A Safety-Cost Based Design-Decision Support Framework Using Fuzzy Evidential Reasoning Approach

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Abstract

This paper illustrates a subjective safety-cost based design-decision support framework using fuzzy logic and evidential reasoning approaches. The framework is divided into three parts. The first one is for safety analysis and synthesis including fuzzy rule-based safety estimation using the fuzzy rule-based evidential reasoning (FRB-ER) approach, as well as the safety synthesis using the evidential reasoning (ER) approach. Considering the cost incurred for the safety improvement in a design option, the second part focuses on synthesising the safety and cost estimates using the evidential reasoning approach to obtain the overall evaluation of the whole system for each design option. The third part is to apply the overall evaluation for design selection. An illustrative example is used to demonstrate the application of the proposed framework.

Keywords: Safety analysis, cost analysis, decision-making, uncertainty, fuzzy rule-base, evidential reasoning

1 Introduction

A decision on implementing a design in a large engineering system depends on that the design satisfies technical and economical constraints [Wang et al. 1996]. In the early design stages of large engineering products such as offshore topsides and offshore support structures, an efficient design option is usually selected from several design options based on the analysis of multiple objectives such as safety and cost. This is usually carried out by several experienced designers, and therefore synthesis of multiple designers' opinions is required. Conventional ap-

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proaches (e.g., probability risk analysis) have been widely used, but often fall short in their ability to permit the incorporation of subjective and/or vague terms as they rely heavily on supporting statistical information that may not be available [Wang and Ruxton 1997]. They may not be well suited for dealing with systems in situations of having a high level of uncertainty, particularly in the feasibility and concept design stages of an engineering system, where there is often inadequate data or imprecise information available when carrying out safety assessments for the system.

Moreover, the cost incurred for the safety improvement in a design option is dependent upon various factors that are often associated with a high degree of uncertainties during the earlier stages of estimation.

In addition, the safety of a structure in an engineering system is often determined by all the associated failure events of each individual component that makes up the structure. The problem may be ultimately generalized to estimate the safety of a system with a hierarchy [Wang et al. 1996].

In order to solve these problem, novel safety and cost analysis methods are therefore required to establish a general framework that provides a basis and tool for safety and cost analysis and synthesis in complex engineering systems, in particular to deal with information that may be imprecise, ill-defined, and incomplete, for which traditional quantitative approaches (e.g., statistical approach) do not give an adequate answer.

One realistic way to deal with imprecision is to use linguistic assessments instead of numerical values. Fuzzy logic approaches [Zadeh 1965] employing fuzzy IF-THEN rules (where the conditional part and/or the conclusions contain linguistic variables [Zimmerman 1991]) can model the qualitative aspects of human knowledge and reasoning process without employing precise quantitative analysis. It does not require an expert to provide a precise point at which a risk factor exists. This actually provides a tool for working directly with the linguistic information, which are commonly used in represents risk factors and carrying out safety assessments [Karwowski and Mital 1986; Keller et al. 1989; Duckstein 1994; Bell and Badiru 1996, Bowles and Pelaez 1995; An et al. 2000, Wang et al. 1995 and 1996, Wang 1997, Sii et al. 2001]. In this context, a safety model using fuzzy rule-based inference system can be more appropriately used to carry out risk analysis associated with incomplete safety information in the initial design stages or a system with high level of innovation.

In engineering safety analysis, intrinsically vague information may coexist with conditions of ‘‘lack of specificity’’ originating from evidence not strong enough to completely support a hypothesis but only with degrees of belief or *credibility* [Binaghi et al. 1999]. Dempster-Shafer (D-S) theory of evidence [Dempster 1968, Shafer 1976] based on the concept of *belief function* is well suited to modeling subjective credibility induced by partial evidence [Smets 1988]. D-S theory enlarges the scope of traditional probability theory, describes and handles uncertainties using the concept of the degrees of belief, which can model incompleteness and ignorance explicitly. It also provides appropriate methods for computing belief functions for combination of evidence [Pearl 1988]. Besides, the D-S theory also shows great potential in multiple attribute decision

analysis (MADA) under uncertainty, where the evidential reasoning (ER) approach for MADA under uncertainty has been developed, on the basis of a distributed assessment framework and the evidence combination rule of the D-S theory [Yang and Singh 1994, Yang and Sen 1994, 1997, Yang 2001, Yang and Xu 2002a, b].

Accordingly, it seems reasonable to extend the fuzzy logic framework to cover credibility uncertainty as well, i.e., to combine fuzzy logic with Dempster-Shafer models to deal with fuzziness and incompleteness in safety analysis. The combination may become substantial when a lack of specificity in data is prevalent [Liu et al. 2003a].

In this perspective, the aim of this paper is to suggest a novel subjective design evaluation framework using fuzzy rule-base and ER approaches [Yang and Xu 2002a, b] for risk analysis, and for safety-cost synthesis of an engineering structure at the initial design stages. The safety analysis is based on fuzzy rule-based evidential reasoning framework – FRB-ER investigated in [Liu et al. 2003b], which is an implementation of the RIMER methodology proposed in [Yang et al. 2003]. In the proposed FRB-ER framework, the parameters used to define the safety level are described using fuzzy linguistic variables; a fuzzy rule-base designed on the basis of a belief structure is used to capture uncertainty and nonlinear relationships between these three parameters and the safety level; an input for each antecedent is transformed into a distribution on the linguistic values of this antecedent. This distribution describes the degree of each antecedent being activated. Moreover, the antecedents of an IF-THEN rule consist of an overall attribute, called *a packet antecedent attribute*. The activation weight of a rule is the aggregation of the degrees to which all the antecedents in the rule are activated. In this context, an IF-THEN rule can be considered as an evaluation problem of a packet antecedent attribute being assessed to an output term in the consequent of the rule with certain degrees of belief. Finally, the inference of the fuzzy rule-based system is implemented using the ER approach [Yang and Xu 2002a, b]. In other words, the inference is formulated as a multi-attribute decision-analysis problem under uncertainty. The final inference result is the safety estimates at the bottom of a hierarchical structure (individual element level).

Then the ER approach is used to obtain the safety synthesis at the higher levels of the hierarchy (element type and the whole system level). Multi safety analysts' judgements can also be synthesised using the framework.

Moreover, the cost incurred by each design option is also evaluated based on fuzzy sets by multiple designers. Utility expressions are defined to map the safety associated with and the cost incurred for each design option onto a utility space. Safety and cost objectives described in terms of the utility expressions are then synthesised again using the ER approach to obtain the preference description of each design option so as to rank the design options.

In the following, Section 2 outlines the safety analysis framework using the FRB-ER approach; Section 3 focuses on the safety synthesis in a hierarchy; The safety and cost synthesis framework is provided in Section 4, where the synthesis result can be used to produce the preference estimate associated with the design

options for ranking purpose. An illustrative example is used to demonstrate the application of the proposed framework in Section 5. Section 6 is the conclusion.

2 Safety Analysis Framework

The proposed framework for modelling system safety consists of four major components, which outline all the necessary steps required for safety analysis evaluation at the bottom level of a hierarchical system (i.e., each cause to technical failure) using the fuzzy rule-based evidential reasoning (FRB-ER) approach proposed in [Liu et al. 2003b].

2.1 Identify Causes/Factors

This can be done by a panel of experts during a brainstorming session at the early concept design stages of a system.

2.2 Identify & Define Fuzzy Input and Fuzzy Output Variables

The three fundamental parameters used to assess the safety level of an engineering system on a subjective basis are the *failure rate* (**FR**)*, consequence severity* (**CS**) and *failure consequence probability* (**FCP**). Subjective assessments (using linguistic variables instead of ultimate numbers in probabilistic terms) are more appropriate for analysis using these three parameters as they are always associated with great uncertainty, especially for a novel system with high level of innovation. These linguistic assessments can become the criteria for measuring safety levels.

The level of linguistic variables used for describing each fundamental parameter is decided according to the situation of the case of interest. The recent literature survey indicates that four to seven levels of linguistic variables are commonly used to represent risk factors in risk analysis [Bell and Badiru 1996]. The typical linguistic variables used to describe **FR**, **CS**, **FCP** of a particular element may be defined and characterized as follows [Sii and Wang 2001, Liu et al. 2003b]:

FR describes failure frequencies in a certain period, which directly represents the number of failures anticipated during the design life span of a particular system or an item. To estimate the **FR**, one may choose to use such linguistic terms as "*very low,*" "*low* (Lo)," "*reasonably low* (RLo)," "a*verage* (A)," "*reasonably frequent* (RF)," "*frequent* (F)," and "*highly frequent* (HF)."

CS describes the magnitude of possible consequences, which is ranked according to the severity of failure effects. One may choose to use such linguistic terms as "*negligible* (N)," "*marginal* (Ma)," "*moderate* (Mo)," "*critical* (Cr)" and "*catastrophic* (Ca)."

For **FCP**, one may choose to use such linguistic terms as "*highly unlikely* (HU)," "*unlikely* (U)," "*reasonably unlikely* (RU)," "*likely* (Li)," "*reasonably likely* (RLi)," and "*Definite* (D)."

The second step in this component is to select the types of fuzzy membership functions used to delineate each input variable. It is possible to have some flexibility in the definition of membership functions to suit different situations. The application of categorical judgments has been quite positive in several practical situations [Schmucker 1984]. It is also common and convenient for safety analysts to use categories to articulate safety information. The belief distribution assessment scheme proposed in sub-section 2.4 provides other alternative ways for inference while no membership function is available.

Safety estimate is the only output fuzzy variable used in this study to produce safety evaluation for a particular cause to technical failure. This variable is also described linguistically, which is described and determined by the above parameters. In safety assessment, it is common to express a safety level by degrees to which it belongs to such linguistic variables as "*poor,*" "*fair,*" "*average,*" and "*good*" that are referred to as safety expressions.

2.3 Construct a Fuzzy Rule-Base with the Belief Structure

Fuzzy logic systems are knowledge-based or rule-based ones constructed from human knowledge in the form of fuzzy *IF-THEN* rules. For example, the following is a fuzzy *IF-THEN* rule for safety analysis:

IF **FR** of a hazard is *frequent AND* **CS** is *catastrophic AND* **FCP** is *likely, THEN* ***safety estimate*** is *Poor.*

In view of the increasing complexity of many knowledge-based systems, the knowledge representation power of fuzzy rule-based systems will be severely limited if only fuzziness is used to represent uncertain knowledge. As we stated in Section 1, there is another kind of uncertainty in representing knowledge, i.e., when the expert is unable to set a strong relation between premise and conclusion.

Accordingly, fuzzy rules for safety analysis can be extended in the following way. In general, assume that the three antecedent parameters, U_1=**FR**, U_2=**CS** and U_3=**FCP** can be described by J_i linguistic terms $\{A_{ij}; j=1,\ldots,J_i\}$, i=1, 2, 3, respectively. One consequent variable ***safety estimates*** can be described by N linguistic terms, i.e., $D_1,\ldots, D_N$. Let A_i^k be a linguistic term corresponding to the ith attribute in the kth rule. Thus the kth rule in a belief rule-base can be written as follows:

R_k: IF **FR** is A_1^k AND **CS** is A_2^k AND **FCP** is A_3^k THEN ***safety estimates*** is $\{(D_1, \bar{\beta}_{1k}), (D_2, \bar{\beta}_{2k}),\ldots, (D_N, \bar{\beta}_{Nk})\}$, $(\sum_{i=1}^{N} \bar{\beta}_{ik} \leq 1)$, with a rule weight θ_k, and the attribute weights δ_1, δ_2, δ_3 (1)

where $\bar{\beta}_{ik}$ ($i\in\{1,\ldots, N\}$; $k\in\{1,\ldots, L\}$; L is the total number of the rules in the rule-base) is a belief degree measuring the subjective uncertainty of the conse-

quent "***safety estimates*** is D_i" drawn due to the antecedent "**FR** is A_1^k AND **CS** is A_2^k AND **FCP** is A_3^k" in the *k*th rule. If $\sum_{i=1}^{N} \overline{\beta}_{ik} = 1$, the output assessment or the *k*th rule is said to be complete; otherwise, it is incomplete. Note that ($\sum_{i=1}^{N} \overline{\beta}_{ik} = 0$) denotes total ignorance about the output given the input. The rule-base with the rules in the form (1) is referred to as *a belief rule-base*.

2.4 Belief Rule-Base Inference Mechanism based on the Evidential Reasoning Approach

Once a rule-base is established, the knowledge contained can be used to perform inference for given input. The inference procedure is basically composed of five steps, summarized as the following sub-sections.

2.4.1 Discretization of the input into the distributed representation of linguistic terms in antecedents

This is to *discretize the input into the distributed representation of linguistic values in antecedents using belief degrees*. By using the distribution assessment approach introduced in [Yang and Singh 1994], in general, we may consider a linguistic term in the antecedent as an evaluation grade, the input for an antecedent attribute U_i can be assessed to a distribution representation of the linguistic terms using belief degrees as follows:

$$S(U_i)=\{(A_{ij}, \alpha_{ij}); j=1,\ldots, J_i\}, i=1, 2, 3 \tag{2}$$

where A_{ij} ($j \in \{1,\ldots, J_i\}$) is the *j*th linguistic term of the ith attribute, α_{ij} the likelihood to which the input for U_i belongs to the linguistic term A_{ij} with $\alpha_{ij} \geq 0$ and $\sum_{j}^{J_i} \alpha_{ij} \leq 1$ (*i*=1, 2, 3), referred to as *the individual matching degree*. α_{ij} in Eq. (2) could be generated using various ways depending on the nature of an antecedent attribute and the available data, which is described in the following three cases:

(1) *Matching function method while the input is in numerical form and the linguistic value for describing antecedent attribute is characterized using fuzzy membership functions* (*suitable for both quantitative and qualitative attribute*);

(2) *Rule-based or utility-based transformation methods while the input is in numerical forms but the fuzzy membership function is not available* (*only suitable for the quantitative attribute*) [Yang 2001, Yang et al. 2003];

(3) *Subjective assessment method* (*suitable for quantitative and qualitative antecedent attribute*).

For more details about the above three cases, we refer to [Yang 2001; Yang et al. 2003; Liu et al. 2003b]. Here we only consider Case (1). Corresponding to rule-base (1), the general input form corresponding to the antecedent attribute in

the kth rule is given as follows:

$$(A_1^*, \varepsilon_1) \text{ AND } (A_2^*, \varepsilon_2) \text{ AND } (A_3^*, \varepsilon_3) \tag{3}$$

where ε_i expresses the degree of belief assigned by an expert to the association of A_i^* (i=1,…, 3), which reflects the uncertainty of the input data.

Finally α_{ij} in Eq. (2) could be formulated in the following way:

$$\alpha_{ij} = \frac{\tau(A_i^*, A_{ij}) \cdot \varepsilon_i}{\sum_{j=1}^{J_i}[\tau(A_i^*, A_{ij})]}, \; i=1, 2, 3; j=1,\ldots, J_i \tag{4}$$

Here (A_i^*, ε_i) is the actual input corresponding to the ith antecedent, τ is a matching function, and $\tau(A_i^*, A_{ij}) = \tau_{ij}$ is a matching degree to which A_i^* belong to A_{ij}. One possible matching function τ is given as follows:

$$\tau(A_i^*, A_{ij}) = \max_x[\min(A_i^*(x) \wedge A_{ij}(x))]. \tag{5}$$

2.4.2 Prioritisation of the Packet Antecedent of a Rule

Considering an input given by Eq. (3) corresponding to the kth rule in (1),

$$\textbf{FR} \text{ is } (A_1^k, \alpha_1^k) \text{ AND } \textbf{CS} \text{ is } (A_1^k, \alpha_2^k) \text{ AND } \textbf{FCP} \text{ is } (A_1^k, \alpha_3^k) \tag{6}$$

where α_i^k is the individual matching belief degree that the input belongs to A_i^k of the individual antecedent U_i appearing in the kth rule.

The global matching weight w_k of the packet antecedent A^k in the kth rule is generated by weighting and normalizing the α_k given by Eq. (6) as follows:

$$w_k = (\theta_k \cdot \alpha_k)/(\textstyle\sum_{i=1}^{L} \theta_i \alpha_i) \tag{7}$$

where θ_k is the relative weight of the kth rule, $\alpha_k = \prod_{i=1}^{3}(\alpha_i^k)^{\delta_i}$, δ_i (i=1, 2, 3) is the weight of the ith antecedent attribute, and L is the number of rules in the rule-base. Note that "AND" connective is used for three antecedents in a rule. In other words, the consequent of a rule is not believed to be true unless all the antecedents of the rule are activated. In such cases, the simple multiplicative aggregation function is used to calculate α_k, which is used in the case study in Section 4. Moreover, note that $0 \le w_k \le 1$ (k=1,…, L) and $\sum_{i=1}^{L} w_i = 1$.

2.4.3 The Degree of Belief in the Consequent of a Rule

An incomplete input for an attribute will lead to an incomplete output in each of the rules in which the attribute is used. In the inference procedure, such incompleteness should be considered. The original belief degree in the ith consequent term of the kth rule in (1) is updated based on the input information as follows:

$$\beta_{ik} = \overline{\beta}_{ik} * [\textstyle\sum_{t=1}^{3}(\tau(t,k) * \sum_{j=1}^{J_t} \alpha_{ij}]/[\sum_{t=1}^{3} \tau(t,k)] \tag{8}$$

where $\tau(t, k)=\begin{cases}1 & \text{if } U_t \text{ is used in defining } R_k \\ 0 & \text{otherwise}\end{cases}$ (t=1, , 2, 3), α_{tj} is given in Eq. (2) with $\alpha_{tj} \geq 0$ and $\sum_{j^t}^{J_t} \alpha_{tj} \leq 1$. $\overline{\beta}_{ik}$ is given in (1) with $0 \leq \sum_{i=1}^{N} \overline{\beta}_{ik} \leq 1$.. Note that $0 \leq \sum_{i=1}^{N} \beta_{ik} \leq 1$ for all k and 1-$\sum_{i=1}^{N} \beta_{ik}$ denotes both the ignorance incurred in establishing R_k and the incompleteness that may exist in the input information.

2.4.4 Rule Expression Matrix for a Fuzzy Rule-Base with Belief Structure

Suppose a fuzzy rule-base with the belief structure is given by R= {R_1, R_2,…, R_L}. The kth rule in (1) can be represented as follows:

R_k: IF U is A^k THEN ***safety estimate*** is D with belief degree B_k

where U represents the antecedent attribute vector (**FR**, **CS**, **FCP**), A^k the packet antecedents { A_1^k , A_2^k , A_3^k }, D the consequent vector (D_1, D_2, …, D_N), β_k the vector of the belief degrees (β_{1k} , β_{2k} , …, β_{Nk}) and $k \in \{1,\ldots, L\}$. Each fuzzy rule with belief structure can be explained in the following way:

The packet antecedent A^k of an IF-THEN rule can be considered as a global attribute, which is considered as being assessed to a linguistic term D_i (the ith possible consequent term in the kth rule) with a belief degree of β_{ik} ($i \in \{1,\ldots, N\}$). This assessment can be represented by

$$S(A^k)=\{(D_i,\ \beta_{ik}\ );\ i=1,\ldots, N\} \tag{10}$$

which is obviously a distributed assessment and is referred to as *a belief structure*, where β_{ik} measures the degree to which D_i is the consequent if the input activates the antecedent A^k in the kth rule, which is given using (8) with $0 \leq \sum_{i=1}^{N} \beta_{ik} \leq 1$ for all k. Here i=1,…, N, k=1,…, L. L is the number of rules in the rule-base and N is the number of the possible consequent terms in the kth rule.

A belief rule-base established using rules given by Eq. (10) can be summarized using the following rule expression matrix shown in Table 1:

Table 1: Rule expression matrix for a belief rule-base

output belief input	D_1	D_2	…	D_i	…	D_N
A^1 (w_1)	β_{11}	β_{21}	…	β_{i1}	…	β_{N1}
⋮	⋮	⋮	…	⋮	…	⋮
A^k (w_k)	β_{1k}	β_{2k}	…	β_{ik}	…	β_{Nk}
⋮	⋮	⋮	…	⋮	…	⋮
A^L (w_L)	β_{1L}	β_{2L}	…	β_{iL}	…	β_{NL}

In the matrix, w_k is the global activation weight of A^k, which measures the degree to which the kth rule is weighted and activated.

In this context, an IF-THEN rule can be considered as an evaluation problem of a packet antecedent attribute being assessed to an output term in the consequent of the rule with certain degrees of belief. Hence, the inference can be formulated as a multi-attribute decision-analysis problem under uncertainty. Finally, the inference of the belief rule-based system is implemented using the ER approach [Yang and Xu 2002a, b] as described in the following subsection.

2.4.5 Rule Combination Using the ER Approach

Having represented each rule using rule expression matrix, the evidential reasoning (ER) approach [Yang 2001, Yang and Xu 2002a,b] can be used to combine rules and generate final conclusions, which can be directly implemented as follows. First, transform the degrees of belief β_{jk} for all j=1,…, N, k=1,…, L into basic probability masses using the equations [Yang 2001, Yang and Xu 2002a]:

$$m_{j,k} = w_k \beta_{j,k}, \ j = 1, \cdots, N;$$

$$m_{D,k} = 1 - \sum_{j=1}^{N} m_{j,k} = 1 - w_k \sum_{j}^{N} \beta_{j,k},$$

$$\overline{m}_{D,k} = 1 - w_k \text{ and } \tilde{m}_{D,k} = w_k (1 - \sum_{j=1}^{N} \beta_{j,k})$$

For all $k = 1, \cdots, L$, with $m_{D,k} = \overline{m}_{D,k} + \tilde{m}_{D,k}$ for all $k = 1, \cdots, L$ and $\sum_{j}^{L} w_j = 1$. The probability mass assigned to the consequent D, which is unassigned to any individual output terms D_j, is split into two parts, one caused by the relative importance of the kth packet antecedent A^k or $\overline{m}_{D,k}$, the other by the incompleteness of the kth packet antecedent A^k or $\tilde{m}_{D,k}$.

Then, aggregate all the packet antecedents of the L rules to generate the combined degree of belief in each possible consequent term D_j in D. Suppose $m_{j,I(k)}$ is the combined degree of belief in D_j by aggregating the first k packet antecedents (A^1,…, A^k) and $m_{D,I(k)}$ is the remaining degree of belief unassigned to any output term. Let $m_{j,I(1)} = m_{j,1}$ and $m_{D,I(1)} = m_{D,1}$. Then the overall combined degree of belief β_j in D_j is generated as follows:

$$\{D_j\}: \ m_{j,I(k+1)} = K_{I(k+1)}[m_{j,I(k)} m_{j,k+1} + m_{j,I(k)} m_{D,k+1} + m_{D,I(k)} m_{j,k+1}]$$

$$m_{D,I(k)} = \overline{m}_{D,I(k)} + \tilde{m}_{D,I(k)}, \ k = 1, \cdots, L$$

$$\{D\}: \ \tilde{m}_{D,I(k+1)} = K_{I(k+1)}[\tilde{m}_{D,I(k)} \tilde{m}_{D,k+1} + \tilde{m}_{D,I(k)} \overline{m}_{D,k+1} + \overline{m}_{D,I(k)} \tilde{m}_{D,k+1}]$$

$$\{D\}: \ \overline{m}_{D,I(k+1)} = K_{I(k+1)}[\overline{m}_{D,I(k)} \overline{m}_{D,k+1}]$$

$$K_{I(k+1)} = \left[1 - \sum_{j=1}^{N} \sum_{\substack{t=1 \\ t \neq j}}^{N} m_{j,I(k)} m_{t,k+1} \right]^{-1}, \ k = 1, \cdots, L-1$$

$\{D_n\}$: $\beta_j = (m_{j,I(L)})/(1-\overline{m}_{D,I(L)})$, $j = 1,\cdots,N$

$\{D\}$: $\beta_D = (\tilde{m}_{D,I(L)})/(1-\overline{m}_{D,I(L)})$.

β_D represents the remaining belief degrees unassigned to any D_j. It has been proved that $\sum_{j=1}^{N}\beta_j + \beta_D = 1$ [Yang and Xu 2002a].

The final conclusion generated by aggregating the L rules, which are activated by the actual input A^* for U=(**FR**, **CS**, **FCP**) can be represented as follows

$$S(A^*)=\{(D_j,\ \beta_j), j=1,\ldots, N\} \tag{11}$$

The inference procedure is based on fuzzy rule-base and evidential reasoning approach, referred to as a *fuzzy rule-based evidential reasoning approach* – FRB-ER approach [Liu et al. 2003b]. The final result is a belief distribution on safety expression, which gives a panoramic view about the safety level for a given input.

3 Safety Synthesis in a Hierarchy

The safety of a structure is often determined by all the associated failure events of each individual component that makes up the structure. A component usually has several failure events. Section 2 mainly focuses on safety assessment of a failure mode at the bottom level of such hierarchy done by an expert. This part is concerned with safety synthesis of a system at various levels such as:

(a) The synthesis of safety estimates of a specific failure event for a component done by a panel of experts; or

(b) The synthesis of safety estimates of various failure events to a component, furthermore, to the safety associated with each sub-system, and finally the safety associated with the system being investigated.

This multi-expert, multi-attribute, and multi-level synthesis safety synthesis can be carried out at the system level to obtain the safety evaluation of the system (i.e. a particular design option in this case).

If we consider several particular events $F=\{F_1,\ldots, F_d\}$ to a technical component, For each particular failure event F_j ($j\in\{1,\ldots, d\}$), the description or the input of its antecedent attribute in the safety rule normally is derived from different information sources or evaluated by different experts. Suppose there are several sources or experts e_i (i=1,…, K), and the input comes from different experts.

Furthermore, we assume that different experts/different sources have different reliability weights, w_{Ei} (i=1,…, K), and $A_{ei} = (A_{ei,1}, A_{ei,2}, A_{ei,3})$ is the input vector derived from the e_i for the antecedent attribute. For each input, we may get the corresponding safety estimate output D_{ei} using the above FRB-ER approach, which can be formulated as follows:

IF U is A_{e1} THEN D_{e1} is $\{(D_1, \eta_{11}),,\ldots, (D_N, \eta_{N1})\}$

…

IF U is A_{ei} THEN D_{ei} is $\{(D_1, \eta_{1i}),\ldots, (D_N, \eta_{Ni})\}$

…
IF U is A_{eK} THEN D_{eK} is $\{(D_1, \eta_{1K}),\ldots, (D_N, \eta_{NK})\}$

Here $\{(D_1, \eta_{1i}), (D_2, \eta_{2i}),\ldots, (D_N, \eta_{Ni})\}$ results from Eq. (11) obtained using the FRB-ER approach. Then the actual output D_c for the safety estimates of a specific cause F_i (i=1,…, d) is in fact a multi-experts synthesis, i.e., the evidence aggregation of $\{D_{e1}, D_{e2},\ldots, D_{eK}\}$ using the ER algorithm considering the expert's weight as

$$S(F_j)=\{(D_i, \eta_i^j); i=1,\ldots, N\}, j=1,\ldots, d \tag{12}$$

Due to several particular failure causes to a technical component, then the final output for the safety estimates of a technical component D is the synthesis of all the assessments $S(F_j)$ (j=1,…, d) for each particular failure event using the ER algorithm again. Then it is desirable to synthesise the evaluations to generate an assessment of the safety associated with each component, furthermore, the safety associated with each sub-system, and finally the safety associated with the system being investigated. The ER approach used here is capable of combining uncertain evaluations at a single level and implementing hierarchical propagation of such evaluations between different levels without any loss of usual information [Yang and Xu 2002a, b].

4 Perform Safety-Cost Synthesis for Each Design Option

This section synthesises the safety and cost estimates for each design option again using the evidential reasoning approach. The steps used in the framework are outlined in the following subsections.

4.1 Cost Assessment Using Fuzzy Sets

When making safety-based design decisions for a marine system, it is necessary to take cost aspects into account. The cost incurred for the safety improvement associated with a design option is usually affected by many factors that often involve uncertainties in estimation. Therefore, it may be more appropriate to model the cost incurred in safety improvement associated with a design option on a subjective basis, like using linguistic variables characterized by fuzzy sets. The costs incurred for a design option can be described using linguistic variable such as {'*very low,*' '*low,*' '*moderately* low,' '*average,*' '*moderately high,*' '*high,*' '*very high*'}, which are referred to as cost expressions that are defined in terms of membership degrees belonging to the seven defined categories as shown in Table 2 [Wang et al. 1996] given in Appendix.

From Table 2, we can see that the cost expressions are not exclusive in the sense that the sum of the membership degrees of the linguistic variables with re-

spect to a category may be greater than 1. Such inclusive expressions may make it more convenient for the safety analyst to judge the costs.

The designers may provide the cost incurred for a design option with reference to Table 2 in Appendix. The cost C_i incurred for designing option i can be described in terms of membership values as follows:

$$C_i = \{\mu_{C_i}^1 / 1, \mu_{C_i}^2 / 2, \mu_{C_i}^3 / 3, \mu_{C_i}^4 / 4, \mu_{C_i}^5 / 5, \mu_{C_i}^6 / 6, \mu_{C_i}^7 / 7\} \tag{13}$$

where each $\mu_{C_i}^j$ (j=1,…, 7) represents a degree to which C_i belongs to the jth category.

(Jun: It would make more sense (more consistent and versatile) to create a belief rule base for combining safety and cost. You could have treated safety as one variable and cost as another variable in the belief rule base. All the following steps would be unnecessary. We can discuss this idea later. Remember the ER approach for MCDA is in essence a special rule based approach.)

4.2 Safety and Cost Descriptions in the Utility Space

The selection of a design proposal relies on safety and cost implications in a particular situation. This requires the synthesis of safety and cost for each design option in a rational manner. Since safety and cost are described using fuzzy linguistic variables, it is appropriate that the ER approach can be used to carry out such a synthesis in order to avoid loss of useful information.

However, as the safety associated with a design option is described in terms of safety expressions, and the cost incurred for each design option is described in terms of membership values describing a corresponding linguistic cost variable, it is necessary to define a utility space to evaluate safety and cost on the same scale to expedite the synthesis process using evidential reasoning. Four exclusive utility expressions {'*greatly preferred;*' '*preferred;*' '*moderately preferred;*' '*slightly preferred*'} are defined as shown in Table 3 [Wang et al. 1996] of Appendix. The safety and cost incurred for each design option are then mapped onto the generalised utility space and expressed in terms of utility expressions.

In assessment, different words may be used to describe equivalent standards. Such equivalence can be established using equivalence rules. Rule based information transformation technique for qualitative assessment proposed in [Yang 2001] can be used here for the safety transformation into the generalized utility expression. For instance, if "*good*" means that the preference degree of a design option is "*greatly preferred*" as far as safety is concerned, then an expression "*good*" in safety assessment is said to be equivalent to a grade "*greatly preferred*" for preference assessment. Similarly "*average*" is equivalent to "*preferred*", "*fair*" to "*moderately preferred*" and "*poor*" to "*slightly preferred.*" In this case, a safety expression S_i associated with the ith design option

S_i= {(β_i^1 , *poor*), (β_i^2 , *fair*), (β_i^3 , *average*), (β_i^4 , *good*)}

can be directly mapped onto the utility space as follows:

$U(S_i)$= {($u_{S_i}^1$, *slightly preferred*), ($u_{S_i}^2$, *moderately preferred*), ($u_{S_i}^3$, *preferred*), ($u_{S_i}^4$, *greatly preferred*)} (14)

Where $u_{S_i}^j = \beta_i^j$ (j=1,…, 4). Each $u_{S_i}^j$ (j=1,…, 4) represents a degree of confidence that the safety expression S_i belongs to the jth utility expression.

Given the membership values of a cost description for a design option with reference to Table 2 and also based on the utility expression in Table 3, the Best-Fit method described in [Wang et al. 1995] can also be used to map the fuzzy cost description onto the defined utility expression. The cost C_i incurred in the ith design option can be evaluated in terms of the utility expressions as follows:

$U(C_i)$= {($u_{C_i}^1$, *slightly preferred*), ($u_{C_i}^2$, *moderately preferred*), ($u_{C_i}^3$, *preferred*), ($u_{C_i}^4$, *greatly preferred*)} (15)

where $u_{C_i}^m$ (m=1, 2, 3 or 4) represents the degree to which the obtained cost C_i is confirmed to the mth (m=1, 2, 3 or 4) utility expression. Suppose the utility expressions {*slightly preferred*, *moderately preferred*, *preferred*, *greatly preferred*} are denoted by U_m {m=1,…, 4}. The extent $u_{C_i}^m$ (m=1, 2, 3 or 4) can be obtained using the Best-Fit method [Wang et al. 1995]:

$$u_{C_i}^m = \frac{a^m}{\sum_{p=1}^{4} a^p} \tag{16}$$

where a_m (m=1, 2, 3 or 4) represents the reciprocal of the relative distance between C_i and the mth utility expression U_m. a_m can be obtained by [Wang et al., 1995]:

$$a^m = \frac{1}{d^m / d^M} \tag{16a}$$

where d_m is the Euclidean distance between C_i and the mth utility expression U_m, and $d^M$is the minimum value of d_m (m=1, 2, 3 and 4). For example, the distance d^m is given as follows:

$$d^m = d(C_i, U_m) = (\sum_{j=1}^{7} (\mu_{C_i}^j - \mu_{U_m}^j)^2)^{\frac{1}{2}} \tag{16b}$$

$\mu_{C_i}^j$ (j=1,…, 7) represents a degree to which C_i belongs to the jth category. $\mu_{U_m}^j$ (j=1,…, 7) is the membership value of U_m in the jth category.

It may be noted that if C_i is completely confirmed to the mth utility expression then $u_{C_i}^m$ is equal to 1 and the others are equal to 0. And $\sum_{m=1}^{4} u_{C_i}^m = 1$. Thus each $u_{C_i}^m$ could be viewed as a degree of confidence that C_i belongs to the mth utility expression.

4.3 Compute the Preference Estimate for Each Design Option

Suppose there are M design options in hand. Given the relative importance of cost w_C against safety w_S, $U(S_i)$ and $U(C_i)$ can be further synthesised using the evidential reasoning approach to obtain a preference estimate associated with design option i in terms of the utility expressions. The synthesised preference estimate $U(i)$ for the ith design option can be expressed as follows:

$U(i)$= {(u_i^1, *slightly preferred*), (u_i^2, *moderately preferred*), (u_i^3, *preferred*), (u_i^4, *greatly preferred*)} (17)

Preference degree P_i associated with the ith design option can be obtained by [Yang and Singh, 1994]:

$$P_i = \sum_{j=1}^{4} \mu_i^j \times K_j + \left(1 - \sum_{j=1}^{4} \mu_i^j\right) \times \frac{1}{4}\left(\sum_{j=1}^{4} K_j\right) \tag{18}$$

where $\left(1 - \sum_{j=1}^{4} \mu_l^j\right)$ describes the remaining belief unassigned after commitment to belief to all U_j (j =1, 2, 3, 4) in safety and cost synthesis, and $(1/4) \times \left(\sum_{j=1}^{4} K_j\right)$ is the average value of the K_js. The numerical values K_j (j=1,…, 4) are assigned here to describe the four utility expressions (i.e. "*greatly preferred*"; "*preferred*"; "*moderately preferred*"; "*slightly preferred*"). It can be calculated by studying the categories and membership values associated with the utility expression in Table 3. For example, suppose K_1^*, K_2^*, K_3^*, K_4^* represent the un-scaled numerical values associated with "*slightly preferred*", "*moderately preferred*", "*preferred*", "*greatly preferred*", respectively. Then K_1^*, K_2^*, K_3^*, K_4^* can be calculated as follows [Wang et al. 1996]:

K_1^*=0.75×6+1×7 =11.5; K_2^*=0.5×4+1×5+0.25×6=8.5

K_3^*=0.25×2+1×3+0.5×4=5.5; K_4^*=1×1+0.75×2=2.5.

The above values give the linear relationship between the utility expressions. The normalized form is then obtained as follows: K_1 = 0.217, K_2 =0.478, K_3=0.739, and K_4=1, where "*greatly preferred*" takes the largest value of 1. Note that K_1 is not normalized to 0. This implies that linguistic variable "*slightly preferred*" still has some preference degree (i.e., 0.217) in the design selection process.

4.4 Rank the Alternative Options in Order of Preference

Design selection can be carried out on the basis of the preference degrees associated with the M design options with regard to the particular considerations of safety and cost using Eq. (18).

It is obvious that a larger P_i means that the ith design option is more desirable. Each P_i represents the extent to which the ith design option is preferred in comparison with others. The best design with the largest preference degree may be selected on the magnitudes of P_i. The preference degrees associated with the design options can be obtained by synthesising the safety associated with and cost incurred for each design options using evidential reasoning approach [Wang et al.

1995]. The attributes of safety and cost can be considered to carry different weights (i.e. having different degrees of important) for different situations while conducting the design selection process.

5 An Illustrative Example

The following example is used to illustrate the practicality of the proposed framework in design option evaluation.

A hydraulic hoisting transmission system of a marine crane consists of five subsystems: a hydraulic oil tank, an auxiliary system, a control system, a protection system and a hydraulic servo transmission system [Wang et al. 1995, 1996]. Each subsystem is associated with several failure modes. Suppose there are four safety analysts. There are four design options in hand. Those options are

Option 1: eliminating no failure modes in the design review process;

Option 2: eliminating failure modes "hoist up limit failure" and "hoist down limit failure" associated with the protection system;

Option 3: eliminating the failure modes involving "major leak" and "no output from the package motor" associated with the hydraulic servo transmission system; and

Option 4: eliminating the two failure modes associated with the protection system in design option 2 and the two failure modes associated with the hydraulic servo transmission system in design option 3.

Suppose four safety analysts make the judgments on each failure mode of each subsystem for design option 1. In this case study, seven levels of linguistic variables may be used for **FR**; five levels for **CS**, seven levels for **FCP** as mentioned in Section 2 and the possible definitions can be seen in [Liu et al. 2003b]. Accordingly, the rule-base with a total number of 245 fuzzy rules with the belief structure can be used. We may have the fuzzy rules with belief structures as follows:

R_{198}: *IF the* **FR** *is* <u>*frequent*</u> *AND the* **CS** *is* <u>*critical*</u> *AND the* **FCP** *is* <u>*unlikely*</u> *THEN the* ***safety estimate*** *is* {(*Good*, 0), (*Average*, 0.2), (*Fair*, 0.7), (*Poor*, 0.1)}

For example, the safety associated with failure modes can be obtained as follows based on FRB-ER approach. The window-based and graphically designed intelligent decision system (IDS), which has been developed based on the evidential reasoning approach [Yang and Xu 1999], is used to implement the ER algorithm:

{(*Good*, 0), (*Average*, 0.0121), (*Fair*, 0.5194), (*Poor*, 0.4685)},

which is a belief distribution representation of the failure modes, representing that we are 51.94% sure that safety level is *Fair*, and 46.85% sure that safety level is *Poor*, and 1.21% sure that safety level is *Average*.

Considering the hydraulic hoisting transmission system of a marine crane consisting of five subsystems each of which is associated with several failure modes, a multi-expert-multi-attribute-multi-level safety synthesis can be imple-

mented using the ER approach. We only give the results summarized in Table 4 of Appendix.

Suppose four safety analysts make the judgments on the cost incurred in each design option. If safety and cost are considered to be of equal importance, then the utility descriptions of the four design options are obtained as given below:

Considering Option #1,

S_1={(0.1119,*"poor"*), (0.1758,*"fair"*), (0.452,*"average"*), (0.2283,*"good"*)}

$U(S_1)$={(0.1119,*"slightly preferred"*), (0.1758,*"moderately preferred"*), (0.452,*"preferred"*), (0.2283,*"greatly preferred"*)}

S_1 is assessed using the FRB-ER approach and the ER approach. The utility description on the safety associated with the option is assessed as "*slightly preferred*" with a belief of 11.19%, as "*moderately preferred*" with 17.58%, as "*preferred*" with 45.20% and as "*greatly preferred*" with 22.83%.

The judgments produced can then be synthesised to obtain the utility description on the cost incurred in design option 1 using the Best-Fit approach.

$U(C_1)$={(0,*"slightly preferred"*), (0,*"moderately preferred"*), (0,*"preferred"*), (1,*"greatly preferred"*)}.

The utility description of the option and the preference degree associated with the description are obtained as follows:

$U(1)$={(0.0495,*"slightly preferred"*), (0.0778,*"moderately preferred"*), (0.2,*"preferred"*), (0.6727,*"greatly preferred"*)}.

P_1=0.8684.

The similar computations are performed for the other three design options separately. The results are summarized in Table 4 of Appendix.

It can be noted that in this case, design option # 1 is ranked first, design option # 3 second, design option # 4 third and design option # 2 last. This implies that if safety and cost are considered equally important then design option 1 should be selected. When the relative importance of safety against cost changes, there may be different ranking orders of the design options. Fig. 1 shows the preference degrees associated with the four design options at different values of relative importance of safety and cost (note that $w_S+w_C=1$). For example, when $w_S=0.6$ and $w_C=0.4$, the ranking of the four design options is option 1, option 3, option 2 and option 4. Given the particular requirements on safety and cost, the ranking of design options can be found in Fig. 1.

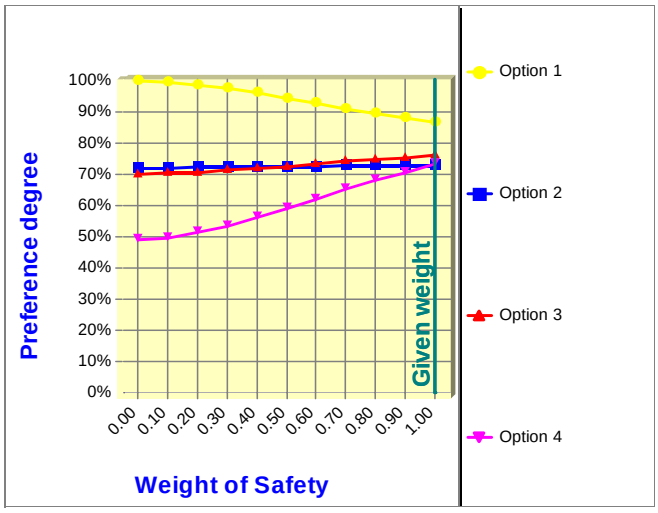

Fig. 1 Ranking of the design options

6 Conclusions

This paper outlines and explains a philosophy for subjective safety-cost modelling for engineering safety assessment using fuzzy logic and the evidential reasoning approach. For each design option, the risk analysis of each element is first carried out using the safety model based on the FRB-ER approach. Then the evidential reasoning approach is used to synthesise safety estimates for each design option. Finally the synthesis of safety and cost is performed using evidential reasoning for each design option and mapped onto a common utility space before generating the preference estimates and ranking the design options.

In the safety analysis, the usual safety rule base was extended to the belief rule-base which can take into account subjective expert judgments with uncertainties of both a probabilistic and fuzzy nature. Multi-attribute and multi-expert decision-making was also realized based on the safety estimate using the ER algorithm. The approach provides a flexible way to represent and a process such hybrid uncertain safety assessment information to arrive at final conclusions.

The proposed framework offered great potential in safety assessment and deign decision support of engineering and management systems, especially in the initial concept design stages where the related safety information is scarce or with various types of uncertainty involved.

The developed approach only deals with safety and cost objectives. However, it can be extended to deal with other objectives such as reliability, availability and maintainability.

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Appendix: Tables mentioned in the main text

Table 2: Cost expressions

μ_C	Categories						
Linguistic Variables	1	2	3	4	5	6	7
Very High	0	0	0	0	0	0.75	1
High	0	0	0	0	0.75	1	0.25
Moderately High	0	0	0	0.5	1	0.25	0
Average	0	0	0.5	1	0.5	0	0
Moderately low	0	0.25	1	0.5	0	0	0
Low	0.25	1	0.75	0	0	0	0
Very Low	1	0.75	0	0	0	0	0

Table 3. Four utility expressions

μ_U	Categories						
Linguistic Variables	1	2	3	4	5	6	7
slightly preferred	0	0	0	0	0	0.75	1
moderately preferred	0	0	0	0.75	1	0.25	0
preferred	0	0.25	1	0.5	0	0	0
greatly preferred	1	0.75	0	0	0	0	0

Table 4: Safety-cost synthesis into utility space and the preference degree

Options	Evaluation terms	Utility expressions			
		slightly preferred	*moderately preferred*	*preferred*	*greatly preferred*
#1	$U(S_1)$	0.1119	0.1758	0.4520	0.2283
	$U(C_1)$	0	0	0	1
	U(1)	0.0495	0.0778	0.2	0.6727
	P_1	0.8684			
#2	$U(S_2)$	0.11	0.1432	0.3956	0.3512
	$U(C_2)$	0.008	0.112	0.8112	0.0688
	U(2)	0.008	0.112	0.8112	0.0688
	P_2	0.7235			
#3	$U(S_3)$	0.1123	0.0956	0.1345	0.6576
	$U(C_3)$	0.0169	0.1234	0.8345	0.0252
	U(3)	0.0612	0.1077	0.5047	0.3264
	P_3	0.7641			
#4	$U(S_4)$	0.013	0.018	0.035	0.934
	$U(C_4)$	0.002	0.96	0.006	0.032
	U(4)	0.0073	0.4861	0.0201	0.4864
	P_4	0.7352			