v_h$ and $v_s = v_e$. And controller’s forces are given by
\[
\begin{align*}
\tau_m &= -K_f F_e, \\
\tau_s &= C_s(\mu x_m - x_s)
\end{align*}
\]
where $C_s$ is controller at the slave side and it is used for motion tracking, $K_f$ and $\mu$ are environmental interaction force and position scaling factors, respectively, $x_m$ and $x_s$ are manipulators’ positions.

\begin{align*}
F_h \quad \Delta_m \quad P_m \quad P \quad F_e \quad \phi_m \quad u_m \quad u_s \quad \tau_s = K_p(\mu x_m - x_s) - K_v v_s
\end{align*}

Based on this architecture different types of controller ($C_s$) can be implemented, yet it is generally designed as a PD-controller such that $C_s(s) = K_p + K_v s$. Also, $P$ controller have been implemented, yet extra damping parameter was inserted to certify stability.
The values of the system’s parameters, which are going to be used in the analyses, are given in Table I [14].

<table>
<thead>
<tr>
<th>Model</th>
<th>Controller</th>
</tr>
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<tbody>
<tr>
<td>$M_m = 0.64$ kg</td>
<td>$M_s = 0.61$ kg</td>
</tr>
<tr>
<td>$B_m = 3.4$ Ns/m</td>
<td>$B_s = 11$ Ns/m</td>
</tr>
<tr>
<td>$K_p = 4000$ N/m</td>
<td>$K_y = 80$ N/s/m</td>
</tr>
</tbody>
</table>

Network representation of the system can be any of the immittance matrices; impedance, admittance or hybrid [34]. In our analyses admittance matrix will be used and we assumed there is no latency in the communication channel. Let $Y_m(s) = (M_m s + B_m)^{-1}$ and $P_p(s) = (s^2 M_s + s(B_s + K_y) + K_p)^{-1}$, admittance matrix representation of damping injected, controlled with (16) and (18), P-F controller is

$$Y_p(s) = \begin{bmatrix} Y_m & -K_f Y_m \\ -K_p \mu Y_m P_p & (K_p \mu Y_m K_f + s) P_p \end{bmatrix}$$

(19)

The system illustrated in Fig. 2 and controlled with (16) and (17) will be called PD-F architecture, and its admittance matrix representation is slightly different such that

$$Y_{PD}(s) = \begin{bmatrix} Y_m & -K_f Y_m \\ -\mu(s K_v + K_p) Y_m P_p & Z_{22} Y_m P_p \end{bmatrix}$$

(20)

where $Z_{22}(s) = s^2 M_m + s(B_m + \mu K_f K_v) + \mu K_f K_p$. These admittance matrices were used in FDI, (9), and it was converted to an equivalent LMI, (15), hereby stability analysis was transformed to a convex optimization search for suitable multipliers. But as mentioned before stability is not the only criterion needs to be considered while designing a teleoperator.

A physical object is said to be transparent if the light passing through it is not being scattered. In bilateral teleoperation, generally speaking, instead of light the environment’s impedance or force is transmitted. And transparency is measured by defining how well the impedance or force is being transmitted; for example in the position-force teleoperation architecture level of the transparency is measured with maximum achievable force and position scaling factors. In ideal condition multiplication of these factors ($K_f \mu$), will be called as transparency index, is equal to 1, but due to the current trade off it is not possible to obtain an absolutely stable and also ideally transparent teleoperator. Our aim is to obtain maximum achievable transparency indexes with P-F and PD-F architectures. Both controllers were analysed by many researches, but to the best of authors’ knowledge there is not any research that uses such an assumption about environmental uncertainty and using Zames-Falb multiplier search as a stability criteria for bilateral teleoperation.

A. Case 1

Firstly, Llewellyn’s stability criterion was used to analyse P-F control architecture with LTI passive operators assumption on both human and the environment with the parameters given in Table I and the admittance matrix given in (19). For the analysis we choose the frequency range as $0 \times 10^6$ rad/s where maximum achievable transparency index is searched without destroying passivity of the 2-port network and for the search bisection algorithm was used. It was concluded that with this hypothesis maximum achievable $K_f \mu$ value is $\approx 0.127$ which is also indicated in [14]. Less conservative results were obtained with Zames-Falb multipliers such that maximum $K_f \mu$ value is 0.132, and this result was obtained without restricting or even adding more properties to uncertainty sets.

B. Case 2

We also analysed stability of the bilateral teleoperation with PD-F controller architecture whose admittance matrix defined in (20) and tried to obtain a stable design as it is highlighted in [14] with this controller it is not possible to fulfil Llewellyn’s stability criteria unless $M_s = 0$. Namely the design is non-absolutely stable with the parameters in the Table I, when both human and environment are assumed to be passive LTI systems. However, the novel assumption on the environment and the use of Zames-Falb multipliers, we concluded that PD-F controller architecture is absolutely stable and maximum achievable transparency index is 0.119. That study shows the conservativeness of the previous stability analyses methods and usefulness of the one highlighted here.

Remark 2: In the numerical examples the dimensions of the transfer function vectors, $\Lambda(j\omega)^{1 \times N_1}$ and $\Xi(j\omega)^{1 \times N+2}$, were incrementally increased and final values were chosen such that increasing $N_1$ and $N$ numbers has no significant effect on the transparency index. And results were obtained while $N_1 = 7$ and $N = 6$. Moreover, the transfer function vector $\Lambda$’s pole ($-\alpha$) was chosen in way to be different from system’s eigenvalues to prevent numerical problems and the results were obtained while $\alpha = 10$.

As mentioned before slave needs to mimic the behaviour of the master manipulator to be able to complete challenging tasks in an high quality manner. In order to evaluate this we simulated the designed two control architectures, P-F and PD-F, with the maximum achievable transparency indexes. A sinusoidal force signal was used to substitute for operator’s force applied to the master manipulator. And environmental interaction force was designed as monotonically increasing function depending on the speed of the slave. It is remarked that P-F architecture’s position error is approximately 7 times higher than the PD-F architecture’s, see Fig. 3. That concludes more complex controllers give better performance specifications; lesser transparency index yet significantly higher position tracking. Present stability analyses, however, are restricting us to design systems controlled by these controllers. Therefore, more powerful and less conservative stability analyses methods need to be used to analyse bilateral teleoperation and evaluate control algorithms that can be implemented.

VI. Conclusion

In this paper, we have analysed stability of the bilateral teleoperation using Zames-Falb multipliers. The key contribution is the use of a different assumption to establish the
absolute stability of the system: The environment is assumed to be a memoryless, monotone, and bounded nonlinearity rather than as traditionally passive operator. Then following the IQC framework proposed in [17] and the Chen and Wen’s parametrisation of the class of Zames-Falb multipliers, the stability analysis is transformed into a convex optimization search expression by LMIs for suitable multipliers. The more sophisticated description of the environment leads to less conservative results than previous results in the literature. The usefulness of this description has been shown in a 2-Channel architecture, where the term $Y_{22}$ in the admittance matrix is no longer restricted to be passive, since the Zames-Falb multiplier can “correct” some lack of passivity for the class of nonlinearities considered here. The performance then can be improved.

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