Part 1. Fibonacci Sequence (10 marks)

Write a program that will, for any positive integer \( N \) input by the user, output the Fibonacci sequence
\[
1, 1, 2, 3, 5, \ldots
\]
up to, but not exceeding, \( N \). (The sequence should be computed recursively; i.e. each term is the sum of the previous two terms.)

Include in your report a full program listing, together with the output for \( N = 1000 \).

Part 2. Power Series (15 marks)

Write a function that will return \( \sin^{-1} x \) by summing the following power series:
\[
\arcsin(x) = \sum_{n=0}^{\infty} \frac{(2n)!}{2^n (n!)^2} \cdot \frac{x^{2n+1}}{2n+1}, \quad |x| < 1
\]

Include also a test program that will invoke this function for a user-specified value of \( x \) and compare results with the standard Fortran intrinsic function \( \text{ASIN} \).

Include in your report a full program listing, together with the results of your power-series evaluations for the following values of \( x \):
\[
0.5 \\
-0.8 \\
0.999
\]

Part 3. Vectors and Matrices (15 marks)

Write separate functions or subroutines to calculate the scalar and vector products of two 3-dimensional vectors and a subroutine to calculate the determinant of a 3x3 matrix.

Write suitable test programs to input vectors or matrices and invoke your scalar-product, vector-product and determinant routines. (Input should be at run-time, not hard-wired in code). Do not use the standard intrinsic function \( \text{DOT} \_\text{PRODUCT} \) to find the scalar product!!

Include in your report full program listings (including test programs), together with the program’s output for the following:
- scalar product of \((1.5, 3.7, -1.5)\) and \((3.2, 5.0, -2.7)\)
- vector product of \((1.5, 3.7, -1.5)\) and \((3.2, 5.0, -2.7)\)
- determinant of
\[
\begin{bmatrix}
2 & 4 & -3 \\
1.5 & 3.5 & 1.7 \\
-3 & -4.3 & -6
\end{bmatrix}
\]