Distribution of information within and across colour spaces

Iván Marín-Franch and David H. Foster; School of Electrical and Electronic Engineering, University of Manchester, Manchester M60 1QD, UK

Abstract

The aim of this work was to investigate how efficiently colour information is distributed, in a least-redundancy sense, across the variables of a colour space, and how this distribution varies across different colour spaces. It was found that among the physiological and psychophysical colour spaces tested, Derrington-Krauskopf-Lennie space was the most efficient, although this result is contingent on assumptions about the pre-processing of cone signals. Among the colorimetric spaces tested, CIECAM02 was the most efficient. For most spaces, the variables associated with chromatic properties carried more information than the variables associated with achromatic properties.

Introduction

There are many different kinds of colour spaces. Some are based on psychophysical data [1] and others on direct physiological measurements [2, 3] or on physiological theories of optimal signal transmission [4]. These spaces are based typically on non-opponent and opponent combinations of signals from long-, medium-, and short-wavelength-sensitive-cones. Analogously, colorimetric spaces designed to specify or describe colour appearance [5, 6], are based on non-opponent and opponent combinations of tristimulus values. The presence of opponency in these representations is not accidental, for it has the effect of increasing independence, most obviously between responses from long- and medium-wavelength-sensitive cones. In general, a representation in which statistical independence between variables is achieved is called an efficient representation [7]. How efficiently, then, is colour information distributed across the variables of a given colour space, and how does this distribution vary across different colour spaces?

The aim of this work was to address these questions for images of natural scenes under different daylight illuminants. To this end, estimates were made, for a given colour space, of the information preserved between the representation of a scene under one illuminant and the representation of the same scene under a different illuminant. Information was interpreted in the sense of Shannon [8, 9], with the principal measure being mutual information, which quantifies the statistical interdependence of the colour representations and which is intimately related to colour constancy. It was found that among the physiological and psychophysical colour spaces tested, Derrington-Krauskopf-Lennie space [2] was the most efficient, and among the colorimetric spaces tested, the CIECAM02 space [6] was the most efficient.

A preliminary study of the distribution of colour information over non-opponent and opponent variables of CIELAB space [5] has been previously reported [10], but without analysis of the interdependencies between variables.

Information-theoretic analysis

All the colour spaces considered here encode spectra in terms of one non-opponent variable, A say, generally associated with achromatic properties, and two opponent variables, P and

Q say, generally associated with chromatic properties. Suppose that, in some colour space, A_1 is the non-opponent representation of a scene illuminated by illuminant 1 and A_2 is the representation of the same scene illuminated by illuminant 2. If the probability density functions of A_1, A_2, and the pair (A_1, A_2) are $f_1(a_1)$, $f_2(a_2)$, and $f(a_1, a_2)$, respectively, then the mutual information $I(A_1; A_2)$ between A_1 and A_2 is given [8, 9] by

$$I(A_1; A_2) = \iint f(a_1, a_2) \log \frac{f(a_1, a_2)}{f_1(a_1) \, f_2(a_2)} \, \mathrm{d}a_2 \mathrm{d}a_1 \, ,$$

where the integrations are taken over the spaces spanned by A_1 and A_2. Mutual information is always positive, except when A_1 and A_2 are independent, i.e. $f(a_1, a_2) = f_1(a_1) f_2(a_2)$, in which case it is zero. It can be expressed in terms of the differential entropies associated with each variable, defined [9] e.g. for A_1 as

$$h(A_1) = - \int f_1(a_1) \log f_1(a_1) \, \mathrm{d}a_1 \, . \tag{1}$$

Thus,

$$I(A_1; A_2) = h(A_1) + h(A_2) - h(A_1, A_2), \tag{2}$$

where $h(A_1, A_2)$ is the entropy of the pair (A_1, A_2). The mutual information between each of the opponent variables P and Q was defined in exactly the same way.

Mutual information can be extended to the triplets (A_1, P_1, Q_1) and (A_2, P_2, Q_2) to give for a particular colour space the total information between the two representations of a scene under the two illuminants. The total mutual information $I(A_1, P_1, Q_1; A_2, P_2, Q_2)$ is denoted here by I_{APQ}, and the marginal mutual information $I(A_1; A_2)$, $I(P_1; P_2)$, and $I(Q_1; Q_2)$ by I_{A}, I_{P}, and I_{Q}, respectively.

An important property of mutual information is that it is invariant under invertible, differentiable transformations of the variables involved [11]. This means that if A, P, and Q are defined by invertible transformations of long-, medium-, and short-wavelength-sensitive-cone responses (or tristimulus values), then the total information is preserved; otherwise information is lost [9]. For instance, in [4], A, P, and Q are linearly related to long-, medium-, and short-wavelength-sensitive cone responses, l, m, and s. Therefore the total mutual information in (A, P, Q) space is the same as in (l, m, s) space. Nevertheless, the distribution of information differs because none of the variables A, P, and Q is an invertible transformation of any of the variables l, m, and s.

The efficiency of the representation provided by a colour space depends on the degree of independence of the variables of that space. This independence can be measured by the multi-information [12, 13], which is a generalization of mutual information and which measures the higher-order dependencies between the variables of that space. The multi-information for A_1, P_1, and Q_1 is given by

$$M(A_1; P_1; Q_1) = h(A_1) + h(P_1) + h(Q_1) - h(A_1, P_1, Q_1), \tag{3}$$

and analogously for (A_2, P_2, Q_2).

The total mutual information I_{APQ} between (A_1, P_1, Q_1) and (A_2, P_2, Q_2) can be factored into the marginal mutual information I_A, I_P, and I_Q and the multi-information between the variables. Let $R_{APQ} = M(A_1; P_1; Q_1) + M(A_2; P_2; Q_2)$ be the redundancy, defined here as the sum of the multi-information between the variables for illuminant 1 and for illuminant 2. Thus,

$$I_{APQ} = I_A + I_P + I_Q - R_{APQ} + D_{APQ}, \qquad (4)$$

where $D_{APQ} = M(A_1, A_2; P_1, P_2; Q_1, Q_2)$ is the higher-order dependency between the variables. Redundancy R_{APQ} and higher-order dependency D_{APQ} terms are always positive. If the variables A, P, and Q are independent, then $R_{APQ} = D_{APQ} = 0$, and $I_{APQ} = I_A + I_P + I_Q$.

A similar factorization can be performed across achromatic and chromatic variables; that is, between the non-opponent variable A and the two opponent variables taken together as, say, $C = (P, Q)$. Thus,

$$I_{APQ} = I_A + I_C - R_{AC} + D_{AC}. \qquad (5)$$

Methods

The physiological and psychophysical colour spaces considered here, all based on recoding cone responses at each point in the image of a scene, were as follows:

1. Derrington-Krauskopf-Lennie (DKL) space [2], as specified in [14];
2. De Valois and De Valois (DD) space [3];
3. Guth ATD1 (G1) space [1, 15];
4. Guth ATD2 (G2) space [1, 15];
5. Buchsbaum-Gottschalk (BG) space [4].

The colorimetric spaces, all based on the tristimulus values at each point, were as follows:

1. CIELAB [5],
2. CIELAB [5] after chromatic adaptation using the CAT2000 transform [16],
3. CIECAM02 space [6].

Data for analysis were drawn from 50 hyperspectral images of natural scenes [17, 18] under three representative daylight changes with correlated colour temperatures as follows: from 4000 K to 6500 K, from 25,000 K to 6500 K, and from 25,000 K to 4000 K. These particular daylight illuminants were chosen for their special role in the CIE specification [5], and were approximated by the CIE method for reconstructing illuminants [19]. The cone responses (l, m, s) or tristimulus values (X, Y, Z) at each point in the image of a scene were obtained from the Stockman and Sharpe cone fundamentals [20] and the CIE 1931 colour-matching functions [5], respectively. The triplets of responses or tristimulus values were then transformed to the colour spaces listed above. All the spaces treated here were taken in their standard forms and were not adjusted for this particular analysis.

For each scene, daylight change, and colour space, the marginal and joint entropies, as in (1), were estimated to obtain the mutual information I_A, I_P, and I_Q as in (2), and the multi-information terms in R_{APQ} and D_{APQ} as in (3). The total mutual information was then obtained with (4). The factorization in (5) was calculated in an analogous way. A modification of the Kozachenko-Leonenko estimator of differential entropy [21] was used for the calculation of marginal and joint entropies. This involved estimating the entropy after previously whitening the data, which gave better estimates. Hence, if $\text{Var}(A, P, Q)$ is the

variance-covariance matrix of the variables A, P, Q, then the differential entropy $h(A, P, Q)$ is given [9] by

$$h(A, P, Q) = h(A^*, P^*, Q^*) + \frac{1}{2} \log |\text{Var}(A, P, Q)|, \qquad (6)$$

where $| \cdot |$ denotes the determinant of a matrix and

$$\begin{pmatrix} A^* \\ P^* \\ Q^* \end{pmatrix} = [\text{Var}(A, P, Q)]^{-1/2} \begin{pmatrix} A \\ P \\ Q \end{pmatrix}.$$

Results and comment

All the colour spaces listed earlier are related to each other by invertible differentiable transformations, except the Guth G1 and G2 spaces [15]. The total mutual information for these two spaces will, in general, be smaller than for any other space. Although the existence of an inverse of the CIECAM02 representation has been reported [6], there are some technical anomalies [22], the most important of which is referred to [23] as the "brightness problem", namely, that the calculated brightness may fail to be a real number. A solution has been proposed [23], which was adopted here, but it leads to non-invertibility. Fortunately, the brightness problem occurs rarely in practice, and was not found here with illuminants with correlated colour temperatures of 4000 K and 6500 K and only rarely with 25,000 K.

Table 1 shows the mean total information estimated for 50 natural scenes and a change in daylight illuminant from a correlated colour temperature of 4000 K to one of 6500 K.

Table 1. Total estimated information from images of natural scenes under a change in daylight illuminant from a correlated colour temperature of 4000 K to one of 6500 K

Colour space	Mean (SD)
Physiological-psychophysical	
DKL	19.98 (1.18)
DD	19.98 (1.18)
G1	18.28 (1.24)
G2	18.96 (1.08)
BG	19.98 (1.18)
Colorimetric	
CIELAB	20.13 (1.14)
CIELAB CAT2000	20.32 (1.14)
CIECAM02	20.04 (1.02)

The mean total information estimated was closely similar over all spaces at ~ 20 bits, except for the G1 and G2 spaces at ~ 18 and ~ 19 bits, respectively. As expected, estimates were the same for DKL, DD, and BG spaces, which are linearly related to each other. For the illuminant change from a correlated colour temperature of 25,000 K to one of 6500 K, the estimates were very similar. For a larger daylight illumination change, from a correlated colour temperature of 25,000 K to one of 4000 K, the mean total information estimated was ~ 3 bits lower.

Figure 1 shows values of the individual quantities I_A, I_P, I_Q, $-R_{APQ}$, and D_{APQ} in (4) averaged over the 50 scenes under a change in daylight illuminant from a correlated colour temperature of 4000 K to one of 6500 K. The sum of all the values for each colour space coincides with the corresponding values in Table 1.

The least-redundant spaces, that is, those with the lowest values of R_{APQ} in (4), were DKL for physiological and psychophysical spaces and CIECAM02 for colorimetric ones, for
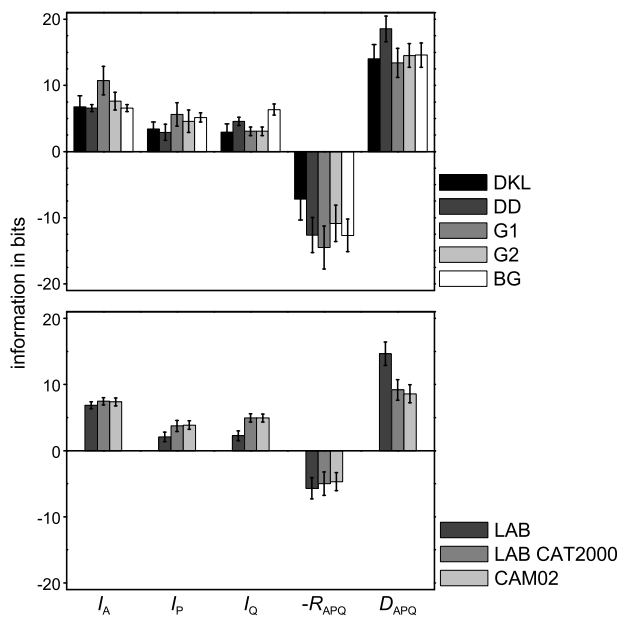
Figure 1. *Distribution of information across non-opponent and opponent variables A, P, and Q within colour spaces for a change in daylight illuminant from a correlated colour temperature of 4000 K to one of 6500 K. The upper plot is for physiological and psychophysical colour spaces and lower plot is for colorimetric spaces. Error bars indicate ± 1 SD.*
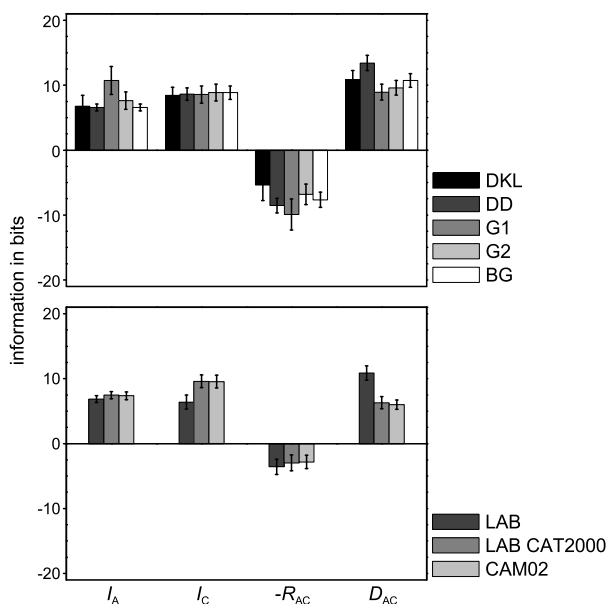

Figure 2. *Distribution of information across achromatic and chromatic variables A and C within colour spaces for a change in daylight illuminant from a correlated colour temperature of 4000 K to one of 6500 K. The upper plot is for physiological and psychophysical colour spaces and lower plot is for colorimetric spaces. Error bars indicate ± 1 SD.*

the three illuminant changes considered. That a colour space is redundant does not imply that there is a loss of information, but that the interdependencies between variables play an important role in information [24], indicated in Fig. 1 by the large values of D_{APQ}.

Given the broad division between achromatic and chromatic variables, it is interesting to compare the information for the non-opponent variable A and the information for the two opponent variables taken together as $C = (P, Q)$. Figure 2 shows the values of the individual quantities I_A, I_C, $-R_{AC}$, and D_{AC} in (5) averaged over the 50 scenes under a change in daylight illuminant from a correlated colour temperature of 4000 K to one of 6500 K. In contrast to Fig. 1, the sum of all the values for each colour space was not exactly equal to the corresponding values in Table 1 because, in (5), differential entropies different from those in (4) were needed, and the estimates were subject to sampling errors. For most spaces, the chromatic information I_C was found to be larger than the achromatic information I_A. In addition, the information I_C was found to be generally larger than the sum of the marginal information $I_P + I_Q$.

Conclusions

The distribution of colour information from natural scenes across the variables of the colour spaces considered here was achieved with varying degrees of efficiency. All the spaces displayed some degree of redundancy and higher-order dependencies between their variables. The spaces with the least redundancy were Derrington-Krauskopf-Lennie space among physiological and psychophysical colour spaces and CIECAM02 space among colorimetric spaces. It is emphasized, however, that these results hold only for the default forms of these spaces. Nevertheless, for most colour spaces, it seems that opponent variables together carry more information than the non-opponent variable.

Acknowledgments

References

[1] S. L. Guth. *J. Opt. Soc. Am. A*, 8:976–993, 1991.
[2] A. M. Derrington, J. Krauskopf, and P. Lennie. *J. Physiol*, 357:241–265, 1984.
[3] R. L. De Valois and K. K. De Valois. *Vision Res*, 33:1053–1065, 1993.
[4] G. Buchsbaum and A. Gottschalk. *Proc. R. Soc. London, B*, 220:89–113, 1983.
[5] CIE. CIE Publ. No. 15:2004. 2004.
[6] CIE. CIE Publ. No. 159:2004. 2004.
[7] W. S. Geisler. *Annu. Rev. Psychol*, 59:167–192, 2008.
[8] C. E. Shannon. *AT&T Tech. J*, 27:379–423, 1948.
[9] T. M. Cover and J. A. Thomas. *Elements of Information Theory*. Wiley series in telecommunications. John Wiley & Sons, Inc, New York, 1991.
[10] D. H. Foster, K. Amano, and S. M. C. Nascimento. In *AIC Colour 05 - 10th Congress of the International Colour Association*, pages 69–72, Granada, 2005.
[11] G. A. Darbellay and I. Vajda. *IEEE Trans. Inform. Theory*, 46:709–712, 2000.
[12] S. Watanabe. *IMB J. Res. Dev*, pages 66–82, 1960.
[13] M. Studený and J. Vejnarová. In *Learning in graphical models*, pages 261 – 297. MIT Press, Cambridge, MA, USA, 1998.
[14] D. H. Brainard. In Kaiser and Boynton, editors, *Human Color Vision*, pages 563–579. Optical Society of America, Washington, DC., 2nd edition, 1996.
[15] S. L. Guth. *J. Opt. Soc. Am.*, 9:344, 1992.
[16] C. Li, M. R. Luo, B. Rigg, and R. W. G. Hunt. *Color Res. Appl*, 27:49–58, 2002.
[17] S. M. C. Nascimento, F. P. Ferreira, and D. H. Foster. *J.*

Opt. Soc. Am. A, 19:1484–1490, 2002.

[18] D. H. Foster, K. Amano, S. M. C. Nascimento, and M. J. Foster. *J. Opt. Soc. Am. A*, 23:2359–2372, 2006.

[19] D. B. Judd, D. L. Macadam, and G. Wyszecki. *J. Opt. Soc. Am.*, 54:1031–1040, 1964.

[20] A. Stockman and L. T. Sharpe. *Vision Res*, 40:1711–1737, 2000.

[21] L. F. Kozachenko and N. N. Leonenko. *Prob. Inf. Trans*, 23:95–101, 1987. Translated from Problemy Peredachi Informatsii, 23:9–16, 1987.

[22] C. J. Li and M. R. Luo. *Color Res. Appl*, 30:99–106, 2005.

[23] M. H. Brill and S. Susstrunk. 2008. (Manuscript available at http://infoscience.epfl.ch/record/111596/files/ BrillS2007_sub.pdf).

[24] B. B. Averbeck, P. E. Latham, and A. Pouget. *Nat. Rev. Neurosci*, 7:358–366, 2006.

Author Biography

Iván Marín-Franch (ivan.marin-franch@postgrad.manchester.ac.uk) received his B.Sc. in Statistics in 1998 and M.Sc. in Statistical Science and Techniques in 2000 from the University of Granada. He then worked in the private sector as a business-applications programmer and consultant and, in 2005, began his research for a Ph.D. in the School of Electrical and Electronic Engineering at the University of Manchester.