Is there a better non-parametric alternative to von Kries scaling?

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Abstract

The effect on cone excitations of a change in illuminant on a scene may be predicted by von Kries scaling, but these predictions are not perfectly accurate. Here, a non-parametric method was used instead, but which preserved the principle of independence of activity in cone or cone-opponent mechanisms. Performance was evaluated over samples taken from 50 hyperspectral images of vegetated and non-vegetated natural scenes under large changes in daylight illuminant. Taking due account of differences in degrees of freedom, the non-parametric model gave significantly better predictions than von Kries scaling of cone or cone-opponent activity.

Introduction

Von Kries scaling refers generally to the idea that the spectral effects of the prevailing light on the sensitivity of each class of cone receptor of the eye depend only on activity in that cone class [1, 2]. Although originally conceived for the adaptation of the eye to stimulus lights, many models of colour constancy, including Land's Retinex models [3, 4], have assumed that von Kries scaling applies also to cone activity in response to lights reflected from surfaces. Thus, if l, m, and s are the excitations of long-, medium-, and short-wavelength-sensitive cones for light reflected from a surface under one illuminant and l', m', and s' are the corresponding excitations for another illuminant, then von Kries scaling models their relationship by a simple multiplication; that is,

$$
\begin{aligned}
l' &= k_{L}\, l, \\
m' &= k_{M}\, m, \\
s' &= k_{S}\, s,
\end{aligned}
\tag{1}
$$

where the coefficients k_{L}, k_{M}, k_{S} are constants, which are dependent only on activity in the corresponding cone class, and which may be estimated from the data by ordinary least squares or from the ratio of excitations of a spectrally neutral surface. [The error terms representing random variation, which would normally be included on the right-hand side of (1), have been omitted for clarity.]

It is emphasized that the statement (1) is not about chromatic adaptation (e.g. [5, 6, 7]) but about how activity in a given cone class—or sensor type—varies with the spectrum of the illumination on a scene [8, 9]. In principle, the latter provides a recipe for the former. Because interactions between different cone classes are not involved in (1), this form of von Kries scaling is referred to as a diagonal-matrix transformation [10].

When tested in computer simulations, von Kries scaling has indeed been found to give a good description of the effects of daylight illuminant change on natural scenes [11], and also with other surfaces and illuminants [12]. The predictions are not, however, perfectly accurate. Figure 1 (left panel) shows an example of the excitations of medium-wavelength-sensitive cones for light reflected from 100 surfaces drawn randomly from a natural

vegetated scene under a daylight of correlated colour temperature 4000 K plotted against the corresponding excitations for light reflected from the same surfaces under a daylight of correlated colour temperature 25000 K. Similar distributions were obtained for short- and long-wavelength-sensitive cones. The straight line is a linear regression passing through zero. Although the fit is good, there are regions where there appears to be a local bias with the data mainly on one side of the curve, as the expanded section in the right panel of Fig. 1 makes clear. Figure 2 shows the scene from which these excitations were calculated.
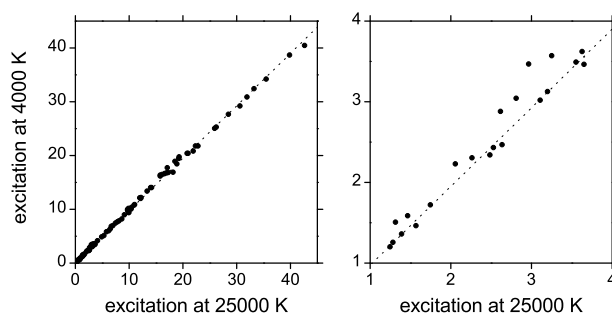

Figure 1. *Modelling cone excitations by von Kries scaling. Left panel shows a linear fit with zero intercept. The right panel is an expanded section near the origin revealing local bias. Data for medium-wavelength-sensitive cones.*

Figure 2. *Image of the scene from which the cone excitations plotted in Fig. 1 were calculated. The small grey sphere at the bottom left was masked in the calculation of cone excitations.*

There are two ways in which the performance of von Kries scaling may be improved (for non-von-Kries approaches, see e.g. [9, 13]). The first way is to generalize the principle of independence of activity in cone mechanisms [1] so that it ap-

plies after opponent interactions between cone signals have taken place [14, 15]. Thus, activity within a cone-opponent mechanism is assumed to be affected only by activity within that mechanism [16, 17]. The accuracy of predictions is usually increased [14, 18], owing to the sharpening of spectral sensitivities that occurs with opponency [16, 19, 17]. For non-biological applications, spectral sensitivities may be optimized directly [20, 21].

The second way of improving performance, but still preserving independence, is to generalize the scaling transformation. This has been done by introducing a non-zero offset so that the multiplications in (1) become affine transformations, which can give a better fit to some post-receptoral combinations of cone excitations [22, 23].

The aim of the present work was to develop the second way of improving performance by allowing the transformations to be any smooth function of the individual cone excitations l, m, s; that is, the excitations l', m', s' are modelled by

$$
\begin{aligned}
l' &= f_{\mathrm{L}}(l), \\
m' &= f_{\mathrm{M}}(m), \\
s' &= f_{\mathrm{S}}(s),
\end{aligned} \tag{2}
$$

where $f_{\mathrm{L}}, f_{\mathrm{M}}, f_{\mathrm{S}}$ are smooth, not necessarily linear, functions of cone excitations, each function dependent only on activity in the corresponding cone class. [As in (1), the error terms that would normally be included on the right-hand side of (2) have been omitted for clarity.] An analogous approach may be taken to predicting cone-opponent excitations.

The problem then is to estimate these smooth functions f_{Q} in (2) for Q = L, M, S. One possible approach is to model the functions by polynomial regression [24], but it is known to have several disadvantages [25], including the fact that degree of the polynomial cannot be controlled continuously.

The approach taken here was non-parametric, that is, not dependent on any particular model of the functions f_{L}, f_{M}, and f_{S}, and was based on local linear regression [26, 25], as explained later. It was found that for both cone and cone-opponent excitations, local linear regression provided a more accurate description than scaling, without violating the independence principle.

To avoid confusion, a distinction should be drawn between this local linear approach to modelling the effects of illuminant change on cone excitations and a previous study of von Kries scaling [27] which used local linear models of surface reflectance and which was concerned with finding the illuminants for which von Kries scaling holds perfectly.

Methods

In computer simulations, spectral reflectances were drawn from 50 hyperspectral images of natural scenes [11]. Each scene was assumed to be illuminated by a spatially uniform daylight illuminant with incident spectral radiance $E(\lambda)$ or $E'(\lambda)$, with correlated color temperatures 25000 K and 4000 K, respectively. Wavelength λ ranged over 400–720 nm in 10-nm steps. Excitations of the long-, medium-, and short-wavelength-sensitive cones at each selected surface in a scene were calculated for the Smith and Pokorny fundamentals [28, 29], $L(\lambda)$, $M(\lambda)$, and $S(\lambda)$. Excitations of the corresponding cone-opponent mechanisms with sharpened spectral sensitivities [30, 16] $L^{\#}(\lambda)$, $M^{\#}(\lambda)$, and $S^{\#}(\lambda)$ were calculated [17] as linear combinations $L^{\#}(\lambda) = 2.46L(\lambda) - 1.97M(\lambda) + 0.075S(\lambda)$, $M^{\#}(\lambda) = -0.66L(\lambda) + 1.58M(\lambda) - 0.12S(\lambda)$, and $S^{\#}(\lambda) = 0.09L(\lambda) - 0.14M(\lambda) + 1.00S(\lambda)$. This particular choice of coefficients was not critical, however, as von Kries scaling and the non-parametric

method were applied at the same level, that is, either to cone excitations or to cone-opponent excitations.

Thus, if the effective spectral reflectance [31] of the selected surface was $R(\lambda)$, then the individual cone excitations $q = l$, m, s for illuminant $E(\lambda)$, and $q' = l'$, m', s' for illuminant $E'(\lambda)$, were obtained from

$$
q = \int Q(\lambda) E(\lambda) R(\lambda) \, \mathrm{d}\lambda, \tag{3}
$$

and

$$
q' = \int Q(\lambda) E'(\lambda) R(\lambda) \, \mathrm{d}\lambda, \tag{4}
$$

where $Q(\lambda) = L(\lambda)$, $M(\lambda)$, $S(\lambda)$. Analogously, cone-opponent excitations $q^{\#} = l^{\#}$, $m^{\#}$, $s^{\#}$ for illuminant $E(\lambda)$, and $q^{\#\prime} = l^{\#\prime}$, $m^{\#\prime}$, $s^{\#\prime}$ for illuminant $E'(\lambda)$, were obtained from

$$
q^{\#} = \int Q^{\#}(\lambda) E(\lambda) R(\lambda) \, \mathrm{d}\lambda, \tag{5}
$$

and

$$
q^{\#\prime} = \int Q^{\#}(\lambda) E'(\lambda) R(\lambda) \, \mathrm{d}\lambda, \tag{6}
$$

where $Q^{\#}(\lambda) = L^{\#}(\lambda)$, $M^{\#}(\lambda)$, $S^{\#}(\lambda)$.

Data for analysis therefore consisted of pairs of cone excitations (q_i, q_i') as in (3) and (4), or pairs of cone-opponent excitations $(q_i^{\#}, q_i^{\#\prime})$ as in (5) and (6), for light reflected from $N = 100$ surfaces, $i = 1, 2, \ldots, N$, drawn randomly from each scene under the two daylight illuminants. The value of N is not critical providing that it is large enough for the distribution of excitations not to be sparse, and similar results were obtained with much larger values of N.

Fitting was based on local linear regression [25], which assumes that the function f_{Q} relating q to q' in (2) can be adequately approximated locally by a Taylor expansion (here of degree one); that is, the value of the function f_{Q} at the point q in the neighbourhood of q_0 is approximated by

$$
f_{\mathrm{Q}}(q) \approx a_0 + a_1 (q - q_0),
$$

where the coefficients a_j, $j = 0, 1$, are related to the zero- and first-order derivatives of the corresponding excitation; in particular $a_0 = f_{\mathrm{Q}}(q_0)$.

For each q, estimates $\hat{a}_j$ of the coefficients a_j were obtained by minimizing the locally weighted sum of squares,

$$
\sum_{i=1}^{N} w_h(q - q_i) \left[q_i' - (a_0 + a_1 (q_i - q)) \right]^2,
$$

where w_h is the weight function, which is characterized by a bandwidth h and which governs the influence of each point q_i on the local linear estimate. The local linear regression estimate $\hat{f}_{\mathrm{Q}}(q)$ of q' is then $\hat{a}_0$. The weight function used here is expressed in terms of a kernel K, so that $w_h(q - q_i) = K((q - q_i)/h)$. As there were regions where data were sparse, a Gaussian function, $K(u) = (2\pi)^{-1/2} \exp(-u^2/2)$, which has unbounded support, was used.

The bandwidth h controls the spread of the weight function. A small bandwidth results in a variable estimate which follows the data very closely, whereas a large bandwidth gives a smooth but biased estimate, with the data falling mainly on one side of the curve. Thus the choice of the bandwidth is crucial. The mean integrated squared error, $\mathrm{MISE} = \int (f_{\mathrm{Q}} - \hat{f}_{\mathrm{Q}})^2 \mathrm{d}q$, is a way to

balance the bias against the variance. This loss function, however, depends on the unknown f_Q and therefore cannot be used directly. By the bootstrap principle [32], the unknown quantities can be approximated by re-sampling from the estimated distribution of the data.

This approach was used here to find an estimate of the bandwidth that minimized MISE [33]. Accordingly, 200 bootstrap samples were obtained by the wild bootstrap [34] and for each of these samples the local linear estimate was calculated as a function of the bandwidth h. An approximation of the MISE was then calculated from these estimates and the value of h that minimized this approximation was used as the estimate of the MISE-optimal bandwidth.

A bootstrap test, based on 500 bootstrap samples, was also used for the comparison of the local linear regression with von Kries scaling. The test statistic was the scaled difference between the residual sums of squares for the two models. It is asymptotically F-distributed with degrees of freedom corresponding to the difference between the degrees of freedom of the local and von Kries scaling models, and the residual degrees of freedom of the local model [35].

Exactly analogous calculations were performed for pairs of cone-opponent excitations $(q_i^{\#}, q_i^{\#\,\prime})$, as in (5) and (6).

Results

Figure 3 shows the fit obtained by local linear regression (continuous line), which apart from the deviation from linearity at larger excitations, looks similar to the fit by von Kries scaling (dotted line), here shown for long-wavelength-sensitive cones. Similar results were obtained for medium- and short-wavelength-sensitive cones. The adjusted R^2 was very high, over 99% with more than 42 of the 50 scenes for von Kries scaling, and more than 46 of the 50 scenes for local linear regression. But these R^2 values are relatively insensitive to local biases, and, on an expanded scale, marked differences between the fits obtained by von Kries scaling and local linear regression become evident. Figure 4 shows two expanded regions of the same graph. Notice that there is little offset near zero excitations (left panel) and that the direction of the bias reverses at large excitations (right panel).

This improvement is more than would have been expected by the simple increase in the degrees of freedom associated with the local model. A summary χ^2 measure of the difference between the local and von Kries scaling estimates over all 50 scenes and each of the three cone classes was calculated based on the p-values derived from the F-statistics. This yielded $\chi^2 > 335$ with d.f. = 50. For cone-opponent mechanisms, the corresponding difference between local and von Kries scaling estimates yielded $\chi^2 > 359$ with d.f. = 50. For both types of excitations, local linear regression was significantly better than von Kries scaling.

Discussion

As a procedure for predicting changes in cone excitations due to changes in reflected light, von Kries scaling is efficient: with just one degree of freedom for each cone class, it accounts for much of the variance in receptor excitations due to changes in illuminant. Nevertheless, as demonstrated here, it does show small but statistically significant failures that can be compensated for by local linear regression, in a way which preserves the principle of independence of cone excitations or cone-opponent excitations. Thus for applications where accurate predictions of excitations under wide ranges of illuminant changes are important, it may be useful to consider methods of fitting other than
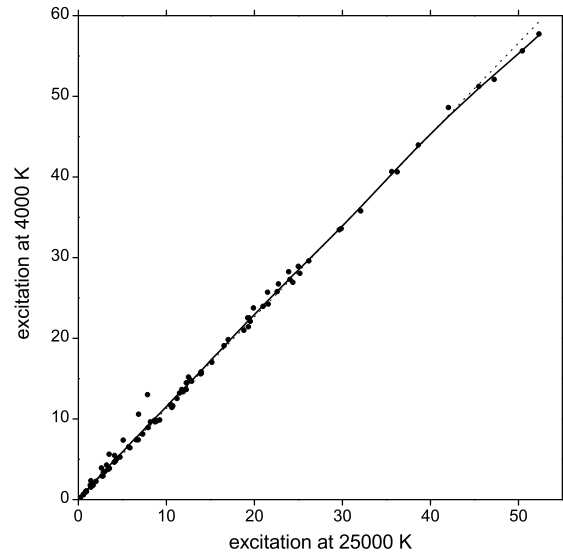

Figure 3. Modelling cone excitations by local fitting (solid line) and by von Kries scaling (dotted line). Data for long-wavelength-sensitive cones.
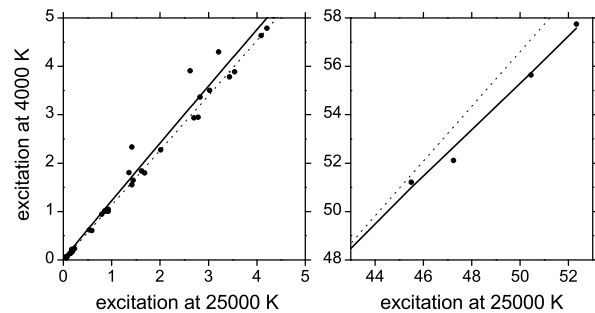

Figure 4. Expanded sections of Fig. 3.

linear transformations.

Acknowledgments

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