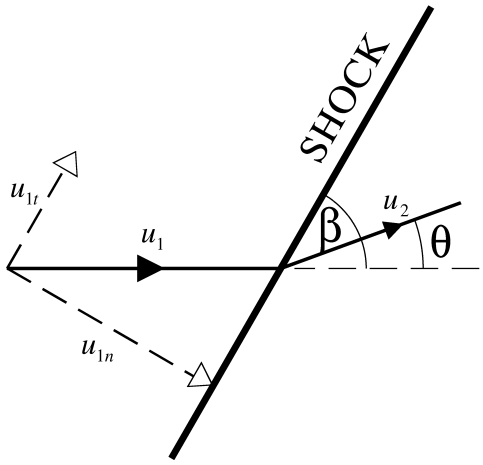
Oblique Shocks



The component of velocity *tangential* to the shock is unchanged.

The ratios of *normal* components of velocity and thermodynamic variables across the shock are the same as for normal shocks ... but with a Mach number based on the component of velocity normal to the shock, i.e. $Ma_1 \rightarrow Ma_1 \sin\beta$.

$$u_{2t} = u_{1t}$$

$$\frac{u_{2n}}{u_{1n}} = \frac{1+\frac{1}{2}(\gamma-1)Ma_1^2\sin^2\beta}{\frac{1}{2}(\gamma+1)Ma_1^2\sin^2\beta}$$

The angles between the shock and the upstream or downstream flow are given

$$\tan\beta = \frac{u_{1n}}{u_{1t}}, \qquad \tan(\beta-\theta) = \frac{u_{2n}}{u_{2t}}$$

The angle of deflection is given by:

$$\tan\theta = \frac{1}{\tan\beta}\left[\frac{Ma_1^2\sin^2\beta - 1}{1+\frac{1}{2}(\gamma+\cos 2\beta)Ma_1^2}\right]$$

For each Ma_1 there is a maximum deflection angle, θ_{max}. This may be found by differentiating with respect to β and setting the derivative equal to 0, from which one obtains a quadratic equation for cos 2β:

$$[\gamma Ma_1^4]\cos^2 2\beta - [(\gamma-1)Ma_1^4 + 4Ma_1^2]\cos 2\beta - [Ma_1^4 + 2(\gamma-1)Ma_1^2 + 4] = 0$$

For all $\theta < \theta_{max}$ there are 2 solutions for β: the smaller corresponding to a *weak* shock (which usually occurs in practice) and the latter to a *strong* shock.

For $\theta > \theta_{max}$ an attached shock is not possible and a curved, detached shock occurs.

For small θ the weak shock angle β tends to the Mach angle μ, where

$$\sin\mu = \frac{1}{Ma_1}$$

and the strong shock tends to the normal shock condition (β = 90°).

Nomenclature

- u = velocity
- Ma = Mach number;
- γ = ratio of specific heat capacities;
- β = angle between shock and upstream flow;
- θ = angle of deflection of the flow velocity through the shock
- μ = Mach angle (limit of an infinitesimally-weak shock).

Subscripts 1 and 2 denote conditions upstream and downstream of the shock, respectively. Subscripts *n* and *t* denote components normal and tangential to the shock.